A Theory of Liquidity Spillover Between Bond and CDS Markets

Batchimeg Sambalaibat

February 2015 (First version: May 2013)

This paper builds a search-theoretic model of bond and CDS markets that features endogenous funding liquidity and interdependent bond and CDS market liquidity. I show that, in the long run, speculative CDS trades attract liquidity into the CDS market than then, due to search frictions, spills over into the bond market and increases bond market liquidity. In the short run, however, speculative CDS buyers instead attract liquidity away from the bond market. In a separate empirical paper, I document how a series of European policies that banned speculative CDS purchases affected bond market liquidity. The implications from the long- and short-run effects help rationalize the observed changes in bond market liquidity.

A large body of work explores, in the context of exchange traded assets, why financial derivatives exist and how they affect the underlying assets. In the context of over-the-counter (OTC) traded assets, however, we have a limited understanding of the effect of derivatives even though the majority of asset classes are traded over-the-counter. In this paper, I study how derivatives affect liquidity and prices of the underlying assets when both the underlying

*Price College of Business, University of Oklahoma. E-mail: bsambala@ou.edu. I thank Laurence Ales, Patrick Augustin, Maria Chaderina, Brent Glover, Rick Green, Burton Hollifield, Lars Kuehn, Artem Neklyudov, Nicolas Petrosky-Nadeau, Guillaume Rocheteau, Bryan Routledge, Stefano Sacchetto, Chester Spatt, Chris Telmer, Pierre-Olivier Weill, Ariel Zetlin-Jones, discussants (Luca Benzoni, Peter Feldhutter, Pete Kyle, Haoxiang Zhu) and participants at the Southwest Search and Matching Meeting (UCLA), Carefin-Bocconi (Milan), 10th Central Bank Microstructure Workshop (Rome), FDIC/University of Maryland Annual Bank Research Conference, NYU Stern Microstructure Meeting, VU Gutmann Center Symposium, McGill University, Bank of International Settlements, Ohio State University, Penn State University, University of Lausanne, AEI, University of Calgary, Notre Dame University, University of Oklahoma. All errors are my own. The current version of the paper can be found at https://sites.google.com/site/sambalaibat/.

1OTC markets include fixed income (corporate bonds, government bonds, municipal bonds, the interbank market), commodity, currency, mortgage and other asset backed securities, structured products, interest rate rate swaps and other derivatives on these underlying.
and the derivative are traded over-the-counter. I explore this, in particular, in the context of sovereign bond and credit default swap (CDS) markets.

The controversy surrounding CDS during the debt crisis in Europe culminated in a series of policies that banned “naked” purchases of CDS, which is the practice of buying CDS protection without actually owning the underlying bonds. These policies serve as exogenous shocks to the CDS market and allow us to empirically identify the effect of naked CDS trading on the underlying bonds. In a separate empirical paper, I document that permanent versus temporary CDS bans had completely opposite effects on bond market liquidity. When the EU voted in October 2011 to permanently ban naked CDS referencing EU countries, countries affected by the ban experienced a decrease in their bond market liquidity. When Germany temporarily banned naked CDS in May 2010, this pattern reversed: bond market liquidity temporarily increased instead.

This paper builds a dynamic search-theoretic model of over-the-counter bond and CDS markets and rationalizes the opposite changes in bond market liquidity with the following theory. In the long-run, shorting the underlying through CDS contracts attracts supply of liquidity than then, due to search frictions, spills over into the bond market and increases bond market liquidity. In the short-run, however, the supply of liquidity is fixed and naked CDS buyers instead compete with and attract liquidity away from the bond market. Permanent and temporary CDS bans reverse the long- and short-run effects, respectively. Liquidity suppliers, forced to exit the CDS market permanently, pull out from the bond market also. But when the ban is only temporary, they temporarily substitute out of the CDS into the bond market.

In the model, I capture the over-the-counter structure of bond and CDS markets using the search and bilateral bargaining mechanism of Duffie, Garleanu, and Pedersen (2005, 2007). A fraction of bond owners are hit by a liquidity shock that requires them to sell their bonds. Locating a buyer, however, involves search costs. When a seller finds a buyer, she takes into account the difficulty of locating a buyer again and ends up selling her bond at a discounted price. Thus, as in the standard search framework, search costs create an illiquidity discount in bond prices.

I study how CDSs affect the illiquidity discount, bid-ask spreads, and bond volume by modeling two novel features. The first is the presence of CDS markets. CDSs are zero net supply derivative assets, while bonds are fixed supply assets, and trading CDS contracts also involves search costs. CDSs exist in the model because they complete markets. In particular, CDSs and bonds pay off in different states (default and non-default states). Investors cannot directly short bonds, so buying naked CDS allows short positions with respect to the underlying bonds that are otherwise not possible. This assumption captures a fundamental difference between bond and CDS mar-

---

2 A buyer of a CDS protection pays a periodic fee until either the contract matures or a default (or a similar event) occurs. In return, the protection seller transfers the purchased amount of insurance in the event of default. The contract specifies the reference entity, the contract maturity date, the insurance amount, and the events that constitute a credit event.
kets: it is cheaper to short credit risk using the CDS market than using the bond market.

The second novel feature is endogenous funding liquidity. In particular, entry decision of investors in the position of supplying liquidity by buying bonds or selling CDS adjusts endogenously to the introduction and the shutting down of the CDS market. The model thereby allows for distinct notions of funding liquidity and market liquidity and both are endogenous.

In this environment, the spillover effect works as follows. Allowing investors to short the underlying bonds by buying CDS attracts investors who want to take the other side of the trade and hold long positions. These are investors in the position of supplying liquidity into either market by buying bonds (from investors trying to liquidate) or selling CDS to speculators. Because bond and CDS markets have search frictions and buying bonds and selling CDS are close substitutes, for long investors there is an increasing returns to scale to searching simultaneously in both markets. The end result is that, by creating demand for liquidity, short investors attract liquidity into the CDS market that then spills over into the bond market.

I refer to the spillover effect as a long-run effect because the number of investors who prefer to hold long positions (and hence supply liquidity) responds to the demand. In the short-run, however, the number of long investors is fixed and CDS instead decreases bond market liquidity. The intuition is that a reallocation of investment resources takes time. In the short-run, investors are (if necessary) substituting only locally between bond and CDS markets. But, in the long-run, they are reallocating their investment funds at a wider scale between credit markets (encompassing both bond and CDS) and non-credit markets (e.g. currency or equity markets).

This paper contributes to the literature by providing, to the best of my knowledge, the first theoretical framework of over-the-counter trading in both the underlying and derivative markets. The model features endogenous funding liquidity and endogenous prices and liquidity that are interdependent across bond and CDS markets. Thus, beyond explaining the ban effects, the structural framework can be used to address questions that have been explored empirically so far and consequently with results limited by potential endogeneity problems. In an extension, I disentangle the determinants of the CDS-bond basis and show how exogenous changes in bond and CDS market liquidity and in funding liquidity affect the basis.

In existing theories of liquidity interaction between multiple markets, the aggregate number of traders is kept fixed and consequently introducing additional markets – by construction – results in a fragmentation and a migration of traders across multiple markets. My results instead show that allowing for an endogenous aggregate number of investors is particularly important when studying the effect of an additional market, a trading venue, or an instrument.

We see the opposite long- and short-run effects in different market contexts for derivative and underlying assets. The overwhelming empirical evidence is that derivatives generally increase liquidity of the underlying
Hew (2000) and references therein). But when the increase in the derivative activity is unexpected, the effect is instead a decrease in liquidity of the underlying (see, for example, Bessembinder and Seguin (1992)). My results show that these opposite effects are not necessarily at odds with one another and can be explained by the fact that capital is limited in the short-run and the reallocation of investment resources takes time.

The academic literature on CDS has so far focused on covered CDS, while the debate surrounding CDS has been about the speculative use of CDS (i.e. naked CDS purchases). This paper fills this gap in the literature and sheds light on the effect of naked CDS purchases which are at the heart of what makes CDS a derivative instrument. First, in contrast to bonds, CDS is a standardized instrument as it is written on the universe of all bonds of the bond issuer and not on individual bonds. As a result, CDS is a popular instrument to trade the overall credit risk of the issuer while avoiding bond specific risk. Second, shorting bonds is inherently limited by the supply of bonds and hence relatively costly. The analogous supply constraint does not exist for CDS. Put together, these two features make CDS a popular instrument to short the sovereign issuer in order to hedge long exposures correlated with the sovereign risk. This type of trade is a naked purchase of CDS. A further evidence suggesting that naked CDS positions are a large and an important part of the CDS market comes from the fact that trades in European sovereign single-name contracts have mostly dried up thanks to the naked CDS bans.

Existing notions of how CDS might affect the bond market cannot on their own explain why different CDS bans would have different effects. It is commonly believed that investors are more willing to buy and hold bonds if they can hedge them with CDS. This suggests that covered CDS positions would increase bond market liquidity. In an extension, I verify that this is the case. Trading the CDS-bond basis would have a similar effect on bond market liquidity. Both covered CDS positions and basis trades necessarily involve a long position (with respect to the underlying) in one market and a short position (again, with respect to the underlying) in the other market. In contrast, in my mechanism, there is an increase in liquidity of the bond market due to traders seeking a long position (with respect to the underlying) in both markets. As for naked CDS trading, a potential effect is that it increases liquidity of the CDS market itself and, consequently, indirectly.

---

3Between the EU’s introduction of the permanent ban in October 2011 and June 2013 (the end of my sample period), the total amount of CDS purchased declined by one third. Since then trades have dried up further according to ISDA (2014). In December 2014, for example, DeutscheBank which used to be one of the biggest dealers in the CDS market has entirely exited the single name CDS market.

4I show, in an extension, that covered CDS positions alone do not give rise to the opposite long- and short-run effects and hence cannot explain the ban effects.

In a basis trade, investors trade on an arbitrage opportunity that arises if bond and CDS markets price the underlying credit risk differently. For example, if the CDS premium is too low relative to bond yields, the CDS market is underestimating default risk relative to what the bond yields suggest. A basis trading strategy would be to buy the underpriced bonds and hedge their default risk with the currently cheap CDS.
increases bond market liquidity by making CDS a cheaper hedging tool. All of these effects cannot on their own explain the opposite effects.

The mechanism proposed in this paper critically relies on search frictions in the CDS market. Previous theories on CDS are unsatisfactory in that they rely on the assumption that CDS is a more liquid instrument than bonds. In the data, however, it is the other way around: for sovereign names, CDS bid-ask spreads are on average ten times larger than bond bid-ask spreads. In my model, CDS exists not because it is more liquid but because it is a cheaper way to short the underlying. In the absence of search frictions in the model, the existence of naked CDS buyers is redundant. Thus, OTC trading frictions in the CDS market are a key ingredient for the spillover effect and hence an important part of the explanation behind the observed changes in bond market liquidity after the bans.

Finally, policy implications of my results are that, first, permanently banning naked CDS trading adversely affected bond market liquidity, depressed bond prices, and thereby increased sovereign’s borrowing cost exactly when governments were trying to avert a liquidity dry-up and credit risk spiral. This result is particularly important in the context of a sovereign debt crisis. Second, my results show that temporary versus permanent and anticipated versus sudden regulations can have very different consequences.

The paper is organized as follows. Section 1 presents the model environment, while Section 2 derives the main theoretical results. Section 3 draws additional testable predictions of the model, Section 4 discusses possible extensions, and Section 5 gives institution details on bond and CDS markets and the bans. Section 6 concludes. All proofs are in the Appendix.

Related Literature

This paper belongs to the search literature of financial assets beginning with the seminal papers Duffie, Garleanu, and Pedersen (2005, 2007). My framework is closely related to the extensions of their environment to multiple assets by Vayanos and Wang (2007), Weill (2008) and, in particular, it is a variant of Vayanos and Weill (2008)’s framework that sheds light on the on-the-run phenomenon of Treasury bonds. I contribute to this literature, first, by modeling over-the-counter trading in derivatives in addition to trading in the underlying asset and, second, by endogenizing the entry decisions of agents into the market for the underlying asset in response to the introduction of the derivative market.

Related papers are Afonso (2011) and Lagos and Rocheteau (2009) who endogenize the entry decisions of traders and of market-makers, respectively, but in a single market setting. My model differs by featuring both multiple markets and endogenous entry and therefore sheds light on the rate of entry into one market as a result of introducing another market and on the mechanism through which traders migrate between different markets.

A search theoretic paper applied specifically to CDS markets is Atkeson, Eisfeldt, and Weill (2012) who in a static setting study how banks’ CDS
exposure arises endogenously depending on their size and their exposure to aggregate risk. In contrast, my paper focuses on naked CDS and studies in a dynamic setting the feedback from the CDS market into the bond market by allowing trade in both the bond and the CDS market as opposed to just the CDS market. Oehmke and Zawadowski (2013) explore how CDS affects bond prices in the type framework with exogenous trading frictions. In contrast, my model features endogenous trading costs.

A related literature is equilibrium asset pricing models with exogenous trading frictions (see, for example, Amihud and Mendelson (1986), Acharya and Pedersen (2005)). My model features endogenous bond market liquidity and thereby allows for an endogenous interaction and a spillover between the underlying and the derivative markets.


Motivated by the theoretical arbitrage relation between how credit risk is priced through bond prices versus through CDS spreads, a growing number of papers study the joint dynamics of bond and CDS spreads, or equivalently the CDS-bond basis, as well as the relative price discovery mechanism in bond and CDS markets. These papers’ findings suggest that on average the arbitrage relation holds. But when it does not and the price of credit risk in these two markets deviate, where the price discovery takes place (determined by which of the two prices leads the other) is state dependent. In particular, one of the important determinants is the relative liquidity in these markets. I add to this literature by providing a tractable theoretical framework with endogenous liquidity interaction between the two markets and, hence, precise implications on liquidity and prices in both markets.

My work is also related to the literature that studies how CDS affects the issuer of the debt security on which the CDS contracts are written. Empirical studies include Ashcraft and Santos (2009) and Subrahmanyam, Tang, and Wang (2011) who study the effect on firms’ cost of borrowing and credit risk, respectively. Also Das, Kalimipalli, and Nayak (2013) document that cor-
porate bond market liquidity did not improve with the inception of the CDS market, while Massa and Zhang (2012) and Shim and Zhu (2010) document that CDS markets increased corporate bond market liquidity. In contrast, my paper identifies the effect of naked CDS trading (as opposed to the CDS market in general) on bond market liquidity and focuses on sovereign bond and CDS markets.

On the theoretical front, Arping (2013) and Bolton and Oehmke (2011) formalize the tradeoffs associated with the empty creditor problem in the context of corporate debt and Sambalaibat (2012) in the context of sovereign debt. Duffee and Zhou (2001) find that credit derivatives alleviate the lemons problem associated with banks having private information on their loans. Thompson (2007) and Parlour and Winton (2009) study the tradeoffs that banks face in selling off versus insuring loans on their balance sheets. Thus, these papers have focused on issues surrounding covered CDS buyers who are directly exposed to the issuer’s default risk. This paper instead focuses on how naked CDS buyers affect the issuer’s cost of borrowing through their effect on bond market liquidity and bond prices.

This paper also contributes to the theoretical literature that studies the distribution of liquidity and trade across multiple markets. Examples that use information-based frameworks are Admati and Pfleiderer (1988), Pagano (1989), and Chowdhry and Nanda (1991), while search-theoretic ones are Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). A typical result in these papers is that traders endogenously concentrate in one market and trade in the other market disappears. Multiple markets can co-exist under additional assumptions of heterogeneous agents and heterogenous markets so that there is a “clientelet” effect. The focus of these papers has been the endogenous cross-sectional distribution of liquidity and trade across markets and assets. This endogeneity is, consequently, on the intensive margin (i.e. the number of traders can vary in the cross-section but the aggregate number of traders is fixed), and the results of these papers are effectively partial equilibrium effects. In my model, if the aggregate number of traders is kept fixed, then (similar to these papers) with the introduction of the CDS market, traders migrate from the bond market to the CDS market borrowing for safer firms but adverse effects for riskier firms as banks may lose the incentive to monitor firms. Subrahmanyan, Tang, and Wang (2011) find CDS increases firms’ credit risk which they attribute to protected creditors’ reluctance to restructure. Berndt and Gupta (2009) find that borrowers, whose loans have been sold off, underperform. Duffee and Zhou (2001) also show that credit derivatives adversely affect the parallel loan sales market.

I do not formally model the issuer’s borrowing cost in the primary debt markets. He and Milbradt (2012) provide a formal treatment of the feedback loop between credit risk, the issuer’s borrowing cost through the primary debt markets, and liquidity of the secondary bond markets.

For example, Pagano (1989) shows that if markets differ in their fixed entry cost, then an equilibrium with multiple markets exists and has the following feature: the more liquid market has a larger fixed cost of entry and is also the market where only large traders (those needing a larger portfolio adjustment) are attracted to. This is because the larger market has a bigger absorbing capacity (i.e. minimal price impact) and the fixed entry cost can be spread over a large transaction size.
and bond market liquidity decreases. However, my model also shows that if the aggregate number of traders is endogenous to the introduction of an additional security (i.e. the endogeneity is on the extensive margin), then the result is the opposite: the number of traders and liquidity in the market for the underlying asset increase.

More broadly, this paper belongs to the literature on the effect of derivatives such as options and futures on the market for the underlying assets. A majority of this literature is empirical. Theoretical frameworks that study the effect of derivatives on liquidity of the underlying asset market include Subrahmanyam (1991), Gorton and Pennacchi (1993), and John, Koticha, Subrahmanyam, and Narayanan (2003) and they also get the “migration” result as the above multiple market information-based models. I add to the literature by endogenizing entry. Also, these papers are based on Kyle (1985) and Glosten and Milgrom (1985) type frameworks where illiquidity arises from asymmetric information. The stylized OTC search framework of my paper is better suited for sovereign bond markets for two reasons. First, trade in sovereign bond markets is fragmented across heterogeneous bonds and, second, asymmetric information and insider trading are less severe with respect to governments than with respect to individual firms.

1 Model

Time is continuous and goes from zero to infinity. Agents are risk neutral, infinitely lived, and discount the future at a constant rate \( r > 0 \). A bond is an asset with supply \( S \), pays coupon flow \( \delta_b \), and trades at price \( p_b \). Investors can also trade a CDS contract in which a buyer of a CDS contract pays a premium flow \( p_c \) to the seller and, in return, benefits from an expected insurance payment of \( \delta_c \). The bond coupon flow can be interpreted as an expected coupon flow: with intensity \( \eta \) the bond defaults but otherwise pays a dollar of coupon. Hence, \( \delta_b = (1 - \eta) \$1 \). Similarly, \( \delta_c \) can be interpreted as an expected insurance payment: a CDS contract pays out a dollar if there is a default on the coupon payment, thus \( \delta_c = \eta \$1 \). As in standard search models, default and credit risk are not endogenous and the focus is instead on changes in asset prices through changes in asset liquidity.

Asset positions are denoted by $\theta_b$ for bond and $\theta_c$ for CDS. Let $\theta_{i \in \{b,c\}} = 1$ indicate a long position (exposed to risk), $\theta_i = 0$ no position, and $\theta_i = -1$ a short position (i.e. bought CDS). CDS allows both long and short positions ($\theta_c \in \{-1, 0, 1\}$): a seller and a buyer of a CDS contract have, respectively, long and short exposures to the underlying credit risk. I assume that bonds allow only long positions and that investors cannot directly short bonds; thus, $\theta_b \in \{0, 1\}$. I restrict the net position to $|\theta_b + \theta_c| \leq 1$; this rules out holding a long position in each market simultaneously. From now on, when I refer to a “long” or “short” position, I will mean with respect to the underlying credit risk. Thus, a long position through the CDS market, for example, does not mean an investor has bought CDS but instead means she has sold CDS and hence is (long) exposed to the underlying default risk.

Investors have heterogeneous valuations of asset cash flows. Specifically, holding $\theta_b$ units of the bond yields a utility flow $\theta_b (\delta_b + x_b^i) - |\theta_b|y$, while $\theta_c$ units of CDS yield $-\theta_c (\delta_c + x_c^i) - |\theta_c|y$. Here, $x_b^i \in \{-x_b < 0, 0, x_b\}$ and $x_c^i \in \{-x_{ch} < 0, 0, x_{ch} > 0\}$ are stochastic processes and $y$ is a cost of risk bearing that is positive for both long and short positions. I define an agent with $\{x_b^i = x_b, x_c^i = -x_{ch}\}$ as a high-valuation agent, $\{x_b^i = 0, x_c^i = 0\}$ as an average-valuation agent, and $\{x_b^i = -x_b, x_c^i = x_{ch}\}$ as a low-valuation agent. This specification is shown in Table 1.

The parameters $x_b, x_{ch},$ and $x_{cd}$ capture, in a reduced form, any reason investors may have to want to trade bonds and CDS. One interpretation is that they are hedging benefits. In particular, investors have an idiosyncratic endowment with heterogeneous correlations with the underlying bond cash flow. The endowment of a low-valuation agent is more correlated with the bond compared to the endowment of a high-valuation investor. Relative to a high-valuation agent then, a low-valuation agent derives less utility from a trade that makes her wealth even more correlated with the bond. Such trades are buying a bond ($\theta_b = 1$) or selling CDS ($\theta_c = 1$). However, a trade that makes an investor’s position less correlated with the bond (such as buying CDS, $\theta_c = -1$) yields a greater utility to a low-valuation investor than to a high-valuation investor. Appendix C provides a micro-foundation and closed form expressions for the hedging benefits $x_b, x_{ch}, x_{cd},$ and $y$ in an environment with risk averse agents and risky assets. It shows that the hedging benefits increase with agents’ risk aversion, the bond cash flow risk, and the correlation of agents’ endowment with the bond cash flow.

I keep $x_b, x_{ch},$ and $x_{cd}$ general but they do not have to be different for the results in the following sections to hold. The difference in $x_{ch}$ and $x_{cd}$ is determined by the difference in the correlation of high and low investor’s

---

13 For a similar setup, see Duffie, Garleanu, and Pedersen (2005) with two types of agents and Vayanos and Well (2008) with three types of agents.

14 In particular, a low-valuation agent gets an extra disutility ($x_b$) from holding a bond ($\theta_b = 1$), while a high-valuation gets an extra utility ($x_b$). As for selling CDS ($\theta_c = 1$), a low-valuation agent experiences a greater dis-utility paying out the insurance payment ($- (\delta_c + x_{cd}) - y$) than a high-valuation agent ($- (\delta_c - x_{ch}) - y$). In terms of buying CDS ($\theta_c = -1$), it benefits a low-valuation agent more ($((\delta_c + x_{cd}) - y)$ than a high-valuation agent ($((\delta_c - x_{ch}) - y)$).
endowment with the bond cash flow. If the endowment of the low-valuation investor is more correlated (in absolute magnitudes) with the underlying credit risk, then $x_{cl} > x_{ch}$. As for $x_b$ versus $x_{ch}$ and $x_{cl}$, suppose $x_{ch} = x_{cl} = x_c$. If the bond cash flow risk maps to a greater level of risk in the CDS cash flow, then $x_c > x_b$; otherwise, $x_c \leq x_b$.

Table 1: Valuation of bond and CDS payments by high-, average-, and low-valuation agents.

Agents are heterogeneous in their valuation of bond and CDS cash flows. As shown in the “Bond Owner” column, high-valuation agents derive a higher utility from a long exposure to the bond, while low-valuation agents derive a disutility from a long exposure to the bond. Conversely, low-valuation agents derive a higher utility from a short position (as shown in the “CDS Buyer” column), while high-valuation agents derive a disutility from a short position. As a result, in equilibrium high-valuation agents search for long positions, while low-valuation agents short credit risk. average-valuation agents are in between.

<table>
<thead>
<tr>
<th>Types</th>
<th>Bond Owner ($\theta_b = 1$)</th>
<th>CDS Buyer ($\theta_c = -1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$\delta_b + x_b - y$</td>
<td>$\delta_c - x_{ch} - y$</td>
</tr>
<tr>
<td>Average</td>
<td>$\delta_b - y$</td>
<td>$\delta_c - y$</td>
</tr>
<tr>
<td>Low</td>
<td>$\delta_b - x_b - y$</td>
<td>$\delta_c + x_{cl} - y$</td>
</tr>
</tbody>
</table>

**Assumption 1.** $x_{ch} + x_{cl} > 2y > x_{ch}$

Assumption 1 ensures that a CDS trade is profitable between high- and low-valuation investors but not between high- and average-valuation investors. To see this, if a low-valuation investor buys CDS from a high-valuation investor, the buyer’s flow surplus from the transaction is $(\delta_c + x_{cl}) - y - p_c$, while the seller’s is $p_c - (\delta_c - x_{ch}) - y$. The total surplus is then $x_{ch} + x_{cl} - 2y$ which is positive from Assumption 1. If, instead, an average-valuation agent buys CDS from a high-valuation investor, the total surplus is $x_{ch} - 2y$, which is negative from Assumption 1. Thus, the cost of risk bearing parameter, $y$, restricts the set of possible trades between different agents and arises in a setting with more than two types of agents. For a CDS trade to be profitable, the difference in the valuations of the involved parties has to be far enough to offset the cost of risk bearing. The derivation providing a micro-foundation for the hedging benefits (Appendix C) gives a closed form expression for $y$ and shows that it is an increasing function of the risk aversion parameter and the bond cash flow risk.

There is an infinite mass of average-valuation agents. Fixed flows of average-valuation agents $F_h$ and $F_l$ respectively switch to high- and low-valuation investors. In turn, high- and low-valuation investors get a liquidity shock and switch back to an average-valuation with Poisson intensities $\gamma_d$ and $\gamma_u$, respectively. What generates trade in equilibrium is these constant flows and liquidity shocks.

I endogenize the entry rate of high-valuation investors as follows. A high-valuation agent enters to trade in bond and CDS markets if the expected value of doing so (denoted by $V_{h(0,0)}$) is at least greater than the value of her
outside option, denoted by $O_h$. It captures the opportunity cost of entering credit markets including the cost of raising capital. A fraction $\rho$ of investors choose to enter according to:

$$\rho = \begin{cases} 
1 & V_{h[0,0]}(\rho) > O_h \\
[0, 1] & V_{h[0,0]}(\rho) = O_h \\
0 & V_{h[0,0]}(\rho) < O_h. 
\end{cases} \quad (1)$$

Thus, the total flow of high-valuation investors actually entering is $\rho F_h$ and the steady state measures of high and low-valuation investors are $\frac{\rho F_h}{\gamma_d}$ and $\frac{F_l}{\gamma_d}$, respectively. I assume that parameter conditions are such that, in the steady state, high-valuation investors are the marginal investors in the bond: $\frac{\rho F_h}{\gamma_d} \geq S + \frac{F_l}{\gamma_d}$.

The specification of hedging benefits and assumptions ensure that, in the steady state equilibrium, high- and low-valuation investors prefer to hold long and short positions, respectively, while average-valuation investors prefer to stay out of the markets.

Since high-valuation investors are in the position of supplying liquidity into bond and CDS markets (by buying bonds from investors looking to liquidate their bond position and selling CDS to short sellers), endogenous entry translates to an endogenous aggregate number of long investors and, consequently, an endogenous aggregate supply of liquidity. I also refer to the supply of liquidity as funding liquidity (see Rocheteau and Weill (2011) for a similar interpretation). In later sections, I characterize separately market liquidity in bond and CDS markets (captured by bid-ask spreads, trading volume and illiquidity discount and premia).

### 1.1 The Bond and the CDS Market

Buyers and sellers in the bond market meet at a rate $M_b \equiv \lambda_b \mu_{bb} \mu_{bs}$, where $\lambda_b$ is the exogenous matching efficiency of the bond market, and $\mu_{bb}$ and $\mu_{bs}$ are the measures of bond buyers and sellers, respectively.\footnote{Fonsø (2011) provides a more general setup in which there is a continuous distribution of agents with different outside values. My setup is a special case of this.} Given the total meeting rate, buyers find a seller with intensity $\lambda_b \mu_{bb}$, and sellers find a buyer with intensity $\lambda_b \mu_{bs}$. Once matched, a buyer and a seller bargain and split the surplus proportional to their respective bargaining powers: $\phi$ and $1 - \phi$.

Analogously, in the CDS market, CDS buyers find a seller with intensity $\lambda_c \mu_{cb}$ and sellers find a buyer with intensity $\lambda_c \mu_{cs}$ where $\mu_{cb}$ and $\mu_{cs}$ are the measures of CDS buyers and sellers, respectively, and the total meeting rate is $M_c \equiv \lambda_c \mu_{cb} \mu_{cs}$.

\footnote{A general functional form for the matching functions is $M_b(\mu_{bb}, \mu_{bs}) = \lambda_b \mu_{bb}^{\alpha_{bb}} \mu_{bs}^{\alpha_{bs}}$ and $M_c(\mu_{cb}, \mu_{cs}) = \lambda_c \mu_{cb}^{\alpha_{cb}} \mu_{cs}^{\alpha_{cs}}$, thus I have implicitly assumed $\alpha_{bb} = \alpha_{bb} = \alpha_{cb} = \alpha_{cs} = 1$. Although constant returns to scale is standard in search models applied to labor markets, in the context of financial markets, the standard assumption is increasing returns to scale. Weill (2008) shows that comparative statics from a model with increasing returns to scale fit better the stylized facts regarding, for example, liquidity and the supply of the asset.}
1.2 Agent Types and Transitions

An agent of type $\tau = i[θ_b, θ_c]$ is composed of his valuation type $i \in \{h, a, l\}$ (high “$h$”, average “$a$”, low “$l$”) and his asset position $[θ_b, θ_c]$. His asset position can be either a non-owner: $[θ_b, θ_c] = [0, 0]$, a bond owner: $[θ_b, θ_c] = [1, 0]$, a CDS seller: $[θ_b, θ_c] = [0, 1]$, or a naked CDS buyer: $[θ_b, θ_c] = [0, -1]$. For now, to focus on the marginal effect of naked CDS positions, I rule out covered CDS position $[θ_b, θ_c] = [1, -1]$ but Section 3.2 will relax this assumption. As shown in Table 2, the conjectured set of agent types is $T ≡ \{h[0, 0], l[0, 0], h[1, 0], a[1, 0], h[0, 1], a[0, 1], l[0, -1]\}$.

### Table 2: Agent Types

<table>
<thead>
<tr>
<th>[θ_b, θ_c]</th>
<th>[0, 0]</th>
<th>[1, 0]</th>
<th>[0, 1]</th>
<th>[0, -1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$µ_{h[0,0]}$</td>
<td>$µ_{h[1,0]}$</td>
<td>$µ_{h[0,1]}$</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>$∞$</td>
<td>$µ_{a[1,0]}$</td>
<td>$µ_{a[0,1]}$</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>$µ_{l[0,0]}$</td>
<td></td>
<td></td>
<td>$µ_{l[0,-1]}$</td>
</tr>
</tbody>
</table>

I describe next the conjectured optimal trading strategies specific to each agent type ($τ$) and other possible transitions between types (see Figure 1).

A high-valuation non-owner ($h[0, 0]$) seeks a long exposure to credit risk by buying a bond or selling CDS. I assume he can simultaneously search in both markets. Before he is even able to find a counterparty, he may switch to an average-valuation agent and exit the economy. But if he finds and trades with a bond-seller first, he becomes a high-valuation bond owner, $h[1, 0]$. He is happy to keep this position until he is hit by a liquidity shock, in which case he becomes an average-valuation agent and will seek to liquidate his bond position (that is, become a bond seller ($a[1, 0]$)). Upon finding a bond buyer, he exits the market.

If a high-valuation non-owner ($h[0, 0]$) instead bumps into a CDS buyer first and sells CDS (which occurs with intensity $λ_cµ_{cb}$), he becomes a $h[0, 1]$ type who is long-exposed to credit risk. He is happy with this position unless he switches to an average-valuation agent and becomes one of $a[0, 1]$. As an average-valuation agent, he will seek to unwind his position by searching for another CDS seller to take over his side of the trade at the original price.\footnote{Thus, the average-valuation investor is effectively intermediating the CDS trade between the high- and low-valuation investors. Whether the high- and average-valuation investors bargain a new price or transact at the original price (that the average-valuation is currently getting paid by the CDS buyer) is immaterial and is only a simplification.}

In practice, this is called “assignment” or “novation.”
Since a low-valuation non-owner ($l[0,0]$) wants to short credit risk, she searches to buy CDS, finds a counterparty with intensity $\lambda_c \mu_{cs}$ and consequently becomes a CDS holder, $l[0,-1]$. If she switches to an average-valuation agent, she terminates her contract. The termination forces her counterparty to search all over again (i.e. become $h[0,0]$ again) if the counterparty was a high-valuation investor. But if the counterparty was an average-valuation investor, it allows the counterparty to simply exit the economy. Thus, CDS contracts are asymmetric in that a CDS buyer can terminate but a seller cannot default and instead have to find an investor to take over his side. I do not need this asymmetry for my results but impose it to be realistic and rule out counterparty risk in CDS contracts.  

Figure 1: Transitions Between Agent Types  
The figure shows the transitions between agent types. Flow of $\rho F_h$ and $F_l$ agents enter the economy as high- and low-valuation investors. High- and low-valuation agents switch to an average-valuation with intensities $\gamma_d$ and $\gamma_u$, respectively. A trader seeking a long position ($h[0,0]$) finds a counterparty in the bond and the CDS market with probabilities $\lambda_b \mu_{bs}$ and $\lambda_c \mu_{cb}$, respectively. A bond seller, $a[1,0]$, finds a buyer with probability $\lambda_b \mu_{bb}$. A trader seeking to establish a short position, $l[0,0]$, by buying CDS finds a counterparty with probability $\lambda_c \mu_{cs}$.

Given the conjectured equilibrium trading strategies, the measure of buyers and sellers in bond and CDS markets are: $\mu_{bb} = \mu_{h[0,0]}$, $\mu_{bs} = \mu_{a[1,0]}$.

$^{18}$Alternatively, a buyer and a seller could be each required to post collateral that then is seized upon default, as in actual CDS contracts. And typically, the collateral required from the seller is larger to prevent counterparty risk. For simplification, my specification is an extreme case of this where the collateral that the buyer is required to post is zero while the collateral required from the seller is infinity. It is not important for the main results whether CDS contracts are symmetric (both can terminate or that both do not terminate) or asymmetric.
\( \mu_{cs} = \mu_{h[0,0]}, \mu_{ch} = \mu_{[0,0]} + \mu_{a[0,1]} \). In the steady state, the measures of types are constant and the in-flow of agents has to equate the out-flow for each type as shown in Table 3.

Table 3: Flow-ins and outs
In the steady state equilibrium, the measure of agent types is constant: a flow of agents turning into a particular type (Flow-in) has to equal the flow of agents switching out of that type (Flow-out).

<table>
<thead>
<tr>
<th>Type</th>
<th>Flow-in = Flow-out:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{h[0,0]} )</td>
<td>( \rho F_h + \gamma_{a \mu_{h[0,1]}} = \gamma_{d \mu_{h[0,0]}} + (\lambda_{b \mu_{bs}} + \lambda_{c \mu_{cb}}) \mu_{h[0,0]} )</td>
</tr>
<tr>
<td>( \mu_{l[0,0]} )</td>
<td>( \bar{F}<em>l = \gamma</em>{a \mu_{l[0,0]}} + \lambda_{c \mu_{cs}} \mu_{l[0,0]} )</td>
</tr>
<tr>
<td>( \mu_{h[1,0]} )</td>
<td>( \lambda_{b \mu_{bs}} \mu_{h[0,0]} = \gamma_{d \mu_{h[1,0]}} )</td>
</tr>
<tr>
<td>( \mu_{a[1,0]} )</td>
<td>( \gamma_{d \mu_{h[1,0]}} = \lambda_{b \mu_{bs}} \mu_{a[1,0]} )</td>
</tr>
<tr>
<td>( \mu_{h[0,1]} )</td>
<td>( \lambda_{c \mu_{cb}} \mu_{h[0,0]} = \gamma_{d \mu_{h[0,1]}} + \gamma_{a \mu_{h[0,1]}} )</td>
</tr>
<tr>
<td>( \mu_{a[0,1]} )</td>
<td>( \gamma_{d \mu_{h[0,1]}} = \gamma_{a \mu_{a[1,1]}} + \lambda_{c \mu_{cs}} \mu_{a[1,1]} )</td>
</tr>
<tr>
<td>( \mu_{l[0,-1]} )</td>
<td>( \lambda_{c \mu_{cs}} \mu_{l[0,0]} = \gamma_{a \mu_{l[0,-1]}} )</td>
</tr>
</tbody>
</table>

Bond market clearing imposes that the total measure of bond owners has to equal the bond supply:

\[
\mu_{h[1,0]} + \mu_{a[1,0]} = S. \tag{2}
\]

CDS market clearing requires that the total number of CDS contracts sold has to equal the number of CDS contracts purchased:

\[
\mu_{h[0,1]} + \mu_{a[0,1]} = \mu_{l[0,-1]} . \tag{3}
\]

1.3 Prices and Bargaining
Prices of bonds and CDS arise from bilateral bargaining between buyers and sellers. Let \( V_{\tau} \) denote the expected utility of type \( \tau \in T \). A bond buyer’s marginal benefit of buying a bond is the increase in his expected utility \( V_{h[1,0]} - V_{h[0,0]} \) and his marginal cost is the bond price \( p_b \). Thus, he is willing to buy as long as the marginal benefit is greater than the marginal cost: \( V_{h[1,0]} - V_{h[0,0]} \geq p_b \), and the smaller the price is, the larger is his surplus. Analogously, for a seller, the marginal benefit of selling her bond is the bond price, \( p_b \), and in return she is giving up the value of being a bond owner, \( V_{a[1,0]} \), which is the marginal cost. Hence, she will sell as long as \( p_b \geq V_{a[1,0]} \). Thus, the bond price has to lie in the interval: \( V_{a[1,0]} \leq p_b \leq V_{h[1,0]} - V_{h[0,0]} \) and the length of this interval is the total surplus from trade. The buyer and the seller split the surplus proportional to their respective bargaining powers: \( \phi \) and \( 1 - \phi \). The greater the bargaining power of the buyer (i.e. higher \( \phi \)), the lower the bond price:

\[
p_b = \phi V_{a[1,0]} + (1 - \phi) (V_{h[1,0]} - V_{h[0,0]}). \tag{4}
\]
Analogously, a CDS seller and a CDS buyer Nash-bargain over the price such that the seller and the buyer get $\phi$ and $1 - \phi$ fractions of the total surplus, respectively. The buyer’s surplus is $V_{[0,1]} - V_{[0,0]}$ and the seller’s is $V_{h[0,1]} - V_{h[0,0]}$. Thus, the CDS price is implicitly defined by:

$$V_{h[0,1]} - V_{h[0,0]} = \phi \left( V_{l[0,1]} - V_{l[0,0]} + V_{h[0,1]} - V_{h[0,0]} \right).$$

(5)

A CDS seller who switches to an average-valuation, $a[0,1]$, will search for another CDS seller to take over his side of the trade (at the original price) and exit with zero utility if $0 - V_{a[0,1]} > 0$.

### 1.4 Value Functions

To characterize investors’ expected utilities, consider, for example, an $h[0,0]$ type. In a small time interval $[t + dt]$, he could (a) switch to an average-valuation (with probability $\gamma_d dt$ and get utility 0), (b) become a bond owner (with probability $\lambda_b \mu_b dt$ and get $V_{h[1,0]} - p_b$), (c) become a CDS seller (with probability $\lambda_c \mu_c dt$ and get utility $V_{h[0,1]}$), or (d) remain an $h[0,0]$ type:

$$V_{h[0,0]} = (1 - rd) \left( \gamma_d dt(0) + \lambda_b \mu_b dt(V_{h[1,0]} - p_b) + \lambda_c \mu_c dt V_{h[0,1]} \right) + (1 - \gamma_d dt - \lambda_b \mu_b dt - \lambda_c \mu_c dt) V_{h[0,0]}.$$

After simplifying and taking the continuous time limit, we get:

$$rV_{h[0,0]} = \gamma_d (0 - V_{h[0,0]}) + \lambda_b \mu_b (V_{h[1,0]} - p_b - V_{h[0,0]}) + \lambda_c \mu_c (V_{h[0,1]} - V_{h[0,0]}).$$

(6)

The flow value equations for the other types are analogously derived and are shown in Appendix A.

### 1.5 Equilibrium

**Definition 1.** A steady state equilibrium is given by types’ measures $\{\mu_\tau\}_{\tau \in T}$, prices $\{p_b, p_c\}$, entry decisions $\{\rho\}$, and value functions $\{V_\tau\}_{\tau \in T}$ such that:

1. $\{\mu_\tau\}_{\tau \in T}$ solve the steady state in-flow and out-flow equations in Table 3.

2. Market clearing conditions (2) and (3) hold.

3. Entry decisions, $\{\rho\}$, solve (1).

4. Bond and CDS prices, $\{p_b, p_c\}$, solve (4) and (5).

5. Agents’ value functions, $\{V_\tau\}_{\tau \in T}$, solve agents’ optimization problem given by (6), and (A.17)–(A.22).

In a dynamic search model with multiple assets and more than two types of agents, we can solve for the equilibrium only numerically and equilibrium
existence cannot be established for general parameter values (see Vayanos and Weill (2008) and Weill (2008)). However, Vayanos and Weill (2008) and Weill (2008) show that the existence of an equilibrium can be established when search frictions are small. Following their methodology, I show in the next proposition that a unique steady state equilibrium exists when search frictions are small (that is, $\lambda_b$ and $\lambda_c$ are large).

**Proposition 1.** Suppose

$$x_b - \frac{x_{ch} + (x_{cl} - 2y) \left( \frac{\lambda_b \mu_{cb} + r + \gamma_u + \gamma_d \lambda_c \mu_{cs}}{\lambda_c \mu_{cb} \phi_l + \lambda_c \mu_{cb} \phi_h} \right)}{r + \gamma_d + \gamma_u + \lambda_c \mu_{cb} \phi_l + \lambda_c \mu_{cb} \phi_h} > 0. \quad (7)$$

Then, for large $\lambda_b$ and $\lambda_c$, there exists a unique equilibrium.

The proof is given in Appendix A. The proof of uniqueness involves the following steps. Given $\rho$, Appendix A shows that the set of equations that characterizes the dynamics of the population measures together with the market clearing conditions has a unique solution. Given this solution to the population measures, a linear system of equations characterizing the agents’ value functions and prices uniquely determines $\{V_\tau\}_{\tau \in T}$. Thus, for any $\rho \in [0, 1]$, $V_{h[0,0]}$ is uniquely determined. The agent’s entry decision can be either an interior solution or one of two corner solutions ($\rho = 0$, $\rho = 1$). To show that the agents’ entry decision has a unique solution, the Appendix shows that if (7) holds, $V_{h[0,0]}$ is a strictly decreasing function of $\rho$.

To show the existence of an equilibrium, I verify that all the conjectured optimal trading strategies are indeed optimal. In particular, I first show that the total surplus from trading the bond is positive: $\omega_b = V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]} > 0$. By construction, this will ensure that a high-valuation agent will optimally choose to buy a bond, while an average-valuation agent will not want to be a bondholder and, if she had previously purchased a bond, she will prefer to sell it. Second, Appendix A shows that the total surplus from trading CDS is positive: $\omega_c = V_{h[0,1]} - V_{h[0,0]} + V_{l[0,1]} - V_{l[0,0]} > 0$. This ensures that high-valuation agents will want to sell CDS, while low-valuation agents will want to buy CDS. Third, I verify that the average-valuation agents will prefer to stay out of the markets completely instead of being a CDS buyer or a CDS seller. Proving that $0 - V_{a[0,1]} > 0$ ensures that an agent who had previously been a high-valuation agent and had sold CDS will prefer to find another seller to take over her side of the trade (at the original CDS price) and exit with zero utility.

The guess and verify approach significantly simplifies derivations because it allows us to solve the set of population measures and the set of value functions separately. The difficulty created with endogenizing entry is that we can no longer solve them separately. For example, showing that $V_{h[0,0]}$ is a strictly decreasing function of $\rho$ is not a trivial step.
2 Theoretical Results

I start by characterizing bond market liquidity variables: the illiquidity discount, the bid-ask spread, and trading volume. Propositions 2 and 3 derive bond prices without search frictions versus with. The difference between the two is the illiquidity discount.

**Proposition 2.** If the bond market is frictionless \((\lambda_b \to \infty)\), the bond price is given by

\[
p_b = \frac{\delta_b + x_b - y}{r} \tag{8}
\]

and the CDS market does not affect the bond market.

Proposition 2 shows that, without search frictions in the bond market, the bond price is given by the present value of high-valuation agents’ valuation of the bond. A bond owner – upon getting a liquidity shock – can sell instantly to another high-valuation trader. As a result, bonds are always in the hands of high-valuation agents and never held by agents who have a lower valuation. From (4) the bond price is the weighted average of the marginal valuations of different types of bond owners. Since high-valuation investors are the only bond holders, the bond price is given by their valuation only. In this frictionless environment, the CDS market does not affect the bond market.

**Proposition 3.** The bond price is given by:

\[
p_b = \frac{\delta_b + x_b - y}{r} - \left[ \frac{\gamma_d x_b}{r k} + \phi \left( \lambda_b \mu_{bs} + r \right) \frac{x_c}{r k} \right] \left( \frac{\lambda_b \mu_{bb} + r}{r k} \right) \lambda_c \mu_{cb} \Delta_b[0,1],
\]

where

\[
\Delta_b[0,1] \equiv \frac{\phi(-\phi \lambda_b \mu_{bs} x_b + k x_c)}{[r + \gamma_d + \gamma_u + \lambda_c \mu_{cs}(1 - \phi) + \phi \lambda_c \mu_{cb}] k - \phi \lambda_c \mu_{cb} \lambda_b \mu_{bs} \phi}, \tag{10}
\]

\[
k \equiv r + \gamma_d + \lambda_b \mu_{bs} \phi + \lambda_b \mu_{bb}(1 - \phi).
\]

Proposition 3 shows that with search frictions in the bond market the bond price is lower than the frictionless price in (8). After receiving a liquidity shock, a bond owner is stuck for some time with a bond that she does not value before she can find a counterparty. When she does find a buyer, she takes into account the difficulty of locating a buyer again and ends up selling at a discounted price. Similarly, a potential bond buyer is only willing to buy at a low price because he anticipates this trading friction when it is his turn to liquidate his bond position.

Thus, search costs create an illiquidity discount in the bond price given by the difference between (9) and the frictionless price (8): the sum of the second and the third terms in (9). In particular, the third term is the additional discount in the bond price due to bond buyers having an outside option of providing liquidity in the CDS market (by selling CDS).
Definition 2. The illiquidity discount, $d_b$, in the bond price is defined by the difference between the frictionless bond price (8) and the bond price with search frictions present in the bond market (9):

$$d_b \equiv \gamma_b \Delta h_{[0,1]} + \phi \left( \lambda_b \mu_{h_b} + r \right) \frac{x_b}{rK} + \left( \lambda_b \mu_{bb} + r \right) \frac{\lambda_c \mu_{cb}}{K} \Delta h_{[0,1]}.$$

Two additional bond market liquidity variables are the bid-ask spread and trading volume. I define the bond bid-ask spread, denoted by $\omega_b$, in a standard way as the total trading surplus between a bond buyer and a seller:

$$\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} + V_{a[1,0]}.$$

It gets at the idea that in an environment with dealers, dealers would capture the entire surplus so that buyers buy at an ask-price equal to their marginal valuation of the bond ($p_{ask} = V_{h[1,0]} - V_{h[0,0]}$) and sellers sell at a bid-price equal to their marginal valuation of the bond ($p_{bid} = V_{a[1,0]}$). Bond volume, denoted by $M_b$, is simply the total number of transactions,

$$M_b \equiv \lambda_b \mu_{h[0,0]} \mu_{a[1,0]}.$$

2.1 The Effect of CDS on Bond Market Liquidity

The next proposition gives the main theoretical result of the paper. It shows that shorting bonds by buying (naked) CDS increases bond market liquidity.

Proposition 4 (The Spillover Effect). In the equilibrium of Proposition 1, the bond bid-ask spread ($\omega_b$) is narrower and the bond trading volume ($M_b$) is larger than the environment without the CDS market. There exists $\bar{\lambda}_c > 0$ such that for all $\lambda_c > \bar{\lambda}_c$, the illiquidity discount ($d_b$) is smaller.

The proof is given in Appendix A and the intuition is as follows. Allowing investors to short the underlying bonds by buying (naked) CDS attracts investors who want to take the other side of the trade and hold long positions. These are investors in the position of supplying liquidity into either market by buying bonds (from investors looking to liquidate) or selling CDS to short sellers.

Because buying bonds and selling CDS are close substitutes for long investors and both involve search costs, there is an increasing returns to scale to searching simultaneously in both markets. Intuitively, resources expended on pricing individual bonds help an investor price CDS relatively quickly and vice versa. Thus, they search and trade in both markets and, as a result, the equilibrium entry rate will be given by an interior solution.

The threshold $\bar{\lambda}_c$ is such that, for $\lambda_c < \bar{\lambda}_c$, the equilibrium entry rate $\rho \in [0,1]$ is given by the corner solution of $\rho = 1$. If search frictions in the CDS market worsen (i.e. $\lambda_c$ becomes even smaller), a greater fraction of high-valuation agents would want to enter and provide liquidity in the CDS market. But at most $\rho = 1$ fraction of high-valuation agents is able to enter, and being at the corner solution is analogous to the entry rate being fixed. A numerical calibration shows that for reasonable parameter ranges of $\lambda_c$ the equilibrium entry rate will be given by an interior solution.

Mechanically, high-valuation investors now have more profitable trading opportunities (in addition to buying bonds, they can now also sell CDS). The value of trading in credit markets as a whole increases. They enter at a greater rate until the marginal entrant is again indifferent. The result is an increase in the equilibrium entry rate and consequently the aggregate number of high-valuation investors.
increase in the aggregate number of long investors translates to an increase in the number of bond buyers. Bond market liquidity consequently increases. This is the spillover effect. Naked CDS buyers by creating demand for liquidity attract liquidity into the CDS market that then spills over into the bond market.

Figure 2 illustrates the result on variables measuring bond market liquidity and Figure 3 shows the effect on the relative number of buyers and sellers.

Model Implication on a Permanent CDS Ban

A permanent CDS ban reverses the spillover effect. Liquidity suppliers are forced to exit the CDS market because their counterparties are banned from buying CDS. By exiting the CDS market, they pull out from the bond market also. As a result, bond market liquidity and bond prices decrease. This model prediction rationalizes the observed decrease in bond market liquidity after the permanent CDS ban.

As discussed in Section 5, the actual bans prevented both speculative trades and trades where CDS would have been used as a macro hedge on long positions correlated with the sovereign. In the model, consistent with the actual bans, both would be considered naked CDS purchases because the CDS buyer does not hold the underlying bonds.

The Importance of CDS Search Frictions

The first key ingredient for the spillover effect and hence in rationalizing the empirical patterns is search frictions in the CDS market. CDS has two opposing effects on bond market liquidity and which effect dominates depends on the extent of search frictions in the CDS market ($\lambda_c$). On the one hand, bond buyers are in a better bargaining position because they now have an outside option of providing liquidity in the CDS market (by selling CDS). This puts a downward pressure on the bond price and on bond market liquidity.\(^{22}\) On the other hand, due to the increase in the number of high-valuation investors, bond sellers have a greater number of potential counterparties and, consequently, are also in a better bargaining position. The marginal effect is an increase in the bond price and bond market liquidity.

If the CDS market is frictionless ($\lambda_c \to \infty$), these two opposite effects exactly offset one another. New high-valuation entrants sell CDS immediately upon entry and hence do not end up simultaneously searching in the bond market.\(^{23}\) The aggregate number of high-valuation investors increases but...
the increase does not translate to an increase in the number of bond buyers. Thus, in the absence of search frictions, CDS is redundant and has no effect on bond market liquidity. Proposition 5 formalizes this result.

**Proposition 5.** \( \lim_{\lambda_c \to \infty} d_b(\lambda_c) = d_b^{\text{nocds}}. \)

### The Importance of Endogenous Entry

The second key ingredient for the spillover effect is endogenous entry. If entry and consequently the aggregate number of high-valuation investors is exogenous, the existence of naked CDS buyers instead decreases bond market liquidity. Some high valuation investors who would have otherwise bought bonds migrate to the CDS market and sell CDS instead. Existing bond sellers effectively compete with naked CDS buyers for the same set of investors who are in the position of providing liquidity in either market. Thus, bond sellers face greater congestion externality and greater search costs and bond market liquidity decreases. When entry is endogenous, however, the increase in high-valuation investors not only replaces bond buyers that migrated to the CDS market but also, due to search frictions in the CDS market, results in an even greater number of potential bond buyers.

### Short-run Versus Long-run Effects

I refer to the results when entry is endogenous (the spillover effect) a long-run effect and the results when entry is fixed a short-run effect. The idea is that downscaling investment resources allocated to credit markets and scaling it back up is too costly (and hence fixed) in the short-run or if the change in one of the markets is only temporary. So any necessary reallocation occurs only locally within credit markets (i.e. at the intensive margin between assets that are close substitutes such as bond and CDS). But in the long-run or with a permanent change to one of the markets, investment resources get reallocated at a wider scope in and out of credit markets as a whole (i.e. at the extensive margin).

#### 2.2 A Temporary Naked CDS Ban

This section specifically models a temporary CDS ban and shows that the implications from the short-run effect rationalize the observed increase in bond market liquidity after the German temporary ban.

I model a temporary naked CDS ban as a one-time unexpected drop in the number of naked CDS buyers. In particular, when the shock hits, the distribution of the measure of types switches to \( \{\bar{\mu}_\tau\}_{\tau \in T} \) where the measure of naked CDS buyers is set zero \( (\bar{\mu}_{[0,0]} = 0) \) while all the other elements of \( \{\bar{\mu}_\tau\}_{\tau \in T} \) are equal to the steady state value. I assume that the flow of differently, new high-valuation entrants replace one-to-one bond buyers that migrate to the CDS market and sell CDS instead.
high-valuation investors remains fixed in the short-run as the economy rebounds back to the steady state equilibrium. Time is relabeled so that \( t = 0 \) corresponds to the time of the shock.

As described in detail in Section 5, the temporary was implemented by the German government and applicable to German institutions. At first, the ban may seem limited in scope. However, it applies to one of the major market players in a market already concentrated among few dealers: Deutsche Bank. Thus, the ban should have affected a non-trivial subset of the market. A more realistic specification would be to set the number of CDS buyers – instead of zero – to \( \bar{\mu}_{[0,0]} = (1-x)\mu \) where \( x \) is the fraction of CDS demand originating from Germany. Results, however, would not change qualitatively. Also, the temporary ban lasted for about two months whereas in the model it lasts only an instant. Again, the direction of the results would be the same if it were to last longer. The focus of this section is to capture the immediate reaction to the ban by looking at the short-run effect (i.e. when the entry margin has not had enough time to adjust).

The time-varying equilibrium measure of \( h[0,0] \) type agents from the shock back to the steady state is given by the solution to the following ordinary differential equation (ODE):

\[
\dot{\mu}_{h[0,0]}(t) = \rho F_h + \gamma_u \mu_{h[0,1]}(t) - \left[ \gamma_d(0 - V_{h[0,0]}(t)) + \lambda_b \mu_{ba}(t) + \lambda_{cb}(t) \mu_{h[0,0]}(t) \right].
\]

The initial condition is given by \( \{\mu_{\tau}(0)\}_{\tau \in T} = \{\bar{\mu}_{\tau}\}_{\tau \in T} \) and the entry rate \( \rho \) is kept fixed at the steady state level. The dynamics for the measures of other agents are analogously characterized in (A.48) – (A.54).

Agent \( h[0,0] \)’s value function evolves according to:

\[
\dot{V}_{h[0,0]}(t) = rV_{h[0,0]}(t) - \left[ \gamma_d(0 - V_{h[0,0]}(t)) + \lambda_b \mu_{ba}(t) \left( V_{h[1,0]}(t) - V_{h[0,0]}(t) - p_b(t) \right) \right. \\
+ \left. \lambda_{cb}(t)(V_{h[0,1]}(t) - V_{h[0,0]}(t)) \right],
\]

where

\[
p_b(t) = \phi V_{a[1,0]}(t) + (1 - \phi) \left( V_{h[1,0]}(t) - V_{h[0,0]}(t) \right)
\]

and

\[
V_{h[1,0]}(t) - V_{h[0,0]}(t) = \phi \left( V_{l[0,1]}(t) - V_{l[0,0]}(t) + V_{h[0,1]}(t) - V_{h[0,0]}(t) \right).
\]

It is analogous for the other agents as shown in (A.55) – (A.61). Defining \( \Delta_{h[1,0]} \equiv V_{h[1,2]} - V_{h[0,0]} \) and \( \omega_c \equiv V_{h[0,1]} - V_{h[0,0]} + V_{l[0,1]} - V_{l[0,0]} \), we can rewrite all the ODEs for the value functions in terms of \( \Delta_{h[1,0]} \), \( \omega_b \) and \( \omega_c \). For example,

\[
\dot{V}_{h[0,0]}(t) = rV_{h[0,0]}(t) - \left[ \gamma_d(0 - V_{h[0,0]}(t)) + \lambda_b \mu_{ba}(t) \phi \omega_b(t) + \lambda_{cb}(t) \phi \omega_c(t) \right].
\]

In turn, solutions for \( \Delta_{h[1,0]} \), \( \omega_b \) and \( \omega_c \) are given in Proposition 6

**Proposition 6.** Given the solution to the time-varying dynamics of agent
measures, the dynamics for $\Delta h_{[1,0]}$ and $V_{a[1,0]}$ are given by:

$$
\Delta h_{[1,0]} = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} \left( (\gamma_d + \lambda_b \mu_{b,\phi}) \omega_b + \lambda_c \mu_{c,\phi} \omega_c \right) ds,
$$

$$
V_{a[1,0]} = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} \lambda_b \mu_{bb} (1 - \phi) \omega_b ds,
$$

where

$$
\begin{bmatrix}
\omega_b(t) \\
\omega_c(t)
\end{bmatrix} = \int_t^\infty e^{-\int_u^t A(u) du} \begin{bmatrix}
x_b \\
x_{cl} + x_{ch} - 2y
\end{bmatrix} ds,
$$

$$
A(t) = \begin{bmatrix}
\lambda_c \mu_{c,\phi} & \lambda_c \mu_{c,\phi} + \lambda_c \mu_{c,\phi} (1 - \phi) \\
\lambda_b \mu_{b,\phi} (1 - \phi) & r + \gamma_d + \gamma_u + \lambda_c \mu_{c,\phi} + \lambda_c \mu_{c,\phi} (1 - \phi)
\end{bmatrix}.
$$

Results

Figures 4 and 5 plot the short-run dynamics of types’ measures and bond market liquidity variables from the CDS ban at $t = 0$ back to the steady state. The sudden drop in the number of naked CDS buyers frees up long investors who would have otherwise sold CDS. Long investors then temporarily substitute providing liquidity in the CDS market with providing liquidity in the bond market. In turn, bonds sellers temporarily benefit from the ban as they now locate bond buyers more quickly and face lower search costs.

Thus, a temporary CDS ban reverses the short-run effect described earlier. Long investors do not exit entirely from credit markets (as was the case with a permanent ban) but instead resort to temporarily trading in the bond market. The immediate effect is an increase in bond market liquidity. This prediction is consistent with the observed increase in bond market liquidity following the temporary German ban.

An Implicit Cost of Entry

The short-run effect arises from keeping the entry rate fixed. This assumption is a reduced form way to capture an adjustment cost of entry. Although I do not explicitly incorporate an adjustment cost of entry, equation (11) illustrates one possible way of incorporating it. Now, in addition to comparing the value of entering $V_{h[0,0]}(\rho)$ with the outside investment opportunity $O_h$, high-valuation agents have to take into account a cost of entry, $c(\rho)$, that varies with the entry rate:

$$
\rho = \begin{cases} 
1 & V_{h[0,0]}(\rho) - c(\rho) > O_h \\
[0,1] & V_{h[0,0]}(\rho) - c(\rho) = O_h \\
0 & V_{h[0,0]}(\rho) - c(\rho) < O_h,
\end{cases}
$$

24As the ban is lifted, the number of traders searching to buy CDS increases until the fraction of CDS buyers who finds a CDS seller equals the flow of new low-valuation agents entering the economy.
where \( c'(\rho) > 0, \ c''(\rho) > 0, \ c(0) \geq 0, \) and \( c(1) < \infty. \)

Figure 6 illustrates an example of such a cost function. The temporary CDS ban leads to a small decrease in the value of trading as a high-valuation. When the scale of entry is already large and due to the convexity of \( c(\rho) \), a tiny decrease in \( \rho \) results in a large decrease in the cost. As a result, the entry rate does not have to change much in response to a temporary ban. In contrast, with a permanent ban, the value of trading as a high-valuation investor decreases by a lot. In addition, due to the convexity, as the entry rate \( \rho \) decreases, the resulting decrease in the cost of entry becomes less responsive. As a result, the entry rate has to decrease by a lot in response to a permanent ban. We can also back out how the short-run dynamics of the cost of entry has to look like from the dynamics of \( V_\rho[0,0](\rho^{ss}) \). This is shown in Figure 6.

3 Additional Results

This section starts by looking at how exogenously changing market liquidity and funding liquidity – through their effect on endogenous liquidity of both markets – affect prices and the CDS-bond basis. Then, section 3.2 extends the model to allow for bond holders to buy CDS and hold covered CDS positions. Section 3.3 calibrates the model and numerically illustrates the marginal effects covered and naked CDS positions.

3.1 Additional Empirical Predictions

I start by characterizing the closed-form solutions for the CDS price and CDS market liquidity.

**Proposition 7.** The price of a CDS contract is given by:

\[
p_c = \delta_c + x_{cl} - y - \frac{1}{\phi} (r + \gamma_u + \lambda_c \mu_{cs}) \Delta_{h[0,1]} \tag{12}
\]

where \( \Delta_{h[0,1]} \) is given in (10).

**Proposition 8.** The CDS price in a frictionless environment \( (\lambda_c, \lambda_b \to \infty) \) is given by:

\[
\lim_{\lambda_c, \lambda_b \to \infty} p_c = \delta_c - x_{ch} + y \tag{13}
\]

**Intuition.** High-valuation investors are the marginal sellers of CDS contracts. As a result, in a frictionless environment, the CDS price is given by the flow cost of providing insurance for high-valuation investors.

It is straight-forward to show that CDS contracts are more expensive with search frictions than without: \( p_c > \lim_{\lambda_c, \lambda_b \to \infty} p_c \). Analogous to the definition of bond illiquidity discount, we define CDS illiquidity as the wedge between the CDS price with search frictions and the price without frictions. In the case of CDS, it is an illiquidity premium instead of an illiquidity discount.
**Definition 3.** The illiquidity premium, $d_c$, in the CDS price is defined as the difference between the frictionless CDS price and the price with search frictions:

$$d_c \equiv p_c - \lim_{\lambda_c, \lambda_b \to \infty} p_c = x_{ch} + x_{cl} - 2y - \frac{1}{\phi}(r + \gamma_a + \lambda_c \mu_{cs}) \Delta h_{[0,1]}$$

The CDS bid-ask spread is defined, analogous to the bond bid-ask spread, as the total trading surplus: $\omega_c \equiv V_{h[0,1]} - V_{h[0,0]} + V_{l[0,-1]} - V_{l[0,0]}$. This definition captures the idea that, in an environment with dealers, dealers would extract rents proportional to the total trading surplus through the bid-ask spread that they charge.

Results 1 and 2 analyze the comparative statics with respect to the matching efficiency of the CDS ($\lambda_c$) and the bond market ($\lambda_b$). Result 3 looks at the effect of exogenously increasing funding liquidity. In particular, it relaxes the entry constraint by decreasing the value of the outside option, $O_h$. These results are derived numerically using parameter values in Table 6.

**Result 1.** *The effect of an exogenous increase in CDS market liquidity ($\lambda_c$).*

1. **Bond market liquidity deteriorates:** the illiquidity discount ($d_b$) and the bid-ask spread ($\omega_b$) increase while the volume of trade ($M_b$) decreases.

2. **CDS market liquidity increases.** The illiquidity premium ($d_c$) and the bid-ask spread ($\omega_c$) decrease while the volume of trade ($M_c$) increases.

3. **The bond price** ($p_b$) decreases.

4. **The CDS price** ($p_c$) decreases.

An increase in the CDS meeting intensity ($\lambda_c$) increases CDS market liquidity, which is intuitive, but it also decreases bond market liquidity. This is because the CDS market had a positive externality on bond market liquidity in the presence of search frictions in the CDS market. Lowering CDS search frictions reverses this positive externality.

Due to the opposite changes in bond and CDS market liquidity, the CDS price and the bond yield also change in opposite directions: CDS becomes cheaper (which in the data may be perceived as a decrease in credit risk) while the bond yield increases. Lastly, it is generally not obvious whether an increase in CDS market liquidity should increase or decrease the CDS price, and hence whether it is the buyer or the seller of a CDS contract that extracts the rent. The model shows that it is the CDS seller that captures the spread.

**Result 2.** *The effect of an exogenous increase in bond market liquidity ($\lambda_b$).*

1. **Bond market liquidity increases.** The illiquidity discount ($d_b$) and the bid-ask spread ($\omega_b$) decrease while the volume of trade ($M_b$) increases.
2. **CDS market liquidity decreases.** The illiquidity premium \((d_c)\) and the bid-ask spread \((\omega_c)\) increase while the volume of transactions \((M_c)\) decreases.

3. **The bond price \((p_b)\) increases.**

4. **The CDS price \((p_c)\) increases.**

Result 2 shows that the effects of an exogenous increase in bond versus CDS market liquidity are exactly opposite.

**Result 3.** *The effect of exogenous increase in funding liquidity (a decrease in \(O_h\)).*

1. **Bond market liquidity increases.** The illiquidity discount \((d_b)\) and the bid-ask spread \((\omega_b)\) decrease while the volume of trade \((M_b)\) increases.

2. **CDS market liquidity generally increases.** The illiquidity premium \((d_c)\) decreases while the volume of trade \((M_c)\) increases. The change in the bid-ask spread \((\omega_c)\), however, is not monotone.

3. **The bond price \((p_b)\) increases.**

4. **The CDS price \((p_c)\) decreases.**

**Implications on the CDS-bond basis**

The CDS-bond basis is the CDS spread (the price of a CDS contract) minus the bond yield after adjusting for the risk-free rate. It captures the pricing difference of the same underlying credit risk by the bond versus the CDS market. In a frictionless world, the difference and consequently the basis should be zero. A large body of empirical studies document a persistent deviation of the basis from zero. Consequently, a focus of the recent CDS literature has been on understanding the determinants of the CDS-bond basis and how the relative liquidity of bond and CDS markets affect it. Empirical analysis, however, is limited by endogeneity problems because asset prices and liquidity are interdependent within as well as across markets. Using results 1–3, we can disentangle how exogenously changing market liquidity and funding liquidity affect the CDS-bond basis. Corollary 1 summarizes the results.

**Corollary 1.** *(Effects of exogenous variations in market and funding liquidity on the CDS-bond basis)*

1. **An exogenous increase in CDS market liquidity decreases the CDS-bond basis (the basis becomes more negative).**

2. **An exogenous increase in bond market liquidity increases the CDS-bond basis (the basis becomes more positive).**
3. Exogenously relaxing the entry constraint decreases the CDS-bond basis
   (the basis becomes more negative).

We saw from Result 1 that an exogenous increase in CDS market liquidity - through its effect on liquidity of both markets - decreases the CDS spread while increasing the bond yield. As a result, the basis declines (Corollary 1.1). The effect of an exogenous increase in bond market liquidity is exactly the opposite: the basis increases (Corollary 1.2). This is consistent with Bai and Collin-Dufresne (2011) who find that bond market liquidity increases the basis (in particular, they find bond illiquidity measured by bid-ask spreads decreases the basis). However, they do not control for CDS market liquidity. Arce, Mayordomo, and Peña (2012) finds that bond market liquidity still increases the basis controlling for CDS market liquidity.

Finally, the model predicts that exogenously increasing funding liquidity will decrease the basis (Corollary 1.3). From Result 3, the CDS spread decreased but the bond yield decreased also. Thus, the effect on the basis can be ambiguous. However, for the parameter values used (which I argue are reasonable in Section 3.3), the model predicts a decrease in the basis.

3.2 Extension: Covered CDS

This section extends the model to allow for bondholders to purchase CDS protection and, consequently, hold “covered” CDS positions $a[1, -1]$. The limitation of the benchmark environment with three types of investor valuations is that covered CDS positions do not naturally arise. The only bondholders who are candidates to buy CDS (and hence hold covered CDS positions) are the average-valuation bondholders. But given Assumption 1 that $2y > x_{ch}$, average-valuation investors do not profit from buying CDS (whether they hold bonds or not) and only low-valuation investors benefit from buying CDS. Thus, to “force” average-valuation bondholders to buy CDS (while they are trying to sell their bond), I assume that there is a flow benefit $y_{1, -1}$ that exists only in particular for the covered CDS position. Such benefit could be due to, for example, relaxed capital requirements from hedging bond positions.

I describe next how allowing for covered CDS positions changes the environment and trading strategies for each type of investor.

**Covered CDS buyers ($a[1, -1]$ type)**

Average-valuation bondholders can now, in addition to searching for a bond buyer, simultaneously search for a CDS seller. If she bumps into a CDS seller before she is able to sell her bond, she buys CDS at price $p_{c1, -1}$ and becomes a covered CDS holder. This matching opportunity is reflected in the value and measure functions of a bond buyer: \[B.4\] and Table \[4\]. She will remain a covered CDS holder until she is able to sell her bond. Upon selling her bond, she will terminate her CDS contract and exit the economy. This is reflected in the value and measure functions of a covered CDS holder: \[B.5\] and Table \[4\].
CDS sellers \((h[0,0] \text{ type})\)

A CDS seller can bump into either a naked CDS buyer (low-valuation non-owner) or a covered CDS buyer (average-valuation bond owner). Depending on the type of the CDS buyer (covered versus naked), trading gains and hence bargained CDS prices can be different. So I will categorize CDS sellers by their counter-parties. A CDS seller (i.e. with position \([\theta_b, \theta_c] = [0, 1]\) who’s counterparty is a naked buyer is denoted by \(\tau = i[0, 1][0, -1]\) for \(i \in \{h, a\}\). A CDS seller who’s counterparty is a covered CDS buyer is denoted by \(\tau = i[0, 1][1, -1]\) for \(i \in \{h, a\}\).

As before, CDS price \(p_c\) bargained between a CDS seller and a naked CDS buyer is determined implicitly by:

\[
V_{h[0,1][0,-1]} - V_{h[0,0]} = \phi \left( V_{i[0,-1]} - V_{i[0,0]} + V_{h[0,1][0,-1]} - V_{h[0,0]} \right) .
\] (14)

CDS price bargained between a CDS seller and a covered CDS buyer, denoted by \(p_c[1,-1]\), is instead determined by:

\[
V_{h[0,1][1,-1]} - V_{h[0,0]} = \phi \left( V_{a[1,-1]} - V_{a[1,0]} + V_{h[0,1][1,-1]} - V_{h[0,0]} \right) .
\] (15)

As before, CDS sellers upon switching to an average-valuation will want to unwind their long position by searching for another CDS seller to take over his position at the original price. Thus, among \(h[0,0]\) investors, who are searching to sell CDS, some will directly bump into a naked CDS buyer. While others will instead indirectly sell to a naked CDS buyer by finding and taking over from an average-valuation investor her previously established long position with a naked CDS buyer. Essentially, the transaction between a high- and a low-valuation agent is being intermediated by an average-valuation agent. For \(h[0,0]\) investor, this match occurs with probability \(\lambda_c \mu_{a[1,0][0,-1]}\). Since in both the direct and indirect matches he gets the same price, with total intensity \(\lambda_c (\mu_{[0,0]} + \mu_{a[0,1][0,-1]})\) his trading surplus is \(V_{h[0,1][0,-1]} - V_{h[0,0]}\). Analogously, \(h[0,0]\) type investor will meet with intensity \(\lambda_c (\mu_{a[1,0]} + \mu_{a[0,1][1,-1]})\) a covered CDS buyer either directly or indirectly by taking over from an average-valuation investor her long CDS position with a covered CDS buyer. In both cases, the trading surplus is \(V_{h[0,1][1,-1]} - V_{h[0,0]}\). Above described matching opportunities (and the corresponding intensities and surpluses) are reflected in the value and measure functions of \(h[0,0]\) investors: \((B.3)\) and Table 4.

Bond buyers \((h[0,0] \text{ type})\)

As in the benchmark environment, in addition to selling CDS, \(h[0,0]\) investors enter the bond market to buy bonds. Now, a bond buyer’s trading surplus and the price he pays for the bond depend on the CDS position of the bond seller. He meets with intensity \(\lambda_b \mu_{a[1,0]}\) a bond seller who has not purchased CDS, in which case the bargained bond price and the trading surplus are the same as before. If he meets a bond seller who has purchased CDS (which
occurs with intensity $\lambda_b \mu_{0[1,-1]}$, the bargained bond price is instead given by
\[ p_{b[1,-1]} = \phi V_{a[1,-1]} + (1 - \phi) (V_{h[1,0]} - V_{h[0,0]}) \] (16)
and trading surpluses to the buyer and the seller are $V_{h[1,0]} - V_{h[0,0]} - p_{b[1,-1]}$ and $0 - V_{a[1,-1]} - p_{b[1,-1]}$, respectively. Again, these matching opportunities (and the corresponding intensities and surpluses) are reflected in the value and measure functions of $h[0,0]$ investors: (B.3) and Table 4.

**CDS buyers (l[0,-1] and a[1, -1] types)**

As before, a naked CDS buyer upon switching to an average-valuation cancels her contract and exits the economy. This forces her counterparty to search all over again if her counterparty was a high-valuation investor. This is reflected in the value and measure equations of $h[0, 1][0,-1]$ investors and the measure of $h[0,0]$ investors. If her counterparty was instead an average-valuation agent seeking to get out of the contract, her counterparty will simply exit the economy as reflected in the value and measure functions of $a[0, 1][0, -1]$.

In contrast, the only reason a covered CDS buyer will terminate her contract is if she was able to sell her bond. If her counterparty was a high-valuation investor, he will have to search all over again (as reflected in the value and measure functions of $h[0, 1][1,-1]$ and the measure of $h[0,0]$); but if her counterparty was an average-valuation agent, he will simply exit the economy (as reflected in the value and measure functions of $a[0, 1][1,-1]$ investors).

Finally, the market clearing conditions for the bond and CDS markets, (B.1) and (B.2), reflect changes in the environment due to the addition of covered CDS positions.

**Results**

I first describe the marginal effect of covered CDS.\(^{25}\) The ability to hedge one’s bond position with CDS makes buying and holding a bond more attractive. This increases the value of the bond and consequently decreases the illiquidity discount. As both the bond seller and the buyer have outside options of trading in the CDS market, the total surplus to trading a bond is lower. This means narrower bond bid-ask spreads. As some transactions that would have taken place in the bond market now take place in the CDS market, bond volume is lower. These results are shown in Figure 7.

Recall that in the environment with naked CDS only, CDS affected the bond market only in the presence of CDS search frictions and was otherwise redundant. In contrast, the effects of covered CDS hold irrespective of search frictions in the CDS market. In particular, bond market liquidity variables all approach a new steady state level as the CDS market becomes frictionless ($\lambda_c \to \infty$). These effects can be seen in Figure 7 by comparing the environment with covered CDS (in thin dashed) with the environment without CDS.

\(^{25}\)These results are derived numerically using the parameter values in Table 6.
The marginal effects of allowing covered CDS positions hold both in the short-run (i.e. when entry is fixed) and in the long-run. Figure 8 shows the marginal effect of allowing covered CDS positions when entry is fixed. Thus, the effect of covered CDS on the bond market cannot on its own explain why different bans had different effects on bond market liquidity. The reason that the short- and long-run effects of naked CDS are different while that of covered CDS are the same is as follows. With the addition of naked CDS buyers, there is entirely a new set of short investors introduced into the economy, hence whether entry of their would-be natural counterparties is endogenous or not matters. Allowing covered CDS positions, in contrast, does not introduce into the economy new investors but only new positions for investors that had already existed in the economy (i.e. bond sellers).

Comparing the setting with both covered and naked positions with the setting with just covered CDS isolates the additional effect of naked CDS positions relative to a benchmark with covered CDS. Figure 7 shows in thin dashed lines the results with just covered CDS and in thin solid lines the results with both. By contrasting the two, we can see that the marginal effect of naked CDS (relative to a benchmark with covered CDS positions) is exactly the same as in Section 2: the bond price is higher and closer to the fundamental, bond bid-ask spreads are narrower, and bond volume is greater. Thus, the results outlined in Section 2 go through if we allow for covered CDS positions.

3.3 Calibration

This section calibrates the model environment with both covered and naked CDS trading and illustrates numerically the marginal effects of naked CDS (Section 2) and covered CDS positions (Section 3.2). Parameters are calibrated to match data moments reported in Sambalaibat (2014) for an average sovereign averaged over the period 2008-2012. So any data moments mentioned below come from Sambalaibat (2014).

Table 6 shows the calibration parameter values. The risk-free rate is set at 4%, bond supply, $S$, is normalized to 1, sellers and buyers have an equal bargaining power of $\phi = \frac{1}{2}$. The bond coupon flow, $\delta_b$, and the flow benefit of buying CDS, $\delta_c$, are both normalized to 1%.

The flow of high-valuation investors, $F_h$, is calibrated so that CDS outstanding as a percent of debt outstanding is around 30%. The flow of low-valuation investors, $F_l$, is set to satisfy $\frac{F_h}{\gamma_d} \geq s + \frac{F_l}{\gamma_u}$. How large this difference (i.e. the inequality) is affects, in part, the difference in the time it takes to buy versus the time it takes to sell in bond and CDS markets. The bigger the difference is, for example, the longer it takes to find a bond seller than it takes a bond seller to find a buyer. This difference, in contrast, has a smaller effect on bond and CDS volume. Switching intensities $\gamma_d$ and $\gamma_u$ are calibrated to match annual bond volume to debt outstanding – which is only available for Italy and is 50% – and annual CDS volume as a percent of CDS...
outstanding of around 80% for an average sovereign.

Matching efficiencies $\lambda_b$ and $\lambda_c$ are calibrated to search times in bond and CDS markets (days it takes, in expectation, to find a buyer or a seller). It is not obvious what these search times should be and there is no empirical evidence about this. Vayanos and Weill (2008) argue it should be within couple hours to few days for U.S. government bonds. Since the U.S. government bond market is the most liquid bond market, its search times serve as a lower bound for an average sovereign. From Sambalaibat (2014), the proportional bond bid-ask spread is about 1% for an average sovereign and 0.03% for US government bonds, thus search times for an average government bond market may be as much as 30 times longer than for the U.S. Treasury market. On the other hand, the expected number of days to trade for a seller (buyer) is the ratio of the number of sellers (buyers) to the number of trades per day. Siriwardane (2015) reports that there are about 1,700 counterparties in the CDS market. Sambalaibat (2014) reports that the number of transactions per day is about 12.37 trades for an average sovereign reference name. If we use 1,700 as the number of potential counterparties, it results in 170 days to find a buyer or a seller. This provides an upper bound. But 10–15 big dealers account for almost half of the transactions in the CDS market. Thus, I calibrate to the lower end of the possible range of few days to 170 days: 30–50 days. Siriwardane (2015) also reports that the sell side of the CDS market is more concentrated than the buy side, thus it should be quicker to sell CDS than to buy CDS. Table 7 shows the resulting calibration moments.

Hedging benefits ($x_b, x_{ch}$, and $x_{cl}$) and the cost of risk bearing ($y$ and $y_{1,-1}$) are set so that the bond expected return is about 6%, the bond bid-ask spread (as a percent of the bond price) is 2%, the CDS bid-ask spread (as a percent of the CDS price) is 13%, the CDS price is 2%, and so that the trading gain for a bond owner to buy CDS is positive ($\omega_{c[1,0]} > 0$).

**Results**

Tables 7 and 8 show the calibration results and the counterfactuals of what bond and CDS market liquidity and prices would have been if there were to be no CDS market at all (column 3), only naked CDS transactions (column 4), and only covered CDS transactions (column 5).

The effects of CDS on bond market liquidity show up more through prices (that is, bid-ask spreads and illiquidity premia) than through trading volume. The bond bid-ask spread (as a percent of the bond price) and the bond expected return are 27% and 5% higher in the no-CDS environment than in the environment with both covered and naked CDS trading. This implies that a permanent CDS ban would widen the bond bid-ask spread by as much 27%. The temporary ban exercise (Figure 5) generated with the above calibration

---

26In column “Both”, the environment with both covered and naked CDS positions, bond-sellers hedged with CDS sell their bond at a price different from bondholders who do not. I show the weighted average of these two prices, weighted by the volume of each transaction. Similarly, the CDS price shown is a weighted average of the prices paid by naked and covered CDS buyers. Bid-ask spreads are treated similarly.
values shows that the bid-ask spread narrowed by about 6.5%. Thus, the calibration of the model with reasonable parameter values generates changes in the bond bid-ask spread that are economically significant and consistent with the magnitudes documented in Sambalaibat (2014).

Comparing the relative importance of covered versus naked CDS positions, we see that most of the decrease in the bond bid-ask spread and the expected return is coming from the effect of naked CDS as opposed to covered CDS positions. This is not surprising given that the calibration results in the majority of CDS outstanding (99.2%) being naked purchases.

4 Discussion

In this section, I discuss assumptions of the paper and how they may or may not affect the main results of the paper.

CDS contracts exist in the model because buying CDS is a cheaper way to short credit risk. In particular, investors cannot directly short bonds. As discussed in the introduction, this assumption is motivated by the derivative versus fixed supply features of CDS and bond and the fact that CDS is a more standardized instrument. In addition, in contrast to shorting the underlying through CDS, shorting a bond (as well as unwinding the short position) involves multiple search processes. An investor first has to first search for a counterparty in the repo market to borrow a bond from. Then she goes to the cash bond market to search for a counterparty to sell the bond to. With CDS, it is just one step search process instead. Unwinding the short bond position requires searching for the specific bond that was borrowed in the first place. With CDS, an investor can simply terminate the contract instead. Allowing bond shorting would simply change the benchmark environment and whether the benchmark environment includes bond shorting or not, the marginal effect of naked CDS buyers should be the same.

In the model, as is standard in search models of financial assets, investors can hold only 0 or 1 unit of the bond and the CDS. Allowing for multiple units of CDS would most likely make the spillover effect even stronger. The previously discussed assumption – that directly shorting bonds is more costly than through the CDS market – is partly motivated by the derivative versus fixed supply features of CDS and bonds. Nevertheless, allowing investors to hold multiple CDS might endogenously result in heterogeneous trading costs of shorting across bond and CDS markets.

I assumed that long investors can simultaneously search in both the bond and the CDS market and, as a result, there is effectively an increasing returns to scale to doing so. This captures the idea that resources expended on pricing individual bonds help an investor price CDS relatively quickly and vice versa. In the model, they transact based on wherever they find a counterparty first and do not have to choose to search in one market over the other or be indifferent between the two markets. Imposing that investors choose one of the markets before they begin searching typically gives rise to market segmentation (see Vayanos and Wang (2007)). Imposing this constraint is
somewhat unrealistic given that we do not observe market segmentation and instead see traders simultaneously participating in both the CDS and the bond market.

Search intensities \((\lambda_b, \lambda_c)\) or equivalently matching efficiencies were exogenous. When the search intensity is endogenous, typically, a multiple equilibrium arises. As long as entry is endogenous, however, in the equilibrium where both markets exist (which is what we see in reality), there should still be an increase in the number of long investors due to naked CDS buyers.

I have assumed that all investors can participate in both the bond and the CDS market. In reality, there are market participants, such as retail investors, who can trade in the bond market but not in the CDS market. To that extent, I am looking at the effect of CDS only on the subset of bond traders who can also trade in the CDS market, thus not directly on retail investors, for example. But if the effects get passed down to the rest of the bond market (i.e. those who cannot trade CDS), then CDS would indirectly affect them also and in the ways described in the paper. As for CDS traders, most if not all CDS participants can also trade in the bond market, thus I am capturing the entire CDS market.

Search frictions were the only underlying frictions in the model and informational frictions did not play a role. CDS might affect the bond market, in addition, through informational channels. For example, CDS might worsen bond market liquidity due information asymmetry between investors and the bond issuer (a sovereign, for example). As an instrument to trade on negative news, shorting credit risk through the CDS market may amplify a “run” on sovereign bond markets which then leads to a further liquidity dry-up in the bond market. I do not rule out this type of effect but this mechanism on its own cannot explain why different bans have different effects on bond market liquidity.

Illiquidity could also arise from asymmetric information amongst traders as in Kyle (1985) and Glosten and Milgrom (1985). The search framework is better suited for OTC markets and in particular sovereign bond markets for two reasons. First, illiquidity in bond markets is more due to fragmentation of trades across heterogenous bonds. Second, asymmetric information and insider trading is less severe with respect to governments than with respect to individual firms.

Finally, the model did not have aggregate shocks which are outside the focus of the paper. Nevertheless, it would be an interesting extension to analyze which market responds faster and more strongly to an aggregate shock and how the relative price discovery is affected by exogenous variations in liquidity of the two markets.

5 Institutional Details

This section describes sovereign bond and CDS markets and the European regulations that banned naked purchases of CDS as discussed in Sambalaibat (2014). For more details, see Sambalaibat (2014).
5.1 Sovereign Bond Market

Generally, government bonds trade in over-the-counter markets. A trader in the U.S., for example, shops for sovereign bonds using phone calls, emails, messages and quotes through Bloomberg. Locating a particular bond issue can be at times impossible. Bond markets of few governments, however, are organized as electronic markets. The U.S. Treasury market, for example, is organized as an electronic limit order market. Most of the Italian government bonds trade on an inter-dealer trading platform, called MTS, that functions similar to an electronic limit order market and is not accessible to individual investors. Despite the fact that the MTS is organized similar to an equity market and is one of the largest and most liquid government bond markets, trade is fragmented across heterogeneous bonds and liquidity per bond is low. According to Pelizzon, Subrahmanyam, Tomio, and Uno (2013), daily trading volume and the number of trades per bond on the MTS are comparable to the US municipal bond and the US corporate bond markets.

5.2 CDS Market

As discussed before, credit default swaps are over-the-counter derivative contracts that resemble insurance protection against a default or a similar event (referred to as a “credit event”) on bonds of a firm or a government (the “reference entity”). A buyer of a CDS protection pays a periodic fee (equivalently, the CDS price, premium, or spread) until either the contract matures or a credit event occurs. In return, the seller pays the buyer the protection amount that was purchased (called “notional”) in the event of default (or a similar event) on any one of the bonds of the reference entity that is covered by the CDS contract. CDS contracts are therefore written on firms and governments as a whole and not at a level of individual bond issues.

CDS contracts specify the reference entity, the contract maturity, the notional amount, the set of bonds of the reference entity that the contract covers, and the default events that constitute a credit event. The standard notional amounts are in the range of $10-20 million. Prices of CDS contracts are paid quarterly and are quoted as annualized percentages of the contract notional.

---


28 See Cheung, Rindi, and De Jong (2005), Dufour and Skinner (2004), and Pelizzon, Subrahmanyam, Tomio, and Uno (2013) for more information on MTS trading platforms.

29 For example, suppose you are a holder of bond “A” of the Greek government and Greece defaults on another bond “B.” If both bonds are covered by the contract, you will be still be paid out even if your bond “A” has not been defaulted on.

30 This is comparable to the most common transaction sizes of 5, 10, 25 million euros in, for example, the MTS Global Market (see Cheung, Rindi, and De Jong (2005)).

31 For example, if the price of a CDS contract with $10 million notional is 200 basis points, the protection buyer pays $0.2 million annually in quarterly installments of $0.05 million. The price of a CDS contract can be thought of as, in its simplest form, the probability of default times one minus the recovery rate. For example, if a one year CDS
The governing body for the CDS market, the International Swaps and Derivatives Association (ISDA), determines whether a credit event has occurred or not. The standard credit events for sovereign CDS are Failure to Pay and Debt Restructuring. Protection buyers get paid the difference between the notional and the recovery value that is determined through a special post-credit-event auction. For example, if an investor bought a CDS contract with a notional of $10 million and the recovery rate is 25%, she receives $7.5 million in cash. The ISDA finalizes the actual list of eligible bonds that can be submitted into the auction and oversees the auction. At the end of the auction, all bonds submitted into the auction are bought and sold at the same final bond price, and this final price is the price or the recovery rate that settles all CDS contracts on that reference entity. The recovery value is effectively the price of the defaulted bonds. Although cash settlements have become standard now, CDS buyers also have the option of physically settling their contracts by selling their bonds during the auction.

5.3 The Permanent CDS Ban

Throughout 2011, market participants faced uncertainty over whether the EU would adopt measures to ban naked CDS. The uncertainty was finally resolved on October 18, 2011 when, after months of negotiations, the European Parliament and the EU states passed a law to permanently ban naked CDS. The legislation applied to all CDS transactions referencing governments of the EU regardless of the geographic location of the transaction or the legal jurisdiction of the financial institution involved in the transaction.

The final draft of the law was published March 2012 (Regulation EU No 236/2012). Although the legislation was to be in effect beginning November 1, 2012, the March 2012 regulation stated that traders who enter new contracts after March 2012 would have to unwind them by November 2012. Contracts entered into before March 2012 could remain in place even beyond November 2012. Figure 9 compares the total CDS purchased referencing EU governments versus countries not affected by the ban. We see that the total amount of CDS purchased on EU sovereigns started to dramatically decrease.
starting around the time that the law was passed and has been declining ever since. This decrease did not occur for countries not affected by the ban. Thus, anticipating the difficulty of renewing contracts beyond March 2012, traders started to decrease their activity already beginning the fall of 2011.

A CDS purchase was considered covered if it was hedging a portfolio of assets that was correlated with government bonds of the reference entity. In particular, the value of the portfolio had to have a historical correlation of at least 70% with the government bond price over a period of at least 12 months prior to the CDS purchase. If a CDS purchase could not satisfy this at the time of the purchase, it would be considered naked and hence prohibited. The underlying portfolio could consist of, for example, long positions in private entities within the reference entity country or even long positions through CDS itself. The correlation requirement would be automatically satisfied if the underlying position being hedged consisted of government bonds (at federal and local levels of the government), the liability of state enterprises, and the liability of enterprises guaranteed by the sovereign. The legislation exempted market making activities.

After the purchase, traders did not have to maintain the correlation throughout the CDS contract to allow for prices of the underlying assets to vary. But the size of the underlying positions had to remain “proportional” to the amount of CDS purchased. In other words, a trader could not buy bonds with the intent of selling them back once she purchases CDS. The regulation was enforced by putting the responsibility on institutions to keep track of their positions. Upon request, institutions were supposed to be able to prove that their CDS purchases for the purpose of hedging.

Figure 9 in the Appendix shows the time series of the total amount of CDS purchased around the ban. Between the EU’s introduction of the permanent ban in October 2011 and the end of my sample period in June 2013, the total amount of CDS purchased referencing governments of the EU declined by one third, while a similar decline did not occur for countries not affected by the ban (in dashed line). Today, trading in European sovereign single-name contracts has essentially dried up according to 7. These observations suggest that naked CDS positions played an important role in the CDS market and possibly constituted a large proportion of the total CDS outstanding.

Figure 10 plots the cross country average of the bond bid-ask spread for the groups of countries (affected versus unaffected). After the regulation was introduced, the countries affected by the ban observed a widening of the bond bid-ask spread compared to countries not affected by the ban. This suggests the ban had a detrimental effect on liquidity of the underlying bonds.

5.4 The Temporary CDS Ban

On Tuesday May 18th 2010, Germany prohibited naked purchases of CDS referencing Eurozone governments. As recent as month prior to the ban

\footnote{Market participants were generally confused about how to actually interpret and satisfy the restrictions of the regulation.}
Germany’s rhetoric had been that there is no need to ban naked CDS trading. The regulation was unexpected by market participants and was implemented within the same day that the media first reported it. News about the ban first appeared around 1 pm on Tuesday May 18, 2010 on Reuters. But the official details of the legislation did not emerge until late in the evening around 9:30 pm. The regulation was effective from midnight the same day (within two and half hours from the release of the official statement) and was to be in effect through March 31, 2011. However, later on July 27, 2010 the regulation was made permanent.

The regulation also banned the naked short selling of 10 leading German financial stocks and the naked short selling of Eurozone governments bonds that were allowed to be listed on Germany’s domestic stock exchange. The naked bond short selling restriction, as a result, applied to only a few German and Austrian bonds.

The May 18th 2010 regulation did not specify the territorial scope of the regulation. So it is not clear whether market participants interpreted the regulation to apply to all transactions regardless of the geographic location and the institution. However, according to Allen & Overy LLP and ISDA’s conversations with BaFin (Germany’s financial regulatory body), BaFin confirmed that the regulation applied to transactions where at least one of the counterparties is located in Germany. It would not, for example, apply to a transaction between the New York branch and the London branch of Deutsche Bank.

Figure 11 plots the cross country average of the bond bid-ask spread. The dashed line shows the average for the EU countries that were not affected by the ban (i.e. naked CDS referencing these countries could still be purchased), while the solid line plots the average for the EU countries affected by the ban (i.e. the Eurozone countries). Two vertical lines are drawn for the week before the ban and the week of the ban. We see that for the countries affected by the ban, there was a large and sudden narrowing of the bond bid-ask spread, while this did not occur for the countries not affected by the ban. Figure 12 in the Appendix demonstrates the time series of CDS net notional around the ban.

6 Conclusion

This paper builds a search-theoretic model of over-the-counter bond and CDS markets that features an endogenous liquidity interaction between the two markets and endogenous funding liquidity.

My model shows that, in the long-run, trading activity and liquidity in the CDS market spills over into the bond market and increases bond market liquidity because there is an increasing returns to scale to searching simultaneously in multiple substitute markets. This spillover effect arises from endogenizing entry and consequently the aggregate number of investors which, in standard search models, is kept fixed. In the short-run, however, the entry rate is sticky and is unaffected by the CDS market and, as a result,
introducing the CDS market decreases liquidity in the bond market.

These effects are economically relevant. Sambalaibat (2014) documents that different naked CDS bans implemented in Europe (one permanent and the other temporary) had completely opposite effects on bond market liquidity. The implications from the long-run versus the short-run effects in the model help rationalize the observed changes in bond market liquidity after these CDS bans.
A Appendix: Proofs

To simplify notation, I define $q_{ca} \equiv \lambda_c \mu_{h[0,0]}$, $q_{cb} \equiv \lambda_c \mu_{a[1,0]}$, $q_{bs} \equiv \lambda_b \mu_{a[1,0]}$, $q_{bb} \equiv \lambda_b \mu_{h[0,0]}$, $\phi_h \equiv \phi$, and $\phi_l \equiv 1 - \phi$.

Agents’ flow value equations are analogously derived to (6):

\[
\begin{align*}
    r V_{l[0,0]} & = \gamma_u (0 - V_{l[0,0]}) + q_{ca} (V_{l[0,1]} - V_{l[0,0]}) \\
    r V_{h[1,0]} & = \delta_h + x_b - y + \gamma_d (V_{a[1,0]} - V_{h[1,0]}) \\
    r V_{a[1,0]} & = \delta_a - y + q_{ba} (0 - V_{a[1,0]} + p_b) \\
    r V_{h[0,1]} & = p_c - (\delta_c - x_h) - y + \gamma_d (V_{a[0,1]} - V_{h[0,1]}) + \gamma_u (V_{h[0,0]} - V_{h[0,1]}) \\
    r V_{a[0,1]} & = -p_c - (\delta_c + x_a) - y + \gamma_u (0 - V_{l[0,1]}) \\
    r V_{l[0,1]} & = -p_c + (\delta_c + x_a) - y + \gamma_u (0 - V_{l[0,1]}) \\
\end{align*}
\]

**Proof of Proposition 1.** The proof of uniqueness is shown in Lemma 1 and the proof of existence is shown in Lemma 2.

**Lemma 1.** Suppose (7) holds, then the steady state equilibrium is unique.

**Proof.** First fix $\rho$, then using the in-flow out-flow equations and the market clearing conditions (2), (3), $\mu_{l[0,0]}, \mu_{h[1,0]}, \mu_{a[1,0]}, \mu_{h[0,1]}, \mu_{a[0,1]}, \mu_{l[0,1]}$ can be solved as a function of $\mu_{h[0,0]}$:

\[
\begin{align*}
    \mu_{l[0,0]} & = \frac{F_l}{\gamma_u + \lambda_c \mu_{h[0,0]}} \\
    \mu_{h[1,0]} & = \frac{S \mu_{h[0,0]}}{\lambda_b \mu_{h[0,0]} + \gamma_d} \\
    \mu_{a[1,0]} & = S - \frac{S \mu_{h[0,0]}}{\lambda_b \mu_{h[0,0]} + \gamma_d} \\
    \mu_{h[0,1]} & = \frac{\lambda_c F_l \mu_{h[0,0]}}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_u)} \\
    \mu_{a[0,1]} & = \frac{\gamma_d F_l \lambda_c \mu_{h[0,0]}}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_u)} \\
    \mu_{l[0,1]} & = \frac{\lambda_c F_l \mu_{h[0,0]}}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_u)} \\
\end{align*}
\]

And $\mu_{h[0,0]}$ itself is a solution to:

\[
\rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S \lambda_b}{\lambda_b \mu_{h[0,0]} + \gamma_d} + \frac{\lambda_c F_l}{\gamma_u (\lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_u)} + 1 \right) = 0 \]

The LHS of (A.29) is positive at $\mu_{h[0,0]} = 0$, decreasing in $\mu_{h[0,0]}$, and is negative for large $\mu_{h[0,0]}$, hence (A.29) has a unique positive solution. Thus, (A.29) uniquely determines $\mu_{h[0,0]}$ and has a positive solution, while other $\mu$’s are uniquely determined by (A.23)-(A.27). Next, once $\mu$’s are solved, the value functions and prices are uniquely determined by a linear system of equations: (6), (A.17)-(A.22), and (4)-(5).

We are left with the endogenous entry decisions:

\[
\rho = \begin{cases} 
1 & V_{h[0,0]} (\rho) > O_h \\
0, 1 & V_{h[0,0]} (\rho) = O_h \\
0 & V_{h[0,0]} (\rho) < O_h \end{cases} \]

There are three cases: two corner solutions $\rho = 0$, and $\rho = 1$, and an interior solution. Next, 1
show that \( V_{h[0,0]} \) is strictly decreasing in \( \rho \), which will imply that under each case the equilibrium is unique. The derivation in the proof of existence shows that:

\[
V_{h[0,0]} = \frac{q_{bs} x_b \phi + \Delta_h[q_b(r + \gamma_d + q_b(1 - \phi))]}{(r + \gamma_d) k},
\]

where

\[
\Delta_{h[1]} = \frac{x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b}{\phi} \frac{1}{r + \gamma_u + q_{cs}} + \frac{1}{k} q_{cb} (r + \gamma_d + (1 - \phi) q_{bb})
\]

None of the \( \mu \)'s other than \( \mu_{h[0,0]} \) directly depend on \( \rho \) but depend only indirectly through \( \mu_{h[0,0]} \), thus we write:

\[
\frac{\partial V_{h[0,0]}(\rho)}{\partial \rho} = \frac{\partial \mu_{h[0,0]}}{\partial \rho} \left( \frac{\partial V_{h[0,0]}}{\partial q_{ab}} \frac{\partial q_{ab}}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}}{\partial q_{bb}} \frac{\partial q_{bb}}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}}{\partial q_{cb}} \frac{\partial q_{cb}}{\partial \mu_{h[0,0]}} + \frac{\partial V_{h[0,0]}}{\partial q_{cs}} \frac{\partial q_{cs}}{\partial \mu_{h[0,0]}} \right)
\]

Next, I derive \( \frac{\partial V_{h[0,0]}}{\partial q_{bs}} \), \( \frac{\partial V_{h[0,0]}}{\partial q_{bb}} \), \( \frac{\partial V_{h[0,0]}}{\partial q_{cb}} \), and \( \frac{\partial V_{h[0,0]}}{\partial q_{cs}} \).

\[
\frac{\partial V_{h[0,0]}}{\partial q_{bs}} = \frac{q_{bs} \phi_h}{(r + \gamma_d) k^2} \phi B
\]

\[
\frac{\partial V_{h[0,0]}}{\partial q_{bb}} = \frac{\phi_h (r + \gamma_d + q_b \phi_l)}{(r + \gamma_d) k^2} B
\]

\[
\frac{\partial V_{h[0,0]}}{\partial q_{cs}} = \frac{q_{cb}}{k (r + \gamma_d + \gamma_u + q_{cs} \phi_l)} \left( \frac{r + \gamma_d + q_{cs} \phi_l}{C} \right),
\]

where

\[
B \equiv x_b + \frac{q_{cb}}{C} \left( \frac{(r + \gamma_d + q_{bb} \phi_l)}{k} \frac{A}{C} - A - \frac{(r + \gamma_d + q_{bb} \phi_l)}{k} \right) x_b
\]

\[
A \equiv x_{ch} + \frac{(x_{cl} - 2y)(q_{cs} + r + \gamma_u + q_{cs} \phi_l)}{q_{cs} + r + \gamma_u} - \frac{q_{bs} x_b \phi_h}{r + \gamma_d + q_b \phi_h + q_{bb} \phi_l}
\]

\[
C \equiv \frac{r + \gamma_d + \gamma_u + q_{cs} \phi_l}{\phi_h} + \frac{q_{cb}}{k} (r + \gamma_d + q_{bb} \phi_l)
\]

From here, \( \frac{\partial V_{h[0,0]}}{\partial q_{bs}} > 0 \) while \( \frac{\partial (\lambda_{e(\mu_{[0,1]} + \mu_{[0,0]})})}{\partial \mu_{h[0,0]}} < 0 \) implying that the third term in (A.31) is negative. Since \( \frac{\partial V_{h[0,0]}}{\partial q_{cs}} < 0 \), the fourth term (A.31) is also negative. But the sign of both \( \frac{\partial V_{h[0,0]}}{\partial q_{cs}} \)
and \( \frac{\partial V_h}{\partial q_{bb}} \) depend on the sign of \( B \). Thus, consider \( B \):

\[
B = x_b + \frac{q_{cb}}{C} \left( r + \gamma_d + q_{bb} \phi_l \right) \frac{A}{k} q_{cb} A - \frac{q_{cb}}{C} \left( r + \gamma_d + q_{bb} \phi_l \right) x_b
\]

\[
= x_b \left( 1 - \frac{q_{cb}}{C} \left( r + \gamma_d + q_{bb} \phi_l \right) \right) - \left( 1 - \frac{q_{cb}}{C} \left( r + \gamma_d + q_{bb} \phi_l \right) \right) \frac{q_{cb}}{C} A
\]

\[
= \left( 1 - \frac{q_{cb}}{C} \left( r + \gamma_d + q_{bb} \phi_l \right) \right) \left( x_b - \frac{q_{cb}}{C} A \right)
\]

First, \( 0 < \frac{q_{cb} \phi_l + r + \gamma_d}{k} \) \( < 1 \) and \( 0 < \frac{q_{cb}}{C} \) \( < 1 \). To see the latter, let \( \phi_l = \phi_h \), then \( C > q_{cs} \). From assumption \( \frac{\rho F_h}{\gamma_d} > S + \frac{F_1}{\gamma_u} \):

\[
\mu_h(0,0) + \mu_h(0,1) + \mu_h(0,1) = \frac{\rho F_h}{\gamma_d} > S + \frac{F_1}{\gamma_u}
\]

But using the CDS market clearing condition, we have \( \frac{F_1}{\gamma_u} = \mu(0,0) + \mu(0,1) = \mu(0,0) + (\mu_h(0,1) + \mu_a(0,1)) \).

Thus,

\[
\mu_h(0,0) + \mu_h(0,1) > S + \mu(0,0) + (\mu_h(0,1) + \mu_a(0,1))
\]

Cancel \( \mu_h(0,1) \):

\[
\mu_h(0,0) + \mu_h(1,0) > S + \mu(0,0) + \mu_a(0,1)
\]

\[
\mu_h(0,0) > (S - \mu(1,0)) + \mu(0,0) + \mu_a(0,1) > \mu(0,0) + \mu_a(0,1)
\]

Hence, \( q_{cs} > q_{cb} \) and \( C > q_{cs} > q_{cb} \). Thus, the term in the first bracket of \( B \) is positive. Now consider the term in the second bracket of \( B \), \( x_b - \frac{q_{cb}}{C} A = x_b - \frac{q_{cb}}{C} \Delta_h(0,1) \):

\[
x_b - \frac{q_{cb}}{C} \Delta_h(0,1) = x_b - \frac{q_{cb}}{C} \left( x_ch + (x_{cl} - 2y) \left( q_{cs} + r + \gamma_d + \gamma_u \right) \frac{\phi_h}{q_{cb} \phi_h} \right) x_b
\]

\[
= x_b - \left( x_ch + (x_{cl} - 2y) \left( q_{cs} + r + \gamma_d + \gamma_u \right) \frac{\phi_h}{q_{cb} \phi_h} \right) x_b
\]

\[
= \left( r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_h \right) x_b - \left( x_ch + (x_{cl} - 2y) \left( q_{cs} + r + \gamma_u + \gamma_d \right) \right)
\]

The sign of the expression depends on the numerator:

\[
\left( r + \gamma_d + \gamma_u + q_{cs} \phi_l + q_{cb} \phi_h \right)
\]

This expression is positive from \( \frac{7}{7} \). Thus, \( \frac{\partial V_h}{\partial q_{bb}} > 0 \) and together with \( \frac{\partial V_h}{\partial q_{bb}} > 0 \) implies that the first term of \( \frac{A.31}{A.31} \) is negative. Also, since \( \frac{\partial V_h}{\partial q_{bb}} < 0 \), the second term of \( \frac{A.31}{A.31} \) is also negative.

Finally, from \( \frac{A.29}{A.29} \) and using the Implicit Function Theorem,

\[
\frac{\partial \mu_h(0,0)}{\partial \rho} = \frac{\partial \mu_h(0,0)}{\partial \rho} \left( \frac{s \lambda_d \gamma_d}{(\lambda_d \mu_h(0,0) + \gamma_d)^2} + \frac{\lambda_c f_l(\gamma_d + \gamma_u)}{\gamma_u (\lambda_a \mu_h(0,0) + \gamma_d + \gamma_u)^2} + 1 \right)
\]

Thus, \( \frac{\partial \mu_h(0,0)}{\partial \rho} > 0 \), and consequently \( \frac{\partial V_h(0,0)}{\partial \rho} < 0 \).

\[ \square \]

**Lemma 2. Existence**
Proof. To show existence we verify that the conjectured optimal trading strategies are in fact optimal. In particular, first, we show that the total surplus from trading the bond is positive: $\omega_b = V_{h[1,0]} - V_{b[0,0]} - V_{a[1,0]} > 0$. By construction, this will ensure that individual surpluses to the buyer and the seller of the bond are positive: a high-valuation agent optimally chooses to buy the bond, and an average-valuation agent prefers to sell her bond. Second, we show that the total surplus from trading CDS is positive $\omega_c = V_{h[0,1]} - V_{b[0,0]} + V_{l[0,-1]} - V_{f[0,0]} > 0$. This will imply that the high-valuation agents will want to sell CDS, while low-valuation agents will want to buy CDS. Third, we verify that the average-valuation agents will prefer quit being a CDS seller: $0 - V_{a[0,1]} > 0$. Thus, agents who have previously sold CDS when they were a high-valuation investor will prefer to find another seller to take over his side of the trade and exit the market with zero utility. I proceed by first deriving $\omega_b$, $\omega_c$, $V_{a[0,1]}$.

Subtracting $rV_{l[0,0]}$ (A.17) from $rV_{l[0,-1]}$ (A.22) and defining $\Delta_l[0,-1] = V_{l[0,-1]} - V_{l[0,0]}$, we get:

$$\Delta_l[0,-1] = \frac{\delta_c + x_d - y - p_c}{r + \gamma_u + q_{cs}}$$

From (5):

$$\Delta_h[0,1] = \frac{\phi}{1 - \phi} \Delta_l[0,-1] = \frac{\phi}{1 - \phi} \frac{\delta_c + x_d - y - p_c}{r + \gamma_u + q_{cs}}$$

Also from the value function of $V_{a[0,1]}$,

$$V_{a[0,1]} = \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}}$$

(A.32)

Using (A.20) and substituting in the expression for $V_{a[0,1]}$:

$$rV_{h[0,1]} = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} - V_{h[0,1]} \right) - \gamma_u \Delta_h[0,1]$$

Add $\gamma_d V_{h[0,1]}$ to both sides:

$$(r + \gamma_d) V_{h[0,1]} = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} \right) - \gamma_u \Delta_h[0,1]$$

Subtract $(r + \gamma_d) V_{h[0,0]}$ from both sides:

$$(r + \gamma_d + \gamma_u) \Delta_h[0,1] = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} \right) - (r + \gamma_d) V_{h[0,0]}$$

Thus, we have three equations and three unknowns, $\Delta_h[0,1]$, $p_c$, $V_{h[0,0]}$:

$$\Delta_h[0,1] = \phi \frac{\delta_c + x_d - y - p_c}{1 - \phi}$$

$$(r + \gamma_d + \gamma_u) \Delta_h[0,1] = p_c - (\delta_c - x_{ch}) - y + \gamma_d \left( \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cs}} \right) - (r + \gamma_d) V_{h[0,0]}$$

$$V_{h[0,0]} = \frac{q_{bos} x_b \phi + \Delta_h[0,1] q_{ch} (r + \gamma_d + q_{bb} (1 - \phi))}{(r + \gamma_d) k}$$

(A.33)

where the latter comes from the solution to the equations for $V_{h[1,0]}$, $V_{a[1,0]}$, and $V_{h[0,0]}$. The solution for $\Delta_h[0,1]$ is given by:

$$\Delta_h[0,1] = \frac{x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_d - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bos} x_b}{\phi (1 - \phi) q_{cs} + r + \gamma_u + \gamma_d} + \frac{1}{k} q_{ch} (r + \gamma_d + (1 - \phi) q_{bb})$$

(A.34)
From here:
\[ p_c = \delta_c + x_{cl} - y - \frac{1 - \phi}{\phi} (r + \gamma_u + q_{cs}) \Delta h_{[0,1]} \]  \hspace{1cm} (A.35)

\[ \omega_c = \frac{1}{\phi} \Delta h_{[0,1]} \]

Using the solution to the equations for \( V_{h[1,0]} \), \( V_{a[1,0]} \), and \( V_{h[0,0]} \):
\[ \omega_b = \frac{x_b}{k} - \frac{q_{bs} \Delta h_{[0,1]}}{k} \]  \hspace{1cm} (A.36)

To consider small search frictions, define \( \epsilon \equiv \frac{1}{\lambda_b} \) and \( n \equiv \frac{\lambda_c}{\lambda_b} \). We show existence for \( \epsilon = 0 \).

Then by continuity, existence is established in the neighborhood of \( \epsilon \equiv 0 \) or for small search frictions. With the change of variables, (A.29) becomes:
\[ \rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S}{\mu_{h[0,0]} + \epsilon \gamma_d} + \frac{n F_j}{\gamma_u (n \mu_{h[0,0]} + \epsilon (\gamma_d + \gamma_u)) + 1} \right) = 0 \]  \hspace{1cm} (A.37)

From (A.37), for any \( \rho \in [0,1], \mu_{h[0,0]} \) asymptotically converges to \( \mu_{h[0,0]} = \frac{\rho F_h}{\gamma_d} - (S + \frac{F_j}{\gamma_u}) \) therefore
\[ 0 < \lim_{\lambda_b, \lambda_c \to \infty} \mu_{h[0,0]} < \infty \]
and \( \lim_{\lambda_b, \lambda_c \to \infty} q_{bb} = \infty \). This also implies from (A.25) that
\[ \lim_{\lambda_b, \lambda_c \to \infty} \mu_{a[1,0]} = 0 \]
and \( q_{bs} \) converges to a finite number. Analogously, \( \lim_{\lambda_b, \lambda_c \to \infty} q_{cs} = \infty \) and from (A.23) and (A.27):
\[ 0 < \lim_{\lambda_b, \lambda_c \to \infty} q_{cb} < \infty. \]

To show \( \omega_c > 0 \) using these limits, consider the numerator of \( \Delta h_{[0,1]} \):
\[ x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b \]

Using the above limits of \( q_{cs}, q_{bs}, \) and \( q_{bb} \), it converges to \( x_{ch} + x_{cl} - 2y \) which is positive by Assumption [1].

From (A.32), in order for \( V_{a[0,1]} < 0 \), the CDS price has to be such that \( p_c < \delta_c + y \). From (A.34) and (A.35):
\[ p_c = (\delta_c + x_{cl}) - y - \frac{(1 - \phi_h) (q_{cs} + r + \gamma_u)}{\phi_h} \left( \frac{x_{ch} + \frac{(x_{cl} - 2y)(q_{cs} + \gamma_d + r + \gamma_u)}{q_{cs} + \gamma_d + r + \gamma_u}}{x_{ch} + \frac{(1 - \phi) q_{cs} + \gamma_d + r + \gamma_u}{\phi}} - \frac{x_{ch} q_{bs} \phi_h}{k} \right) \]

This converges to \( \delta_c + y - x_{ch} \), which is less than \( \delta_c + y \). Thus, \( V_{a[0,1]} < 0 \).

Average-valuation agents will not want to buy CDS because the flow utility would be \( \delta_c - y - p_c \).

Given that \( p_c \to \delta_c + y - x_{ch} \), the flow utility \( \delta_c - y - p_c \) converges to \( x_{ch} - 2y \) which is negative by Assumption [1].

To show \( \omega_b > 0 \), consider the numerator of (A.36): \( x_b - q_{cb} \Delta h_{[0,1]} \). Since \( 0 < \lim q_{cb} < \infty \) and \( \Delta h_{[0,1]} \) converges to zero, \( x_b - q_{cb} \Delta h_{[0,1]} \) converges to \( x_b > 0 \). The above results show existence for \( \epsilon = 0 \). By continuity, existence is also established near \( \epsilon = 0 \).

**Proof of Proposition [2]** The bond price is \( p_b = \phi (V_{h[1,0]} - V_{h[0,0]}) + (1 - \phi) V_{a[1,0]} \). Solving \( V_{h[1,0]} \) and \( V_{a[1,0]} \):
\[ V_{h[1,0]} = \frac{\delta_b + x_b - y}{r} - \frac{\gamma_d (x_b + q_{cb} (1 - \phi) V_{h[0,0]})}{r (r + \gamma_d + q_{bb} (1 - \phi))} \]  \hspace{1cm} (A.38)

\[ V_{a[1,0]} = \frac{\delta_b + x_b - y}{r} - \frac{(r + \gamma_d) (x_b + q_{cb} (1 - \phi) V_{h[0,0]})}{r (r + \gamma_d + q_{bb} (1 - \phi))} \]  \hspace{1cm} (A.39)
where from the earlier derivation:

\[ V_{h[0,0]} = \frac{q_{bs} x_b \phi + \Delta_{h[0,1]} q_{cb} (r + \gamma_d + q_{cb}(1 - \phi))}{(r + \gamma_d) k} \]  

(A.40)

Thus, we derive the limits of \( q \)'s, and \( \Delta_{h[0,1]} \) as \( \lambda_b \to \infty \) for an arbitrary \( \lambda_c \). With the change of variable, \( \epsilon = \frac{1}{\lambda_b}, \) (A.29) becomes:

\[ \rho F_h - \gamma_d \mu_{h[0,0]} \left( \frac{S}{\mu_{h[0,0]} + \epsilon \gamma_d} + \frac{\lambda_c F_l}{\gamma_a \left( \lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_a \right)} + 1 \right) = 0 \]

For \( \epsilon = 0, \)

\[ \frac{\rho F_h}{\gamma_d} - S - \mu_{h[0,0]} \left( \frac{\lambda_c F_l}{\gamma_a \left( \lambda_c \mu_{h[0,0]} + \gamma_d + \gamma_a \right)} + 1 \right) = 0 \]  

(A.41)

For any \( \rho \in [0,1], \) the LHS of (A.41) is positive at \( \mu_{h[0,0]} = 0, \) decreasing in \( \mu_{h[0,0]}, \) and is negative for large \( \mu_{h[0,0]}. \) Hence, (A.41) has a positive finite solution, \( 0 < \lim_{\lambda_b \to \infty} \mu_{h[0,0]} < \infty, \) and this implies \( \lim_{\lambda_b \to \infty} q_{cb} = \infty, \) and \( k \to \infty. \) This also implies from (A.25) that \( \lim_{\lambda_b \to \infty} \mu_{a[1,0]} = 0 \) and \( q_{bs} \) converges to a finite number. Analogously, \( \lim_{\lambda_b \to \infty} q_{cs} = \infty \) and from (A.23) and (A.27): \( 0 < \lim_{\lambda_b \to \infty} q_{cd} < \infty. \)

Then as discussed above, the numerator of \( \Delta_{h[0,1]} \) converges to a finite number, while the denominator converges to \( \infty, \) thus, \( \Delta_{h[0,1]} \to 0. \) So \( V_{h[0,0]} \to 0, \) hence \( V_{h[1,0]} \to 0 \), \( V_{h[0,0]} \to \frac{b_h + x_b - y}{r} \) and \( p_b \to \frac{b_h + x_b - y}{r}. \)

**Proof of Proposition 3**

Combining (A.38)-(A.40), we get the bond price.

**Proof of Proposition 4**

Consider parameter conditions such that \( V_{h[0,0]}(\rho_{\text{nocs}}) = V_{h[0,0]}(\rho_{\text{cds}}) = O_h. \) Since the bond price is \( p_b = \phi(V_{h[1,0]} - V_{h[0,0]}) + (1 - \phi)V_{a[1,0]}, \) for an interior solution \( (V_{\text{nocs}}^{h[0,0]} = V_{\text{cds}}^{h[0,0]} = O_h) \) it is sufficient to show that \( V_{h[1,0]}(\rho_{\text{cds}}) > V_{h[0,0]}(\rho_{\text{nocs}}) \) and \( V_{a[1,0]}(\rho_{\text{cds}}) > V_{a[1,0]}(\rho_{\text{nocs}}). \) From (A.38) and (A.39), the derivative with respect to \( q_{bb} \):

\[ \frac{\partial V_{h[1,0]}}{\partial q_{bb}} = \frac{-\gamma_d ((r + \gamma_d)V_{h[0,0]} - x_b)(1 - \phi)}{r (r + \gamma_d + q_{bb}(1 - \phi))^2} \]

\[ \frac{\partial V_{a[1,0]}}{\partial q_{bb}} = \frac{-(r + \gamma_d)((r + \gamma_d)V_{h[0,0]} - x_b)(1 - \phi)}{r (r + \gamma_d + q_{bb}(1 - \phi))^2} \]

Thus, the condition for both \( V_{h[1,0]} \) and \( V_{a[1,0]} \) to be increasing in \( q_{bb} \) at \( q_{bb} = q_{bb}^{\text{cds}} \) is: \( (r + \gamma_d)V_{h[0,0]} - x_b < 0 \) evaluated at \( q_{bb} = q_{bb}^{\text{nocs}}. \) The solution for \( V_{h[0,0]} \) without the CDS market:

\[ V_{h[0,0]} = \frac{q_{bs} x_b \phi}{(r + \gamma_d)(r + \gamma_d + q_{bs} \phi + q_{cb}(1 - \phi))} \]  

(A.42)

Rearranging (A.42) we get:

\[ (r + \gamma_d)V_{h[0,0]} = \frac{q_{bs} \phi}{(r + \gamma_d + q_{bs} \phi + q_{bb}(1 - \phi))} x_b < x_b \]

Next, we show that \( q_{bb} = \lambda_b \mu_{h[0,0]} \) increases with CDS. Consider the solution for \( V_{h[0,0]} \) with CDS:

\[ V_{h[0,0]}^{\text{cds}} = \frac{x_b q_{cds} \phi_h}{k_{\text{cds}} (\gamma_d + r)} + \frac{q_{cb} \Delta_{h[0,1]} (q_{bb} \phi_t + \gamma_d + r)}{k_{\text{cds}} (\gamma_d + r)} \]  

(A.43)
Compare this with (A.42). The fact that $V_{h[0,0]}^{nocds} = V_{h[0,0]}^{cds} = O_h$ and that the second term of (A.43) is asymptotically positive implies that:

$$\frac{x_b q_{bs}^{cds} \phi_h}{k^{cds} (\gamma_d + r)} < \frac{x_b q_{bs} \phi_h}{k (\gamma_d + r)}$$

The term $\frac{x_b q_{bs} \phi_h}{k (\gamma_d + r)}$ is strictly decreasing in $\mu_{h[0,0]}$. Thus, it has to be the case that $\mu_{h[0,0]}^{cds} > \mu_{h[0,0]}^{nocds}$.

Now consider the effect on the bond bid-ask spread, $\omega_b$, characterized in (A.36). The first term $x_b/k$ in (A.36) is strictly decreasing in $\mu_{h[0,0]}$, while the second term is asymptotically positive and arises due to CDS. Thus, it follows that $\omega_{b}^{cds} < \omega_{b}^{nocds}$.

Now consider bond volume, $\lambda_b \mu_{h[0,0]} \mu_{a[1,0]}$. Using the expression for $\mu_{a[1,0]}$ from (A.25), and taking the derivative with respect to $\mu_{h[0,0]}$, we get:

$$\frac{\lambda_b \gamma_d^2}{(\gamma_d + \lambda_b \mu_{h[0,0]})^2} > 0$$

Thus, bond volume is increasing in $\mu_{h[0,0]}$ and hence volume is greater with the existence of the CDS market.

Proof of Proposition 5 Consider what (A.29) limits to as $\lambda_c \rightarrow \infty$ for an arbitrary $\lambda_b$:

$$\frac{\rho F_h}{\gamma_d} - \left( \frac{S \lambda_b \mu_{h[0,0]} + \gamma_d}{\lambda_b \mu_{h[0,0]} + \gamma_d} + \frac{F_l}{\gamma_u} + \mu_{h[0,0]} \right) = 0 \quad (A.44)$$

The LHS of (A.44) is positive at $\mu_{h[0,0]} = 0$, decreasing in $\mu_{h[0,0]}$, and is negative for large $\mu_{h[0,0]}$. Thus, for any $\rho$, $\mu_{h[0,0]}$ is finite as $\lambda_c \rightarrow \infty$. As a result, $\mu_{a[1,0]}$, $q_{bs}$ and $q_{bb}$ are finite. Since $\mu_{h[0,0]} + \mu_{a[1,0]} \rightarrow 0$, $q_{cb}$ is also finite. But $q_{cds} = \lambda_c \mu_{h[0,0]} \rightarrow \infty$. Thus, $\Delta_{h[0,1]} \rightarrow 0$.

When the solution is interior, $V_{h[0,0]}^{cds} = V_{h[0,0]}^{nocds} = O_h$ (A.45).

Then, using $\Delta_{h[0,1]} \rightarrow 0$ and (A.33):

$$\frac{x_b q_{bs}^{cds} \phi_h}{k^{cds} (\gamma_d + r)} = \frac{x_b q_{bs}^{nocds} \phi_h}{k^{nocds} (\gamma_d + r)} \quad (A.46)$$

Since this expression is uniquely determined by $\mu_{h[0,0]}$, it has to be that:

$$\mu_{h[0,0]}^{cds} = \mu_{h[0,0]}^{nocds} \quad (A.47)$$

Thus, $q_{bb} = \lambda_b \mu_{h[0,0]}$ is the same as without CDS. Consequently, from (A.38), (A.39) and (A.45), $V_{h[0,1]}$ and $V_{a[1,0]}$ are the same with or without CDS. Thus, when $\lambda_c \rightarrow \infty$, the bond price is the same as in the benchmark environment without CDS. For (A.47) to hold, from (A.44), the entry rate (hence the measure of high-valuation investors) increases enough to exactly offset the total measure of low-valuation investors $\frac{F_l}{\gamma_u} \cdot (\rho^{cds} - \rho^{nocds}) F_h = \frac{F_l}{\gamma_u}$.

If entry is exogenous, $\lim_{\lambda_c \rightarrow \infty} p_b(\lambda_c) < p_b^{nocds}$ because the measure of high-valuation investors (hence the measure of bond buyers) decreases due the existence of low-valuation investors.

\[\square\]
Proof of Proposition 6. The population measures evolve according to:

\[
\begin{align*}
\dot{\mu}_{h[0,0]}(t) &= p F_h + \gamma_u \mu_{h[0,1]}(t) - \left[ \gamma_d \mu_{h[0,0]}(t) + (q_{bs}(t) + q_{cb}(t)) \mu_{h[0,0]}(t) \right] \\
\dot{\mu}_{l[0,0]}(t) &= F_l - \left[ \gamma_u \mu_{l[0,0]}(t) + q_{cs} \mu_{l[0,0]}(t) \right] \\
\dot{\mu}_{h[1,0]}(t) &= q_{bs} \mu_{h[0,0]}(t) - \gamma_d \mu_{h[1,0]}(t) \\
\dot{\mu}_{a[1,0]}(t) &= q_{cb} \mu_{h[0,0]}(t) - \left[ \gamma_d \mu_{h[1,0]}(t) + \gamma_a \mu_{h[0,1]}(t) \right] \\
\dot{\mu}_{a[0,1]}(t) &= \gamma_d \mu_{h[1,0]}(t) - \gamma_a \mu_{a[0,1]}(t) + q_{cs} \mu_{a[0,1]}(t) \\
\dot{\mu}_{l[0,1]}(t) &= q_{cs} \mu_{h[0,0]}(t) - \gamma_a \mu_{l[0,1]}(t)
\end{align*}
\]

Value functions evolve according to:

\[
\begin{align*}
\dot{V}_{h[0,0]}(t) &= r V_{h[0,0]}(t) - \left[ \gamma_d (0 - V_{h[0,0]}(t)) + q_{bs}(t) \phi_{b}(t) + q_{cb}(t) (V_{h[0,1]}(t) - V_{h[0,0]}(t)) \right] \\
\dot{V}_{l[0,0]}(t) &= r V_{l[0,0]}(t) - \left[ \gamma_u (0 - V_{l[0,0]}(t)) + q_{cs}(t) (V_{l[0,1]}(t) - V_{l[0,0]}(t)) \right] \\
\dot{V}_{h[1,0]}(t) &= r V_{h[1,0]}(t) - \left[ \delta_b + x_b - y + \gamma_d (V_{a[1,0]}(t) - V_{h[0,1]}(t)) \right] \\
\dot{V}_{a[1,0]}(t) &= r V_{a[1,0]}(t) - \left[ \delta_b - y + q_{bb}(t) (1 - \phi) \omega_b(t) \right] \\
\dot{V}_{h[0,1]}(t) &= r V_{h[0,1]}(t) - \left[ p_c(t) - (\delta_c - x_{cl}) - y + \gamma_d (V_{a[0,1]}(t) - V_{h[1,0]}(t)) + \gamma_u (V_{h[0,0]}(t) - V_{h[0,1]}(t)) \right] \\
\dot{V}_{a[0,1]}(t) &= r V_{a[0,1]}(t) - \left[ p_c(t) - \delta_c - y + q_{cs}(t) (0 - V_{a[0,1]}(t)) + \gamma_u (0 - V_{a[0,1]}(t)) \right] \\
\dot{V}_{l[0,1]}(t) &= r V_{l[0,1]}(t) - \left[ -p_c(t) + (\delta_c + x_{ch}) - y + \gamma_u (0 - V_{l[0,1]}(t)) \right]
\end{align*}
\]

Using the ODE for \( V_{h[1,0]} \) and \( V_{h[0,0]} \):

\[
\Delta_{h[1,0]} = r \Delta_{h[1,0]} - [\delta_b + x_b - y - (\gamma_d + q_{bs} \phi) \omega_b - q_{ch} \phi \omega_c]
\]

Together with the ODE for \( V_{a[1,0]} \):

\[
\dot{\omega}_b = -x_b + (r + \gamma_d + q_{bs} \phi + q_{bb} (1 - \phi)) \omega_b + q_{cb} \phi \omega_c
\]

Analogously, we get the ODE for \( \omega_c \):

\[
\dot{\omega}_c = -x_{cl} + q_{bs} \phi \omega_b + (r + \gamma_d + \gamma_u + q_{cb} \phi + q_{cs} (1 - \phi)) \omega_c
\]

To solve for \( \omega_b \) and \( \omega_c \), we write (A.62) and (A.63) in this form:

\[
\begin{bmatrix}
\dot{\omega}_b(t) \\
\dot{\omega}_c(t)
\end{bmatrix} = -\begin{bmatrix} x_b \\ x_{cl} + x_{ch} - 2y \end{bmatrix} + A(t) \begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix},
\]

where

\[
A(t) = \begin{bmatrix} r + \gamma_d + q_{bs} \phi + q_{bb} (1 - \phi) \\ q_{bs} \phi \end{bmatrix} \\
\begin{bmatrix} q_{cb} \phi \\ r + \gamma_d + \gamma_u + q_{cb} \phi + q_{cs} (1 - \phi) \end{bmatrix}
\]

Thus, the solution is:

\[
\begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix} = \int_t^\infty e^{-\int_t^s A(u)du} \begin{bmatrix} x_b \\ x_{cl} + x_{ch} - 2y \end{bmatrix} ds
\]

\[45\]
From here, the solutions to the ODE for \( \Delta_{h[1,0]} \) and \( V_{a[1,0]} \) are given by:

\[
\Delta_{h[1,0]} = \frac{\delta_b + x_b - y}{r} - \int_t^{\infty} e^{-r(s-t)} \left( (\gamma_d + q_{bs}\phi) \omega_b + q_{cb}\phi \omega_c \right) ds
\]

\[
V_{a[1,0]} = \frac{\delta_b - y}{r} + \int_t^{\infty} e^{-r(s-t)} q_{bb} (1 - \phi) \omega_b ds
\]

\[\square\]

**Proof of Propositions 7 & 8** The characterization of the CDS price in an environment with and without frictions come as a corollary of Lemma 2 and are shown in the proof of Lemma 2. \[\square\]

## B Appendix: Covered CDS

Table 4: Measures of Investors in an Environment with Covered CDS

<table>
<thead>
<tr>
<th>Type</th>
<th>Flow-in = Flow-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{h[0,0]} )</td>
<td>( \rho F_h + \gamma_a \mu_{h[0,1]^{[0,-1]}} + q_{bb} \mu_{h[0,1]^{[1,-1]}} = \gamma_d \mu_{h[0,0]} + (q_{bs} + q_{cb}) \mu_{h[0,0]} )</td>
</tr>
<tr>
<td>( \mu_{l[0,0]} )</td>
<td>( \hat{F}<em>l = \gamma_a \mu</em>{l[0,0]} + q_{cs} \mu_{l[0,0]} )</td>
</tr>
<tr>
<td>( \mu_{h[1,0]} )</td>
<td>( q_{bs} \mu_{h[0,0]} = \gamma_d \mu_{h[1,0]} )</td>
</tr>
<tr>
<td>( \mu_{a[1,0]} )</td>
<td>( \gamma_d \mu_{h[1,0]} = (q_{bb} + q_{cs}) \mu_{a[1,0]} )</td>
</tr>
<tr>
<td>( \mu_{h[0,1]^{[0,-1]}} )</td>
<td>( \lambda_b \left( \mu_{l[0,0]} + \mu_{a[0,1]^{[0,-1]}} \right) \mu_{h[0,0]} = (\gamma_d + \gamma_a) \mu_{h[0,1]^{[0,-1]}} )</td>
</tr>
<tr>
<td>( \mu_{a[0,1]^{[0,-1]}} )</td>
<td>( \gamma_d \mu_{h[0,1]^{[0,-1]}} = (q_{cs} + \gamma_a) \mu_{a[0,1]^{[0,-1]}} )</td>
</tr>
<tr>
<td>( \mu_{h[0,1]^{[1,-1]}} )</td>
<td>( \lambda_c \left( \mu_{a[0,1]^{[1,-1]}}, \mu_{h[0,0]} \right) \mu_{h[0,1]^{[1,-1]}} = (\gamma_d + q_{bb}) \mu_{h[0,1]^{[1,-1]}} )</td>
</tr>
<tr>
<td>( \mu_{a[0,1]^{[1,-1]}}, \mu_{a[0,1]^{[1,-1]}} )</td>
<td>( q_{cs} \mu_{h[0,0]} = \gamma_a \mu_{a[0,1]^{[1,-1]}} )</td>
</tr>
<tr>
<td>( \mu_{a[1,-1]} )</td>
<td>( q_{cs} \mu_{a[1,0]} = q_{bb} \mu_{a[1,-1]} )</td>
</tr>
</tbody>
</table>

\[
\mu_{h[1,0]} + \mu_{a[1,0]} + \mu_{a[1,-1]} = S. \tag{B.1}
\]

\[
\mu_{h[0,1]^{[0,-1]}} + \mu_{a[0,1]^{[0,-1]}} + \mu_{h[0,1]^{[1,-1]}} + \mu_{a[0,1]^{[1,-1]}} = \mu_{a[1,-1]} + \mu_{l[0,-1]}. \tag{B.2}
\]

**Value Functions**

The flow benefit for holding covered CDS position can be seen in (B.5).

\[
r V_{h[0,0]} = \gamma_d (0 - V_{h[0,0]}) + \lambda_b \mu_{a[1,0]} (V_{h[1,0]} - p_b - V_{h[0,0]}) + \lambda_b \mu_{a[1,-1]} (V_{h[1,0]} - p_b - V_{h[0,0]}) - \lambda_c (\mu_{a[0,1]^{[1,-1]}}, \mu_{a[0,1]^{[1,-1]}}) (V_{h[0,1]^{[1,-1]}} - \hat{V}_{h[0,0]} - \lambda_c (\mu_{a[0,1]^{[1,-1]}}, \mu_{a[0,1]^{[1,-1]]}} + \mu_{l[0,0]}) (V_{h[0,1]^{[0,-1]}} - V_{h[0,0]}). \tag{B.3}
\]
\[
\begin{align*}
\text{rV}^{[0,0]}_t &= \gamma_u(0 - V^{[0,0]}_t) + q_{cs}(V^{[0,1]}_t - V^{[0,0]}_t) \\
\text{rV}^{[1,0]}_t &= \delta_b + x_b - y + \gamma_d(V^{[1,0]}_t - V^{[0,1]}_t) \\
\text{rV}^{[a,0]}_t &= \delta_b - y + q_{bb}(0 - V^{[a,1]}_t + p_b) + q_{cs}(V^{[a,1]}_t - V^{[a,0]}_t) \\
\text{rV}^{[a,1]}_t &= \delta_b - y + (\delta_c - p_{[1]} - y + y_{1,1} + q_{bb}(0 - V^{[a,1]}_t + p_{[1,1]}) \\
\text{rV}^{[b,0]}_t &= p_{[\text{cln}]} - (\delta_c - x_{ch}) - y + \gamma_d(V^{[b,0]}_t - V^{[b,1]}_t) + \gamma_u(V^{[b,0]}_t - V^{[b,1]}_t) \\
\text{rV}^{[b,1]}_t &= p_{[\text{cln}]} - (\delta_c - x_{ch}) - y + \gamma_d(V^{[b,0]}_t - V^{[b,1]}_t) + q_{bb}(0 - V^{[b,1]}_t + p_{[b,1]}) \\
\text{rV}^{[a,0]}_t &= p_{[\text{cln}]} - (\delta_c - x_{ch}) - y + q_{cs}(0 - V^{[a,0]}_t) + \gamma_u(0 - V^{[a,0]}_t) \\
\text{rV}^{[a,1]}_t &= -p_{[\text{cln}]} + (\delta_c + x_{cl} - y) + \gamma_u(0 - V^{[a,1]}_t)
\end{align*}
\]

C Appendix: A Micro Foundation for the Hedging Benefits

In this section, I derive in a CARA setting a micro foundation for the liquidity shocks \(x_b, x_c\), and the cost of risk bearing parameter \(y\). I simplify the notation by denoting the continuous time dependence \(y(t)\) as \(y_t\).

Agents have CARA utility preferences with risk aversion parameter \(\alpha\): \(u(c) = -e^{\beta_c}\) and time preference rate \(\beta\). The cash flow of the risky bond is given by a cumulative dividend process, \(D^b_t\):

\[
dD^b_t = \left(\delta^b dt + \sigma^b_d dB_t\right)
\]

where \(B_t\) is a standard Brownian motion, \(\delta^b > 0\), and \(\sigma^b_d > 0\) are constants. The cash flow of a CDS buyer is given by the process, \(D^D_t\):

\[
dD^D_t = (\delta^c dt - \sigma^c_d dB_t)
\]

Thus, \(D^b_t\) and \(D^D_t\) are perfectly negatively correlated.

Agents have an idiosyncratic cumulative endowment process:

\[
de_t = \sigma_e \left[\rho_d dB_t + \sqrt{1 - \rho^2_d} dZ_t\right]
\]

where \(\sigma_e > 0\) is a constant. \(Z_t\) is a standard Brownian motion independent of \(B_t\), and \(\rho_e\) is the instantaneous correlation process between the bond cash flow and agents’ endowment process. The correlation process \(\rho_t\) is a three-state Markov chain with states \(\rho_t \in \{\rho_l, 0, \rho_h\}\) where \(\rho_l > 0 > \rho_h\). If \(\rho_l = \rho_l\), the agent is currently a low-valuation investor and her endowment process is positively correlated with the bond’s cash flow, \(D^b_t\). If \(\rho_l = 0\), an agent’s endowment process has no correlation with the bond cash flow, and if \(\rho_l = \rho_h < 0\), an agent is a high-valuation type as her endowment process is negatively correlated with the bond cash flow (hence, she would be more willing to be exposed to the bond relative to a low- or an average-valuation investor). Analogous to the baseline model, a low-valuation agent switches to a high-valuation with intensity \(\gamma_u\), a high-valuation to a low-valuation with intensity \(\gamma_a\), and for an average-valuation agent the intensity of switching to either a high- or a low-valuation is zero.

We restrict the agent’s asset position in the bond market to \(\theta^b_t \in \{0, 1\}\) and in the CDS market to \(\theta^D_t \in \{-1, 0, 1\}\). The set of agent types \(\mathcal{T}\) is the same as in the baseline model.

An agent’s optimization problem is:

\[
J(W_0, \tau_0) = \max_{\{c_t\}} \mathbb{E} \left[ \int_0^\infty e^{-\beta t} u(c_t) dt \right]
\]

(C.3)
subject to: 

\[ dW_t = (rW_t - c_t) \, dt + \sigma_t \, dW_t + \sigma_t^b \, \theta_t^b - p_t \, dt \]

where \( W_t \) is the agent’s wealth process, \( W_0 \) is given, \( p_t \) is the bond price, and \( p_c \) is the CDS price.

**Deriving the Hamilton-Jacobi-Bellman (HJB) Equation**

Next, we derive the Hamilton-Jacobi-Bellman (HJB) equation. Equation (C.3) can be written recursively as

\[ J(W_t, \tau_t) = \max_{c_t} \left[ u(c_t) \Delta t + (1 - \beta \Delta t) \mathbb{E} J(W_{t+\Delta t}, \tau_{t+\Delta t}) \right]. \tag{C.4} \]

Subtract \( (1 - \beta \Delta t) J(W_t, \tau_t) \) from both sides and divide by \( \Delta t \):

\[ \beta J(W_t, \tau_t) = \max_{c_t} u(c_t) + (1 - \beta \Delta t) \mathbb{E} \left[ \frac{J(W_{t+\Delta t}, \tau_{t+\Delta t}) - J(W_t, \tau_t)}{\Delta t} \right]. \]

As \( \Delta t \to 0 \), it limits to

\[ \beta J(W_t, \tau_t) = \max_{c_t} u(c_t) + \mathbb{E} \left[ \frac{dJ(W_t, \tau_t)}{dt} \right]. \tag{C.5} \]

The next step is deriving the expectation of the total differential of \( J(W_t, \tau_t) \). Approximating the total differential \( dJ(W_t, \tau_t) \) by a Taylor-series expansion and taking its expectation, we get:

\[ \mathbb{E} dJ(W_t, \tau_t) = J_W(W_t, \tau_t) \mathbb{E} [dW_t] + \frac{1}{2} J_{WW}(W_t, \tau_t) \mathbb{E} [dW_t^2] + \mathbb{E} [J_{W\tau}(W_t, \tau_t) d\tau_t]. \]

Using the expressions for the bond and CDS cash flows and the endowment process, \( \mathbb{E} [dW_t] \) and \( \mathbb{E} [dW_t^2] \) are given by:

\[ \mathbb{E} [dW_t] = \left( rW_t - c_t + \delta^b \theta_t^b + (p_c - \delta^c) \theta_t^c \right) dt, \]

\[ \mathbb{E} [dW_t^2] = \left( \left( \sigma_D^b \theta_t^b + \sigma_D^c \theta_t^c \right)^2 + 2 \sigma_c \rho_t \left( \sigma_D^b \theta_t^b + \sigma_D^c \theta_t^c \right) + \sigma_c^2 \right) dt. \]

Substituting the above expressions for \( \mathbb{E} [dW_t] \) and \( \mathbb{E} [dW_t^2] \) back into \( \mathbb{E} dJ(W_t, \tau_t) \), we get

\[ \mathbb{E} dJ(W_t, \tau_t) = J_W(W_t, \tau_t) \left[ (rW_t - c_t + \delta^b \theta_t^b + (p_c - \delta^c) \theta_t^c) dt \right] + \mathbb{E} [J_{W\tau}(W_t, \tau_t) d\tau_t] \tag{C.6} \]

\[ + \frac{1}{2} J_{WW}(W_t, \tau_t) \left( \left( \sigma_D^b \theta_t^b + \sigma_D^c \theta_t^c \right)^2 + 2 \sigma_c \rho_t \left( \sigma_D^b \theta_t^b + \sigma_D^c \theta_t^c \right) + \sigma_c^2 \right) dt. \]

\(^{35}\)This comes from observing that over a small time interval \([0, \Delta t] \), (C.3) can be written as:

\[ J(W_0, \tau_0) = \mathbb{E} \int_0^\infty e^{-\beta t} u(c_t^*) dt = u(c_0^*) \Delta t + e^{-\beta \Delta t} \mathbb{E} \left[ \int_0^\infty e^{-\beta (t-\Delta t)} u(c_t^*) dt \right] \]

where \( \{c_t^*\} \) is the optimal consumption path. The term inside the expectations operation is \( J(W_{\Delta t}, \tau_{\Delta t}) \), thus \( J(W_0, \tau_0) = \max_{c_0} u(c_0) \Delta t + e^{-\beta \Delta t} \mathbb{E} J(W_{\Delta t}, \tau_{\Delta t}) \). Similarly if we start at \( \{W_t, \tau_t\} \) and approximate \( e^{-\beta \Delta t} \approx 1 - \beta \Delta t \), we get (C.4).

\(^{36}\)\( dJ(W_t, \tau_t) = J_W(W_t, \tau_t) dW_t + \frac{1}{2} J_{WW}(W_t, \tau_t) dW_t^2 + J_{W\tau}(W_t, \tau_t) d\tau_t + \frac{1}{2} J_{\tau\tau}(W_t, \tau_t) d\tau_t^2. \)
We will consider the steady state. Substituting (C.6) into (C.5), the Hamilton-Jacobi-Bellman (HJB) is given by

\[ \beta J(W, \tau) = \max_c u(c) + J_W(W, \tau) \left[ rW - c + \delta b + (p_c - \delta^c) \theta_c \right] \]

\[ + \frac{1}{2} J_{WW}(W, \tau) \left( \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right)^2 + 2 \sigma_c \rho \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right) + \sigma_c^2 \right) + \frac{E [J_r(W, \tau) d\tau]}{dt}. \]

Proposition 9. Let \( P(\tau, \tau') \) be the instantaneous payoff associated with a transition of a type \( \tau \) investor to type \( \tau' \) and let \( \gamma(\tau', \tau) \) denote the intensity of switching from type \( \tau \) to type \( \tau' \). The solutions to the HJB equations, \( J(W, \tau) \), are given by:

\[ J(W, \tau) = -e^{-\alpha(W_t + V_t + \bar{a})} \]  

where \( \bar{a} = \frac{1}{r} \left( \frac{\log(r)}{\alpha} - r \frac{\beta}{\alpha} - \frac{1}{2} r \alpha \sigma_e^2 \right) \). The constant \( V_t \) is given by

\[ rV_t = \left( \delta^b - x_b(\tau) \right) \theta_b - y(\tau) (\theta_b)^2 + (p_c - (\delta^c + x_c(\tau))) \theta_c - y(\tau) (\theta_c)^2 \]

\[ + \sum_{\tau \neq \tau'} \gamma(\tau', \tau) \frac{1}{r\alpha} \left( 1 - e^{-\alpha(W_t - V_t + P(\tau', \tau))} \right) \]

where \( x_b(\tau) = r \alpha \rho \sigma_e \sigma_D^b, x_c(\tau) = r \alpha \rho \sigma_e \sigma_D^c, \) and \( y(\tau) = \frac{r}{2} (\sigma(\tau))^2 \).

Proof. Using the guessed functional form, \( J(W, \tau) = -e^{-\alpha(W_t + V_t + \bar{a})} \), and the first order condition of (C.7), we can solve for the optimal consumption rate for agent \( \tau \) \[37\]

\[ c_\tau = -\frac{\log(r)}{\alpha} + r (W_t + V_t + \bar{a}) \]

Inserting the optimal consumption back into (C.7) and using \( J(W, \tau) = -e^{-\alpha(W_t + V_t + \bar{a})} \), \( J_W = r \alpha e^{-\alpha(W_t + V_t + \bar{a})} \) and \( J_{WW} = -r^2 \alpha^2 e^{-\alpha(W_t + V_t + \bar{a})} \), we get:

\[ -\beta e^{-\alpha(W_t + V_t + \bar{a})} = -e^{\log(r) - \alpha(W_t + V_t + \bar{a})} + r \alpha e^{-\alpha(W_t + V_t + \bar{a})} \left( \frac{\log(r)}{\alpha} - r (V_t + \bar{a}) + \delta^b \theta_b + (p_c - \delta^c) \theta_c \right) \]

\[ - \frac{1}{2} r^2 \alpha^2 e^{-\alpha(W_t + V_t + \bar{a})} \left( \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right)^2 + 2 \sigma_c \rho \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right) + \sigma_c^2 \right) \]

\[ + \frac{E [J_r(W, \tau) d\tau]}{dt}. \]

Dividing both sides by \( -\frac{1}{r^2} e^{-\alpha(W_t + V_t + \bar{a})} \) and rearranging, we get

\[ 0 = rV_t - e^{\alpha(W_t + V_t + \bar{a})} \frac{E [J_r(W, \tau) d\tau]}{r \alpha dt} + r \bar{a} - r \left[ \frac{\log(r)}{\alpha} - \frac{r - \beta}{r \alpha} - \frac{1}{2} r \alpha \sigma_e^2 \right] \]

\[ - \left( \frac{\sigma_D^b \theta_b}{2} \right)^2 \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right)^2 + 2 \sigma_c \rho \left( \sigma_D^b \theta_b + \sigma_D^c \theta_c \right) + (p_c - \delta^c) \theta_c \).

\[37\]The F.O.C. with respect to \( c_t \) is: \( 0 = \alpha e^{-ac} - J_W(W_t, \tau_t) \). Using \( J_W = r \alpha e^{-\alpha(W_t + V_t + \bar{a})} \), \( re^{-\alpha(W_t + V_t + \bar{a})} = e^{-ac} \). Rewrite it as: \( e^{\log(r)} e^{-\alpha(W_t + V_t + \bar{a})} = e^{-ac} \).
Defining $\bar{a} = \frac{1}{\tau} \left( \frac{\log(r)}{\alpha} - \frac{r - \beta}{\alpha} - \frac{1}{2} r \alpha \sigma^2 \right)$ and using $\theta^b \theta^c = 0$,

$$ rV_{\tau} = \left( \delta^b - r \alpha \sigma \rho \sigma_D^2 \right) \theta_b - \frac{1}{2} r \alpha \left( \sigma_D^2 \theta^b + (\sigma_D^2)^2 \theta^c \right) + (p_c - (\delta^c + r \alpha \sigma \rho \sigma_D)) \theta_c $$

$$ + e^{r \alpha (W + V_{\tau})} \frac{E [J_{\tau}(W, \tau) d\tau]}{r \alpha dt}. $$

Define $x_b(\tau) = r \alpha \sigma \rho \sigma_D^2$, $x_c(\tau) = r \alpha \sigma \rho \sigma_D^2$, $y(\tau) = \frac{r \alpha \sigma}{2} \sigma^2$, and $\sigma(\tau) = \sigma_D^2 \theta^b (\theta_b(\tau))^2 + \sigma_D^2 (\theta_c(\tau))^2$. Given these definitions of $x_b(\tau)$, $x_c(\tau)$, and $y(\tau)$,

$$ rV_{\tau} = \left( \delta^b - x_b(\tau) \right) \theta_b - y(\theta_b)^2 + (p_c - (\delta^c + x_c(\tau))) \theta_c - y(\theta_c)^2 + e^{r \alpha (W + V_{\tau} + a)} \frac{1}{r \alpha} \frac{E [J_{\tau}(W, \tau) d\tau]}{dt}. $$

Consider, for example, how (C.9) looks for $\tau = h[0, 0]$ type. $EJ_{\tau}(W, \tau) d\tau$ for $\tau = h[0, 0]$ is:

$$ \frac{EJ_{\tau}(W, \tau) d\tau}{dt} = \gamma d dt \left( J(W, \infty) - J(W,h[0,0]) \right) + q_{bs} dt \left( J(W - p_b \left( \theta^b - \theta^b \right), h[1,0]) - J(W, h[0,0]) \right) $$

Using $J(W, \tau) = -e^{-r \alpha (W + V_{\tau} + a)}$ and multiplying both sides by $e^{r \alpha (W + V_{\tau} + a)}$, we get

$$ e^{r \alpha (W + V_{\tau} + a)} \frac{E [J_{\tau}(W, \tau) d\tau]}{dt} = \gamma d \left( 1 - e^{-r \alpha (V_{\infty} - V_{\tau})} \right) + q_{bs} \left( 1 - e^{-r \alpha (-p_b + V_{h[1,0]} - V_{\tau})} \right) + q_{cb} \left( 1 - e^{-r \alpha (V_{h[0,1]} - V_{\tau})} \right). $$

Thus, using the fact that $\theta_c = \theta_b = 0$ for $\tau = h[0, 0]$ type, (C.9) for $\tau = h[0, 0]$ type is given by

$$ rV_{\tau} = \gamma d \left( 1 - e^{-r \alpha (V_{\infty} - V_{\tau})} \right) + q_{bs} \left( 1 - e^{-r \alpha (-p_b + V_{h[1,0]} - V_{\tau})} \right) + q_{cb} \left( 1 - e^{-r \alpha (V_{h[0,1]} - V_{\tau})} \right). $$

It is analogous for the other types. Table 5 gives all the possible switching intensities $\gamma(\tau', \tau)$ from $\tau \in \mathcal{T}$ to $\tau' \in \mathcal{T}$.

<table>
<thead>
<tr>
<th>$\tau'$</th>
<th>$h[0, 0]$</th>
<th>$l[0, 0]$</th>
<th>$h[1, 0]$</th>
<th>$a[1, 0]$</th>
<th>$h[0, 1]$</th>
<th>$a[0, 1]$</th>
<th>$l[0, -1]$</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h[0, 0]$</td>
<td>0</td>
<td>0</td>
<td>$q_{bs}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_d$</td>
</tr>
<tr>
<td>$l[0, 0]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$q_{cs}$</td>
<td>$\gamma_d$</td>
</tr>
<tr>
<td>$h[1, 0]$</td>
<td>0</td>
<td>0</td>
<td>$\gamma_d$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_u$</td>
</tr>
<tr>
<td>$a[1, 0]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$q_{cb}$</td>
<td>$\gamma_u$</td>
</tr>
<tr>
<td>$h[0, 1]$</td>
<td>$\gamma_u$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_u$</td>
<td>0</td>
<td>0</td>
<td>$q_{cs}$</td>
</tr>
<tr>
<td>$a[0, 1]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_u$</td>
<td>$q_{cs}$</td>
<td>$\gamma_u$</td>
</tr>
<tr>
<td>$l[0, -1]$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_u$</td>
<td>$\gamma_u$</td>
</tr>
</tbody>
</table>

Table 5: Switching intensities from $\tau$ to $\tau'$, $\gamma(\tau', \tau)$

The instantaneous payoff, mentioned in Proposition, is plus or minus the bond price depending on whether it is a sell or a buy position. The instantaneous payoff for a CDS transaction is zero as the CDS premium is paid continually and is hence imbedded in $V_{\tau}$. Thus, $P(\tau, \tau')$ is:

$$ P(\tau, \tau') = \begin{cases} 
-p_b & \text{if } \tau = h[0,0], \tau' = h[1,0] \\
p_b & \text{if } \tau = a[1,0], \tau' = h[1,0] \\
0 & \text{otherwise.}
\end{cases} $$

50
Comparison to the Baseline Model

In the limit as $\alpha \to 0$, the general value function \((\text{C.10})\) satisfies the value functions with risk-neutral agents of the baseline model of in the text of the paper. To see this, linearizing \((\text{C.10})\) using $e^{z} - 1 \approx z$ for small $\alpha$, we get:

$$r V_{h[0,0]} = \gamma d (0 - V_{h[0,0]}) + q_{bs} (-p_{b} + V_{h[1,0]} - V_{h[0,0]}) + q_{cb} (V_{h[0,1]} - V_{h[0,0]})$$  \hspace{1cm} (C.11)

Equation \((\text{C.11})\) is analogous to the value functions of the baseline model with risk-neutral agents.

The baseline model is essentially a reduced form approximation of a more general specification with risk-averse agents and risky assets. The hedging benefits $x_{b}(\tau) = r \alpha \rho_{h} \sigma_{b} \sigma_{c}^{2}$ and $x_{c}(\tau) = r \alpha \rho_{l} \sigma_{e} \sigma_{c}^{2}$ increase with agents’ risk aversion ($\alpha$), bond cash flow risk ($\sigma_{b}$), endowment volatility ($\sigma_{e}$), and the correlation between the bond cash flow and the endowment process, $\rho_{h}$. The parameter $x_{ch}$ from the baseline model is equivalent to $x_{ch} = -x_{c}(\tau) = -r \alpha \rho_{h} \sigma_{e} \sigma_{c}^{2}$ if $\tau$ is a high-valuation investor, while $x_{cl} = r \alpha \rho_{l} \sigma_{e} \sigma_{c}^{2}$ for low-valuation investor. Under a special case where $\sigma_{b}^{2} = \sigma_{D}^{2} = \sigma_{D}$, $x_{b}(\tau) = x_{c}(\tau)$ and $y(\tau) = \frac{r_{c}}{2} (\sigma_{b}^{2})^2 = \frac{r_{c}}{2} (\sigma_{D}^{2})^2$ for all $\tau$. In this case, $x_{b} = x_{ch}$, while $x_{cl}$ is not necessarily equal to $x_{ch}$. If, in addition, $-\rho_{h} = \rho_{l}$, then $x_{b} = x_{ch}$.

D Appendix: Model Figures

Figure 2: The Effect of Naked CDS Purchases on Bond Market Liquidity
The figures compare bond market liquidity (in terms of illiquidity discount ($d_{b}$), the bid-ask spread ($\omega_{b}$), and trading volume ($M_{b}$)) with CDS (solid lines) versus without CDS (dashed lines) as a function of the CDS market efficiency ($\lambda_{c}$). Although it cannot be directly seen from the plots, the variables converge to the no-CDS environment as the CDS market frictions decrease ($\lambda_{c} \to \infty$). The parameter values used to generate the plot are in Table [??].
Figure 3: The Effect of Naked CDS Purchases on Bond Market Composition

The plots compare the composition of buyers and sellers in the bond market with (solid lines) versus without CDS (dashed lines) as a function of the CDS market efficiency ($\lambda_c$). Although it cannot be directly seen from the plots, the variables converge to the no-CDS environment as the CDS market frictions decrease ($\lambda_c \to \infty$). The parameter values used to generate the plot are in Table 6.

Figure 4: The Short-run Dynamics of Types’ Measures After a Temporary CDS Ban

The figures plot the time varying equilibrium path of the number of naked CDS buyers (the left panel), bond buyers (the middle panel), and bond sellers (the right panel) from a temporary ban back to the steady state. A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero (as can be seen in the left panel). The parameter values used to generate the plot are in Table 6.

Figure 5: Bond Market Liquidity Short-run Dynamics after a Temporary CDS Ban

The figure plots the short-run dynamics of the illiquidity discount, the bid-ask spread, and trading volume from a temporary CDS ban back to the steady state. A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero. The parameter values used to generate the plot are in Table 6.
The left panel illustrates an example of a cost of entry function as discussed in Section 2.2. The right panel shows the implicit short-run dynamics of the cost of entry $c(\rho(t))$.

![Figure 6: Cost of Entry](image)

The plots compare the marginal effects of allowing naked (in solid bold) versus covered CDS purchases (in thin dashed) on bond market liquidity. They also show the marginal effect of allowing both types of purchases (in thin solid). Parameter values used to generate the plot are in Table 6.

![Figure 7: The Effects of Covered and Naked CDS on Bond Market Liquidity.](image)

The plots show the marginal effect of allowing covered CDS purchases on bond market liquidity when the entry rate of high-valuation investors is fixed. Parameter values used to generate the plot are in Table 6.

![Figure 8: The Effect of Covered CDS when Entry is Fixed](image)
E Appendix: Data Figures

Figure 9: Permanent Naked CDS Ban and CDS Purchased, Jan 2011 - Aug 2012
The solid line plots the total CDS purchased (CDS net notional, $bln) referencing governments subject to the EU ban (that is, EU governments). The dashed line plots the total CDS purchased referencing governments not affected by the ban (that is, non-EU governments). The vertical line is drawn on October 18, 2011 when the EU passed the naked CDS ban legislation.

Figure 10: Permanent Naked CDS Ban and Bond Bid-Ask Spreads, Jan 2011 - Aug 2012
The solid line plots the cross-country average bond bid-ask spread (% of the mid price) across the countries subject to the ban (EU countries). The dashed line plots the average bond bid-ask spread across countries that were not affected by the ban (outside the EU). The vertical line is drawn on October 18, 2011 when the EU passed the naked CDS ban legislation.
Figure 11: Temporary Naked CDS Ban and Bond Bid-Ask Spreads, Mar 2010 - Aug 2010
The solid line plots the cross-country average bond bid-ask spread (% of the mid price) across countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the average bond bid-ask across countries not affected by the ban (i.e. naked CDS referencing these countries could still be purchased) but countries still within the EU. The vertical lines are drawn the week before and after the German ban is instituted.

Figure 12: Temporary Naked CDS Ban and CDS Purchased, Mar 2010 - Aug 2010
The solid line plots the time series of the total CDS net notional ($billion) across EU countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the total for EU countries that were not affected by the ban (i.e. naked CDS referencing these countries could still be purchased). The vertical lines are drawn the week before and after the German ban is instituted.
Appendix: Tables

Table 6: Calibration Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond coupon blow</td>
<td>( \delta_b )</td>
<td>0.01</td>
</tr>
<tr>
<td>Flow value of insurance protection</td>
<td>( \delta_c )</td>
<td>0.01</td>
</tr>
<tr>
<td>Hedging benefit of high types through bonds</td>
<td>( x_b )</td>
<td>0.078</td>
</tr>
<tr>
<td>Hedging benefit of high types through CDS</td>
<td>( x_{ch} )</td>
<td>0.074</td>
</tr>
<tr>
<td>Hedging benefit of low types through CDS</td>
<td>( x_{cl} )</td>
<td>0.079</td>
</tr>
<tr>
<td>Cost of risk bearing</td>
<td>( y )</td>
<td>0.06</td>
</tr>
<tr>
<td>Exogenous flow of high types</td>
<td>( F_h )</td>
<td>0.854</td>
</tr>
<tr>
<td>Exogenous flow of low types</td>
<td>( F_l )</td>
<td>0.035</td>
</tr>
<tr>
<td>Switching intensity of low types</td>
<td>( \gamma_u )</td>
<td>0.1</td>
</tr>
<tr>
<td>Switching intensity of high types</td>
<td>( \gamma_d )</td>
<td>0.5</td>
</tr>
<tr>
<td>Matching efficiency in the bond market</td>
<td>( \lambda_b )</td>
<td>550</td>
</tr>
<tr>
<td>Matching efficiency in the CDS market</td>
<td>( \lambda_c )</td>
<td>120</td>
</tr>
<tr>
<td>Supply of bonds</td>
<td>( s )</td>
<td>1</td>
</tr>
<tr>
<td>Bargaining power of buyers and sellers</td>
<td>( \phi )</td>
<td>0.5</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>( r )</td>
<td>0.04</td>
</tr>
<tr>
<td>Value of the outside option</td>
<td>( O_h )</td>
<td>0.0361</td>
</tr>
</tbody>
</table>

Table 7: Calibration Results: Volume of Trade and Search Times

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>No CDS</th>
<th>Naked</th>
<th>Covered</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond volume (% of debt)</td>
<td>( M_b/S )</td>
<td>49.1</td>
<td>49.2</td>
<td>49.1</td>
<td>49.2</td>
</tr>
<tr>
<td>CDS volume (% of notional)</td>
<td>( M_c/(\mu_{0,-1} + \mu_{a1,-1}) )</td>
<td>56.</td>
<td>2760.</td>
<td>80.9</td>
<td></td>
</tr>
<tr>
<td>CDS notional (% of debt)</td>
<td>( (\mu_{0,-1} + \mu_{a1,-1})/S )</td>
<td>34.5</td>
<td>0.319</td>
<td>34.8</td>
<td></td>
</tr>
<tr>
<td>Days to buy a bond</td>
<td>250/(\lambda_{b}\mu_{bs})</td>
<td>25.9</td>
<td>29.4</td>
<td>25.5</td>
<td>29.</td>
</tr>
<tr>
<td>Days to sell a bond</td>
<td>250/(\lambda_{b}\mu_{bb})</td>
<td>8.94</td>
<td>7.86</td>
<td>9.08</td>
<td>7.96</td>
</tr>
<tr>
<td>Days to buy CDS</td>
<td>250/(\lambda_{c}\mu_{cs})</td>
<td>36.</td>
<td>142.</td>
<td>50.7</td>
<td></td>
</tr>
<tr>
<td>Days to sell CDS</td>
<td>250/(\lambda_{c}\mu_{cb})</td>
<td>74.8</td>
<td>41.6</td>
<td>36.5</td>
<td></td>
</tr>
<tr>
<td>Entry rate</td>
<td>( \rho )</td>
<td>0.605</td>
<td>0.799</td>
<td>0.606</td>
<td>0.8</td>
</tr>
<tr>
<td>Naked CDS (% of total)</td>
<td>( \mu_{0,-1}/(\mu_{0,-1} + \mu_{a1,-1}) )</td>
<td>99.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Calibration Results: Prices and Bid-Ask Spreads

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>No CDS</th>
<th>Naked</th>
<th>Covered</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond bid-ask (%)</td>
<td>( \omega_b/p_b )</td>
<td>2.51</td>
<td>2.14</td>
<td>2.33</td>
<td>1.98</td>
</tr>
<tr>
<td>Expected bond return (%)</td>
<td>( \delta_b/p_b )</td>
<td>6.23</td>
<td>6.00</td>
<td>6.15</td>
<td>5.93</td>
</tr>
<tr>
<td>CDS bid-ask (%)</td>
<td>( \omega_c/p_c )</td>
<td>13.03</td>
<td>3.32</td>
<td>13.01</td>
<td></td>
</tr>
<tr>
<td>CDS price (%)</td>
<td>( p_c )</td>
<td>1.98</td>
<td>3.01</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

56
Table 9: Anecdotal Evidence of How the EU Ban Affected the Bond Market


<table>
<thead>
<tr>
<th>Source</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>The German Banking Industry Committee:</td>
<td>“The market has become less liquid; the bid-offer spread has widened. Volatility is unchanged, but has tended to shift to the spot/cash markets.”</td>
</tr>
</tbody>
</table>
| The Association for Financial Markets in Europe (AFME) and the International Swaps and Derivatives Association (ISDA): | “Market participants have already observed that seemingly due to the SSR Regulation (restrictions it imposed on sovereign debt and sovereign CDS markets), Asian participation in the European bond market fell by around 50% immediately after the introduction of the SSR, thus demonstrating neatly one adverse impact of the SSR in general in driving investors away.”
“Some buy side market participants have already remarked that even though there is still liquidity in sovereign debt, it is more difficult to source this liquidity.” |
| Alternative Investment Management Association (AIMA) and Managed Funds Association (MFA): | “Some of our members have reported that they have stopped trading European sovereign CDS and bonds, given the regulatory and reputational risks.”
“Restrictions on CDS positions over the medium term will generally make it more difficult for sovereign issuers to borrow through long-dated securities, leading to a shortening of the average maturity profile of sovereign issuance as investors seek to limit their risk exposure, thereby increasing the vulnerability of sovereigns to short term liquidity and funding crises. This sentiment is reflected in the responses to AIMA and MFA’s poll of their members.”
“At worst, the ban could ultimately undermine liquidity in the underlying sovereign debt markets, undermining the ability of sovereigns to raise finance through debt issuance.” |
| Deutsche Bank | “We observed anecdotally that as investors began to understand the details of the regulation, cash volumes reduced with a resultant increase in volatility, although this was not significant.” |
References


Augustin, Patrick, 2014, Sovereign credit default swap premia, *Journal of Investment Management (Forthcoming)*.


Delatte, Anne Laure, Mathieu Gex, and Antonia López-Villavicencio, 2011, Has the CDS market influenced the borrowing cost of European countries during the sovereign crisis?, *Working Paper*.


ISDA, 2014, Adverse liquidity effects of the EU uncovered sovereign CDS ban.


Oehmke, Martin, and Adam Zawadowski, 2013, Synthetic or real? The equilibrium effects of credit default swaps on bond markets, Working Paper.


Tang, Dragon Yongjun, and Hong Yan, 2007, Liquidity and credit default swap spreads, Working Paper.

Thompson, James R, 2007, Credit risk transfer: To sell or to insure, Queen's University working paper pp. 1195–1252.

