

# Risky Utilities

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## Abstract

We develop a theory of “risky utilities”, i.e. private firms that manage an infrastructure for public service, and that may be tempted to engage in excessively risky activities, such as reducing maintenance expenditures (at the risk of provoking a break-down of the system) or in speculation (at the risk of incurring massive losses it cannot bear). These risky utilities include financial utilities like exchanges, clearinghouses or payment systems, as well as standard utilities like electricity transmission networks. Continuation of service is essential, so risky utilities cannot be liquidated. The optimal regulatory contract minimizes the social cost among the contracts that steer the firm away from risky activities. It is simple and implemented with a capital (equity) adequacy requirement and a resolution mechanism when that requirement is breached. The social cost function is explicitly computed and comparative statics can be simply derived.

**Keywords:** moral hazard, dynamic contract, speculation, capital requirements. JEL Classification: D82, D86, G28, L43.

## 1 Introduction

Utilities are private firms that maintain infrastructure for a public service. Classical examples are distribution networks for electricity, natural gas or water, or generators. Utilities are traditionally considered “safe” in several dimensions. First they are often monitored by public authorities, who are supposed to ensure that the physical infrastructure is well maintained. Second, utilities often

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benefit from the implicit guarantee of the government should they encounter financial difficulties. Third, they are typically regarded as a safe investment. For example the US electricity company Con-Edison is renowned for steadily increasing its dividend over the last 40 years.<sup>1</sup>

Several scandals have altered this perception. The California rolling blackouts of 2000-2001 showed that utilities can fail in spectacular ways. Not only can they go bankrupt, as in the case of Pacific Gas and Electric, but power can altogether stop flowing to users. This crisis is said to have cost \$40 to \$45 billion to consumers and businesses in the form of higher power prices (Weare, 2003). This does not include deadweight losses, nor social costs incurred because of electricity disruptions and blackouts, nor the bankruptcy of some distributors and retailers. At the same time Enron's downfall exposed speculative activities at the source of both its failure and the California crisis. These events are not unique: the 2003 American Northeast blackout affected an estimated 10 million people in Ontario and 45 million in eight US states. Its origin is attributed to a lack of pruning of trees, which interfered with the transmission lines. In 2009, a power line fell to the ground in Kilmore East (Victoria, Australia) and started a fire that killed 119 people. The ensuing settlement amounted to AUD 500 million, of which 400 million had to be paid by the State of Victoria because the liability of the private operator had been capped. The transmission line fell because of a faulty conductor that was lacking a protective cap costing \$10. Simultaneously, the Horsham (Victoria) fire started also because of a fallen power line, the screws of which had become loose thanks to inadequate maintenance. The class action was settled for AUD 40 million.

Moreover the Global Financial Crisis (GFC) of 2007-09 has renewed interest in the notion of "utility banking", i.e. banking activities (such as deposit taking, management of payments and loans to small businesses) that are viewed as essential to the economy.<sup>2</sup> This notion has led to several proposals by Volcker in the US ("How to Reform Our Financial System". The New York Times, January 30, 2010 – now part of the Dodd-Frank Act), Vickers in the UK (2011) and Liikanen

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<sup>1</sup>Con Edison investor relations: <http://investor.conedison.com/phoenix.zhtml?c=61493&p=irol-dividends>.

<sup>2</sup>Goodhart (2013) even includes investment banking in these utility activities. He writes: "The provision of access to financial markets, which is what investment banks do, though primarily for large clients, is as much a utility as the provision of retail services to smaller customers". In his Guardian article "*Taming the financial casino. We need to restore narrow banking – to ensure that risky bets cannot again jeopardize the utility*", of March 24, 2009, John Kay claimed: "*We attached a casino – proprietary trading activity by banks – to a utility – the payment system, together with the deposits and lending that are essential to the day-to-day functioning of the non-financial economy.*"

in the EU (2012) for (i) separating or at least ring-fencing the “utility” activities of banks from speculative activities such as proprietary trading and (ii) designate some institutions as systemically significant and thus subject to enhanced regulatory scrutiny and possibly to drastic intervention.<sup>3</sup>

An important consequence of the GFC was the adoption of special regulations for financial institutions whose interruption of service would entail important social costs. The Dodd-Frank Act introduced the new notion of “Financial Utility”: financial infrastructures that are vital for the US economy, such as securities or derivative exchanges, large-value payment systems and clearinghouses.<sup>4</sup> According to Paul Tucker (Deputy Governor, Bank of England) the consequence of the failure of such an institution is “mayhem”, as he witnessed in 1987.<sup>5</sup> The Hong Kong Futures Exchange clearinghouse failed as a consequence of the stock crash of 1987. It resulted in Hong Kong’s futures market and its stock market closing also for a time; they re-opened 45.5% lower. Such a fall does not just affect sophisticated securities market participants. It erodes the savings and retirements accounts of large fractions of the population, and thereby affects their consumption and retirement decisions. It also affects the value of other assets, potentially used as collateral (e.g. housing) and typically engenders disruptive liquidity crises. The Hong Kong failure owed to the pursuit of trading volumes (generating fee income) at the expense of the creditworthiness of the participants. Tucker argues “this episode warrants more study than it has received.”<sup>6</sup> The recent push to centrally clear derivative adds to that impetus (Tucker, 2014).

This article proposes a theory of the regulation of these “Risky Utilities”. Most utilities are already subject to regulation, but its object is mostly to curb their market power. A vast academic literature has studied this form of utility regulation starting with Hotelling (1938), Dupuit (1952) and expanded since by Baron and Myerson (1982), Sappington (1983) and Laffont and Tirole (1986) (see also Laffont and Tirole, 1993). Instead we lay the emphasis on the problem of risk management and continuation of service that is essential to the economy.

Our model is general enough to address both the maintenance problems of traditional utilities and the speculation problems of financial utilities. We label a risky utility any private company

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<sup>3</sup>Goodhart (2013) analyzes in detail why this ring-fencing may be difficult to implement. We will not explore this direction here.

<sup>4</sup>Systemically important financial market utilities (SIFMU) are entities whose failure or disruption could threaten the stability of the US financial system. As of September 2014 eight entities in the U.S. have been designated SIFMUs.

<sup>5</sup>Financial Times, 16 April 2012.

<sup>6</sup>FT.com, 2 June 2011.

that manages an infrastructure for public service, and that may be tempted to reduce maintenance expenditures (at the risk of provoking a break-down) or to engage in speculative activities (at the risk of incurring massive losses it cannot bear). There is already a large literature on the regulation of To Big To Fail banks and Systemically Important Financial Institutions. We focus on the “pure” utility problem and capture the notion of a risky utility by three simple features:

- the company can secretly engage in risky activities (lack of maintenance or speculation) that increase profit but may provoke huge losses (a catastrophe);
- shut-down would exert large negative externalities, so the firm cannot be liquidated and public authorities must intervene following the catastrophe;<sup>7</sup>
- operating profits are stationary so that we abstract from the questions of size and investment policy. This fits a public exchange, a clearinghouse, an electricity transmission company or a generator (after construction).

Public authorities have the power to regulate the company *ex ante* and restructure it *ex post*, should a catastrophe occur. The object of the article is to determine the best regulation contract. To this end we develop a model of risk-taking under moral hazard in continuous time that is tractable enough to allow for a quasi-explicit solution. Comparative statics are then easy to derive.

A regulated firm (agent) can engage in two types of socially wasteful activities: cash-flow diversion and risky activities (speculation) that improve short term profitability but may trigger large losses governed by a Poisson process. The firm is protected by limited liability. An incentive-compatible regulation contract deters both, and the optimal contract minimizes the social cost of regulation among incentive-compatible contracts. This contract is very simple: it is a termination rule associated with restructuring, that is, expropriation of the firm’s owners (with compensation) and on sale to new investors. That intervention is triggered when the value of the firm falls below a threshold; this is interpreted as, and implemented with, a minimal capital requirement. This threshold corresponds to the lowest continuation values that deters speculation. Equity here does not just absorb losses, it also guarantees the firm has enough to lose to not speculate.

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<sup>7</sup>A SIFMU is not subject to bankruptcy law and so cannot be liquidated; instead it is to be placed under receivership under the administration of the FDIC.

We also show there is a connection between cash flow diversion and speculation through the incentive contract, although these activities are independent. The more efficient is diversion the more attractive is the instantaneous return on speculation, and so the more difficult it is to deter. Deterring speculation thus requires a higher equity threshold; so more efficient cash flow diversion induces a higher social cost (because of the option to speculate).

Our paper is related to four strands of the literature. First, we use the continuous time contracting techniques as developed by DeMarzo and Sannikov (2006) and Sannikov (2008). They are particularly appropriate in our context: the decision to speculate can be altered at any point in time, restructuring naturally corresponds to a stopping time and a large loss can arise at any moment with minute probabilities. The model can be viewed as an extension of DeMarzo and Sannikov (2006) to speculative activities, as in DeMarzo, Livdan and Tschisty (2013). To guarantee existence of an optimal contract these papers assume that the shareholder (agent) is less patient than the regulator (principal). Instead we let the agent be subject to an exogenous liquidation shock: with probability  $\delta$  she must sell-off. This bounds her payoffs and allows us to work with the total (private) surplus function. We depart from DeMarzo et al (2013) in other ways: (i) a contract cannot be conditioned on an exogenous observable event (a crisis), so “relative performance” evaluation is not possible; and (ii) we do not rely on public randomization for termination because of the very large losses. Second, we connect to the regulation of equity capital. Our optimal contract is implemented with a minimal equity requirement imposed on the firm, the purpose of which is to ensure the shareholder has enough at stake not to engage in excessive risk-taking. VanHoose (2007) provides a survey that suggests a persistent lack of consensus as to the role and benefit of capital requirements in banking. Furlong and Keeley (1989) establish that asset risk decreases when the capitalization of a bank increases. Milne (2002) observes that a bank’s portfolio choice depends on its capitalization. Our model accords well with both; capital requirements induce the institution to choose the less risky path because breaching the capital requirement triggers restructuring and expropriation. Morrison and White (2005) propose a model of adverse selection and moral hazard in which capital requirements are also used to solve the moral hazard problem and to screen out bad banks (or bankers). In Diamond and Rajan (2000) bank capital acts as a buffer against credit losses and thus curtails bank runs by providing depositors with a measure of insurance. Similarly in Tian, Yang and Zhang (2013) capital buffers protect the banks against contagion arising from

credit losses. That capital may be long-term debt; not so in our model, where it has to be equity to deter speculation.

Third, the paper is related to the literature on financial structure and risk taking, as in Biais and Casamatta (1999). They model an agency problem with two actions however (i) it is static and (ii) the goal is to determine the optimal financial structure of the firm; there are no externalities. As in our paper, equity is necessary to overcome the risk-taking (risk-shifting) problem. Last, we connect to a more recent literature on interventions and bailouts. Zentefis (2014) shows the nature of the rescue matters: if the institution is burdened by excessively large repayments ex post (as a debtor, for example) it has incentives to default. In our model there is no default but early intervention that is final. Panageas (2010) and Kacperczyk and Schnabl (2013) show that an implicit guarantee, or more capital (respectively), enhances return volatility. This is empirically echoed in Mariathasan, Merrouche and Werger (2014). Volatility is fixed in our model – except for the Poisson jump; more importantly, the agent is protected by limited liability and so would *always* like more risk. To deter her, our resolution mechanism expropriates preemptively.

Section 2 presents the model. Section 3 characterizes incentive compatible regulation contracts and suggests an intuitive implementation. Section 4 studies the social cost function in details. We present a discussion in Section 5 and then conclude. All proofs are relegated to the Appendix.

## 2 Model

We adapt the model of DeMarzo, Livdan and Tschisty (2013) to the case of an infrastructure providing a public service that must be continued in all circumstances. The government auctions off the right to operate that infrastructure among a pool of potential investors/managers who have limited wealth  $\omega$ .<sup>8</sup> Operating cash flows follow the process

$$dx_t = \mu dt + \sigma dZ_t \tag{2.1}$$

where  $\mu > 0$ ,  $Z \equiv \{Z_t, \mathcal{F}_t; 0 < t < \infty\}$  is a standard Brownian motion associated with a filtration  $\mathcal{F}_t$  on a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ . At any point in time the infrastructure is operated by a particular investor/manager: the shareholder. There are also passive investors who can participate

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<sup>8</sup>Either their wealth is exogenously limited to  $\omega$  or there are competitive markets for investment and  $\omega$  is the fraction they devote this opportunity.

in the financing of the infrastructure. All agents are risk-neutral and discount future payments at rate  $r > 0$ .

A regulation contract  $\Xi$  specifies the flow of payment (dividends)  $dL_t$  to the shareholder, as well as the termination rule represented by a stopping time  $\tau$ . At date  $\tau$  the firm is restructured at cost  $\gamma$ : the incumbent shareholder receives a payment  $w_\tau$  and the firm is sold to a new shareholder. Since the environment is stationary the terms of the new regulation contract  $\Xi = (L, \tau, w_\tau)$  remain the same. The objective of the government is to minimize the expected present value of the public funds that need to be expended in order to guarantee the continuity of the service provided by the infrastructure.

There are two sources of frictions. First, in the spirit of DeMarzo and Sannikov (2006) the operating cash flow at any moment  $t$  can be diverted by the shareholder: a dollar diverted brings  $\eta \leq 1$  dollars to the shareholder. Second, the shareholder can secretly engage in excessively risky (“speculative”) activities that generate an additional cash-flow  $\Delta\mu$  per unit of time but expose the firm to catastrophic losses  $K$  that wipe it out.<sup>9</sup> For example, the firm sells (but does not buy) CDS or issues options. Or an electricity network may save  $\Delta\mu$  on its maintenance, and thereby expose itself to network failure. Such a catastrophe is governed by a Poisson process of intensity  $\Delta\lambda$ .<sup>10</sup> To simplify the exposition we let the shareholder be subject to an exogenous shock (e.g. liquidity shock or investment opportunity) governed by an independent Poisson process of intensity  $\delta$ . Whenever hit by this shock the shareholder must divest; the associated stopping time is  $\tau_L$ . Beyond the simplification this also maps well into the fact that investors in public infrastructure do not hold their assets forever, and that these divestments occur randomly. Thus restructuring may be triggered either for exogenous reasons or for insufficient performance. The latter is the contractual restructuring associated with the stopping time  $\tau_R$ . Hence the stopping time  $\tau = \tau_L \wedge \tau_R$ : it is the minimum of either stopping time.

A regulation contract is incentive compatible if it is designed in such a way that the shareholder

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<sup>9</sup>These may be financial losses as experienced during the GFC or social losses associated with business interruption for a more traditional utility. Then the regulatory contract forces the shareholder to internalize these externalities.

<sup>10</sup>In full generality (see the appendix) the stochastic process (2.1) writes

$$dx_t = \mu(a)dt + \sigma dZ_t - K[dN_t - \lambda(a)dt], \text{ with } a \in \{0, 1\} \text{ and } \mu(0) = \mu, \mu(1) = \mu + \Delta\mu, \lambda(0) = 0, \lambda(1) = \Delta\lambda$$

never finds it optimal to divert cash, nor to engage in speculative activities.

### 3 The Optimal Contract

Following the recursive approach of Spear and Srivastava (1987) we can characterize any contract by the stochastic process  $w$  describing the continuation payoff of the agent when the contract  $\Xi$  is executed. The agent's continuation utility at date  $t$  takes the form

$$w_t(\Xi) = \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} dLs + e^{-r(\tau-t)} w_\tau \middle| \mathcal{F}_t \right]. \quad (3.1)$$

Using the martingale representation theorem, as in Sannikov (2008), the dynamics of  $w$  write

$$dw_t = rw_t dt + \frac{\beta_t}{\sigma} (dx_t - \mathbb{E}[dx_t]) - P_t (dN_t - \mathbb{E}[dN_t]) - dL_t, \quad (3.2)$$

where  $\beta_t/\sigma$  represents the sensitivity of the agent's continuation payoffs to cash flows, and  $P_t$  is the penalty incurred in case of a large loss. The power of Sannikov's approach (2008) is that incentive compatible contracts can be directly characterized by simple conditions on these sensitivity parameters  $\beta_t$  and  $P_t$ . We now proceed to completely describe these conditions.

#### 3.1 Incentive compatibility

Recall that the process  $L$  of payments to the shareholder satisfies the limited liability constraint  $dL_t \geq 0$ . From the definition of  $w_t$  this implies

$$w_t \geq 0.$$

**Proposition 1** *No cash is diverted if and only if*

$$\beta_t \geq \beta \equiv \eta\sigma \quad (3.3)$$

*and there is no speculation if and only if*

$$P_t \geq \frac{\beta_t}{\sigma} \frac{\Delta\mu}{\Delta\lambda} \quad (3.4)$$

*Combining these two conditions gives the necessary*

$$P_t \geq \eta \frac{\Delta\mu}{\Delta\lambda} \equiv w_m$$

To deter the agent from diverting funds to her own use, the principal specifies a share  $\beta_t/\sigma$  that the agent can keep. Then she prefers (at least weakly) receiving  $\beta_t dZ_t$  from the principal to appropriating the usable fraction  $\eta$  of  $\sigma dZ_t$ . Similarly, engaging in speculation generates an additional  $\Delta\mu$  but may trigger a sufficiently large penalty:  $\Delta\lambda P_t \geq \eta\Delta\mu$ . Incentive compatibility requires that the expected loss from speculation ( $\Delta\lambda P_t$ ) be at least as large as the expected gain ( $\eta\Delta\mu$ ). The incentives are maximized when  $P_t \equiv w_t$ : the shareholder must be wiped out after a catastrophe. Any further penalty would violate limited liability, thus  $w_t \geq w_m$  so as to preserve incentive compatibility. The firm must be restructured or recapitalized when  $w_t$  reaches  $w_m$ .

### 3.2 Characterization of the optimal contract

Our first Proposition outlines the set of incentive compatible contracts. Now we turn to the best contract among all incentive compatible contracts.

**Proposition 2** *The optimal contract is such that*

- $\beta_t \equiv \beta$  (minimum cash flow sensitivity that prevents cash diversion);
- $P_t \equiv w_t$  (the shareholder is wiped out in case of a catastrophe),<sup>11</sup>
- $\tau = \tau_L \wedge \tau_R$ , where  $\tau_R = \inf\{t | w_t \leq w_m\}$  (termination occurs at the earliest of the exogenous retirement and the regulatory intervention threshold for insufficient performance.)
- $L_t \equiv 0$  (compensation is deferred to date  $\tau$ ).

These conditions are easy to interpret. Since regulator and shareholders have the same discount factor it never helps to disburse any cash in the form of early (that is, before termination) payments  $dL_t$ . This is an extreme form of back-loading payments, in order to provide maximum incentives at minimum cost. Moreover it is strictly better to increase the agent's continuation value: it facilitates incentive compatibility. In addition, it is optimal for the regulator to allow for the smallest fraction  $\beta_t$  of the volatile component of the cash flow  $dx_t - \mathbb{E}[dx_t] = \sigma dZ_t$  to be left to the shareholder. Last, the limited liability constraint on  $w_t$  implies that  $P_t \leq w_t$ ; thus imposing a higher penalty may only trigger earlier restructuring without altering the shareholder's incentives. Hence, along

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<sup>11</sup>Since catastrophes do not occur along the equilibrium path in this model, any  $P_t \geq w_m$  is also optimal.

the optimal path,  $w_t$  is subject to the dynamics

$$dw_t = rw_t dt + \beta dZ_t.$$

**Remark 1** *The need to deter cash diversion has a perverse effect: absent the cash diversion problem, a flat (state-independent) compensation is sufficient and clearly also deters speculation. But speculation may be attractive when the firm (or its owners) pockets a fraction of the earnings, which is prescribed by Condition (3.3).*

### 3.3 Implementation of the optimal contract

In line with Biais, Mariotti, Plantin and Rochet (2007, hereafter BMPR) we propose implementing the optimal contract using a well-selected financing and cash management policy.<sup>12</sup> The fundamental principle underlying this implementation is that the firm is required to maintain cash reserves

$$m_t \equiv \frac{w_t}{\eta}$$

that stay proportional to the continuation payoff  $w_t$ . At date 0 the winning bidder (who becomes the shareholder) invests  $\omega$ . The firm issues riskless debt (to the passive investors) paying a constant coupon  $\mu$ , which is guaranteed by the government. The government initially injects

$$m_0 - \omega + I - \frac{\mu}{r} \geq 0,$$

where  $I \geq 0$  is the once-and-for-all investment necessary to start the infrastructure. The remaining  $(1 - \eta)v_0$  is issued either to outside equity holders or held by public authorities – not doing so amounts to giving away too much to the shareholder.<sup>13</sup> A typical balance sheet is shown below.

Productive Assets A	Debt D
Cash reserves $m_t$	Equity $v_t$

<sup>12</sup>The implementation is not unique. DeMarzo and Sannikov (2006) suggest an implementation using credit lines instead.

<sup>13</sup>This would be sleeping participation: the government is not actively engaged in the management of the firm – except when the regulator restructures it.

Under the optimal contract there is no speculation so the cash reserves of the firm follow the dynamics

$$dm_t = \underbrace{rm_t dt}_{\text{interest}} + \underbrace{\mu dt + \sigma dZ_t}_{\text{earnings}} - \underbrace{\mu dt}_{\text{coupon}}$$

Therefore the process

$$dm_t = \frac{dw_t}{\eta} = rm_t dt + \sigma dZ_t \quad (3.5)$$

is also a martingale for any value of  $\eta$ . When  $\eta = 1$  the wealth of the shareholder invested in the firm is exactly its cash reserves and their evolutions also exactly coincide.

Restructuring takes place at time  $\tau = \tau_L \wedge \tau_R$ , where  $\tau_R$  is the first time the cash reserves fall below  $\Delta\mu/\Delta\lambda$  – recall  $m_t = w_t/\eta$ . The government removes the management and expropriates the shareholder of the utility. Nonetheless there is no breach of contract: these actions are part of the contract and the shareholder is paid  $w_\tau$ . The net injection of public funds at that time is thus contingent on  $w_\tau$ . However the social cost is independent of these transfers, even if  $\eta < 1$ ; it is simply  $\gamma > 0$ .

The total value  $v_t$  of the equity of the firm, including the shares accruing to the government, is just equal to  $m_t$ . The private shareholder holds a fraction  $\eta$  of the equity

$$w_t = \eta v_t$$

while the government's participation is  $(1 - \eta)v_t$ . Hence the stopping time can also be regarded as the first time the value of the firm's equity falls below  $v_m \equiv \Delta\mu/\Delta\lambda$ . This is a capital adequacy rule. Here it is particularly simple in that the value of the equity is exactly the value of the cash reserves.

**Remark 2** *The optimal contract uses a combination of debt and equity to mitigate the two frictions. Debt solves the cash diversion problem by appropriating expected earnings  $\mu dt$ . Equity is used to prevent speculation: the minimum capital requirement ensures that the shareholder has enough “skin in the game” to not engage in excessively risky activities.*

**Remark 3** *Our result is also related to the efficiency wage model of Shapiro and Stiglitz (1984). They study firms-workers relationships when workers can “shirk” but can be detected with some exogenous probability, in which case they are fired. The efficiency wage is the minimum wage that*

deters workers from shirking. In our model, the analogue of the efficiency wage is the minimum market value of equity below which the firm starts engaging in excessive risk taking. Then its shareholders are “fired”.

From now on we use the equity value  $v_t$  ( $\equiv m_t$ ) as state variable instead of  $w_t$  ( $\equiv \eta v_t$ ). This variable determines the expected cost of public intervention through the cost function  $C(v)$  that we now study in details.

## 4 The social cost of public intervention

Here we analyze in detail the determinants of the social cost of restructuring the firm, for which we need to characterize the social cost function. The cost of public intervention is related to the optimal regulation contract through the recursive formulation

$$C(v) = [\gamma + C(v_0)] \mathbb{E} [e^{-r\tau} | v], \quad (4.1)$$

where  $\tau = \tau_L \wedge \tau_R$ ,  $\tau_L$  is exogenous, independent of  $Z$  and follows a Poisson process with intensity  $\delta$  and  $\tau_R$  is the first time the equity of the firm falls below the threshold  $v_m$ :

$$\tau_R = \inf \{t | v_t \leq v_m\}.$$

Finally the value  $v_t$  of the firm follows a discounted martingale:

$$dv_t = rv_t dt + \sigma dZ_t. \quad (4.2)$$

Transfers between the government and shareholders (incumbent and new) do not appear in the social cost formula (4.1). Since the utility is never discontinued, expected future cash flow amounts to a constant  $\mu/r$  that can be ignored (this is paid out to debt holders). Therefore social costs at any point in time  $t$  are simply the expected present value of future restructuring costs. The recursive formulation expresses it as the sum of the expected present values of the cost of the next restructuring  $\gamma e^{-r\tau}$  and the continuation cost  $C(v_0) e^{-r\tau}$ . The regulator is constrained by the limited wealth  $\omega = \eta v_0$  of new investors.

### 4.1 Characterization of the social cost function

We begin by outlining a complete characterization of the function  $C(v)$  under the optimal contract.

**Lemma 1** *The function  $C(v)$  is the unique solution of the differential equation*

$$(r + \delta)C(v) = rvC'(v) + \frac{\sigma^2}{2}C''(v) + \delta[C(v_0) + \gamma], \quad v \geq v_m \quad (4.3)$$

*with boundary conditions*

$$C(v_m) = C(v_0) + \gamma \quad (4.4)$$

*and*

$$\lim_{v \rightarrow \infty} C(v) = \frac{\delta}{r + \delta} [C(v_0) + \gamma] \quad (4.5)$$

The first boundary condition is the optimal termination condition. When  $v_t < v_m$  speculation can no longer be prevented (by Condition (3.4)). In light of Remark 1, just offering a flat compensation is an invitation to divert cash. Therefore the firm must be restructured. The shareholder is expropriated when  $v_t = v_m$ ; the regulator incurs a cost  $\gamma$  and resets the firm's continuation at  $v_0$ . The second condition comes from the fact that

$$C(v) \geq \mathbb{E} [e^{-r\tau_L} (C(v_0) + \gamma)],$$

that is, the social cost is at least the cost of occasional (exogenously given) restructures when shareholders divest because of their exogenous shock. Compulsory restructuring may arise before  $\tau_L$ . Because

$$\mathbb{E} [e^{-r\tau_L} (C(v_0) + \gamma)] \equiv \frac{\delta}{r + \delta} [C(v_0) + \gamma],$$

Condition (4.5) follows.

These boundary conditions are not standard: the function  $C(v)$  appears on both sides of (4.4) and (4.5). It is nonetheless quite easy to show there exists a unique solution  $C(v)$ .

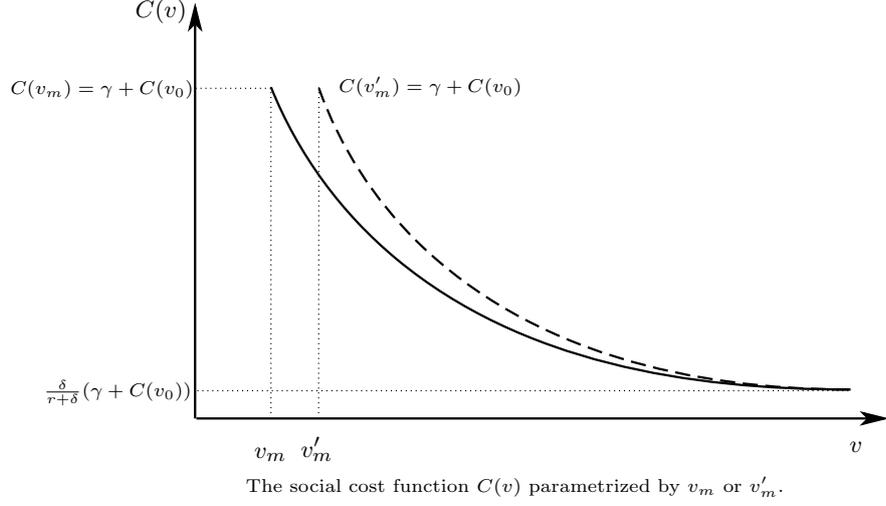
**Proposition 3** *Let  $A(v)$  be the unique solution to the homogenous equation*

$$(r + \delta)A(v) = rvA'(v) + \frac{\sigma^2}{2}A''(v)$$

*such that  $A(0) = 1$  and  $A(\infty) = 0$ . Then, the function*

$$C(v) = \frac{\gamma}{A(v_m) - A(v_0)} \left[ \frac{\delta}{r} A(v_m) + A(v) \right] \quad (4.6)$$

*is the unique solution of the differential equation (4.3) with boundary conditions (4.4) and (4.5). It is decreasing and convex on  $[v_m, \infty)$ .*



The function  $A(\cdot)$  can be expressed as a linear combination of confluent hypergeometric functions of the first kind  $M(a, b; z)$ :<sup>14</sup>

$$A(v) = M\left(-\frac{1}{2}\left(1 + \frac{\delta}{r}\right), \frac{1}{2}; -\frac{rv^2}{\beta^2}\right) - \frac{2v\sqrt{r}}{\beta} \frac{\Gamma\left(\frac{3}{2} + \frac{\delta}{2r}\right)}{\Gamma\left(1 + \frac{\delta}{2r}\right)} M\left(-\frac{\delta}{2r}, \frac{3}{2}; -\frac{rv^2}{\beta^2}\right),$$

where  $\Gamma(\cdot)$  denotes the Gamma function. Function  $C(v)$  is depicted in Figure 1.

The convexity of  $C(v)$  is the reason why it is indeed optimal to set the sensitivity  $\beta_t$  of the shareholder's continuation value  $w_t$  to its minimum value:  $\beta_t \equiv \eta\sigma$ . In addition, any early payment to the agent ( $dL_t$ ) would decrease  $v_t$  and therefore the survival probability of the firm. Similarly any increase in the penalty  $P_t$  beyond  $w_m$  increases the probability of restructure; more precisely it triggers a restructure before it is actually necessary.

## 4.2 Properties of the social cost function

The quasi explicit characterization of the social cost function allows to derive easily several comparative statics results:

**Proposition 4** *The social cost of public intervention*

1. increases with the minimum capital requirement  $v_m = \Delta\mu/\Delta\lambda$ ;
2. decreases with the wealth  $\omega = \eta v_0$  of potential shareholders;

<sup>14</sup>See Abramowitz and Stegun (1964).

3. *increases with the efficiency  $\eta$  of the cash diversion technology;*
4. *is proportional to the restructuring cost  $\gamma$ ;*
5. *increases with the intensity  $\delta$  of the exogenous shocks to shareholders.*

Some of these comparative statics deserve commentary. When investors are effectively less able to commit ( $\delta$  increases) the frequency of restructuring can only increase. Indeed we can see from the boundary condition (4.5) that the lower bound on the social cost  $C(v)$  increases. It is mathematically easy to see that the social cost decreases with  $\omega$ . The intuition is equally simple: if the initial equity injection  $v_0$  of the shareholder were unbounded the firm would never reach the termination threshold  $v_m$  – which is independent of  $v_0$ .

That  $C(v)$  increases with  $v_m$  and  $\eta$  may not be so immediate, for we learned to expect that better-capitalized firms are more resilient. These comparative statics are connected, and their impact relates to Proposition 2. Increasing  $v_m$  (say, above  $\eta\Delta\mu/\Delta\lambda$ ) increases the frequency of costly restructures. But it does not change the shareholder’s incentives to (not) engage in risky activities. So this too validates the earlier claim that  $w_m \equiv \eta\Delta\mu/\Delta\lambda$ . The effect of  $\eta$  is a little more subtle: for any  $w_t$ ,  $\eta$  decreases  $v_t$  (starting at  $v_0$ ). So it is as if the shareholder were committing less of her wealth to the operation of the firm at any point in time. Then the penalty  $P_t = w_m$  has less bite.

## 5 Discussion

The optimal regulation contract can be implemented using an appropriate combination of debt and equity with an appropriate termination rule. As in other papers, debt has a disciplining effect: it is used to extract the firm’s free cash flow, which prevents cash diversion. But it is not sufficient and leaves open the problem of speculation, so equity is necessary too. It takes the form of a minimal equity requirement, which guarantees that the shareholder keeps enough at stake to not engage in excessive risk-taking. This equity requirement is complemented with restructuring (which includes expropriation and compensation at market value of the shareholder) that is triggered every time the capital requirement is violated.

Thus the equity requirement has a quite a different role than the “buffer against losses” often advocated in the banking regulation literature. Instead of absorbing losses and reducing the cost

(and frequency) of public intervention, a higher capital requirement increases them! The reason is that a higher requirement  $v_m$  corresponds to a higher expected return on speculative activities  $\Delta\mu/\Delta\lambda$ . Then restructuring is bound to occur more frequently for any bounded wealth  $\omega$  (to prevent speculation). It would be cheaper in the short run for the regulator to ignore the breach of capital requirement (and possibly only restructure upon insolvency). But this violates incentive compatibility and therefore is socially too costly. So the capital requirement can also be seen as necessary to prompt early corrective action. This action must be drastic here, since social losses can be very large.

Our resolution mechanism is termination and sale to a new shareholder. It very much differs from a bailout: termination occurs not because of financial distress but to preserve incentive compatibility. Yet it guarantees continuation of service, as is socially desirable for SIFMUs and many other utilities. This differs from the proposals of Tucker (2014), who advocates orderly wind-down of financial utilities in distress. If these financial utilities are indeed essential, orderly wind-down is not credible and that regulation is toothless.

We show that the cash diversion parameter  $\eta$  influences the social cost of public intervention (Proposition 4). This is not obvious, for cash flow and speculation are independent actions in the model. To best see the connection, notice that the efficiency  $\eta$  of cash flow diversion enters the restructuring threshold  $w_m = \eta(\Delta\mu/\Delta\lambda)$  (here expressed in terms of the agent's wealth). The reason is that if she speculates, the agent appropriates  $\eta\Delta\mu$ ; so the higher  $\eta$ , the more profitable is speculation and the harder it is to deter it. This exactly translates into a higher capital requirement that we know to increase social costs. Remark 1 tells us that the incentives to speculate are generated by the solution of the cash flow diversion problem. We now also know (i) how costly this is, thanks to the function  $C(v)$ , (ii) that cost increases with the severity of the cash flow diversion problem and (iii) what it implies in terms of capital requirements.

Monitoring is a standard remedy to moral hazard. Here one has to be careful as to *what* is monitored. Monitoring that somehow results in reducing the change  $\Delta\mu$  in the drift is uniformly positive: it reduces the threshold  $v_m$  by curtailing the incentives to engage in risky activities. In contrast, monitoring to reduce the incidence of catastrophes  $\Delta\lambda$  is uniformly bad(!). It increases  $v_m$  in that it is a license to speculate: a large loss is even less likely. An immediate implication of this model in terms of risk management is that, to the extent it is possible, it is better to reduce

the magnitude of losses ( $K$ ) than their frequency  $\Delta\lambda$ .

## 6 Conclusion

We have characterized the optimal regulation contract for a “risky utility” in a dynamic model of risk-taking under moral hazard. The model is relevant for a broad range of applications, ranging from standard utilities to the newly designated financial utilities. The emphasis is laid on the survival risk of these businesses, and on the externalities their failure (either financial or operational) generates.

Regulation is needed to alleviate two frictions. Shareholders can divert some of the earnings to their benefit. They can also engage in excessively risky activities to increase those earnings in the short run at the expense of catastrophic losses (in the longer term).

The optimal contract can be implemented by an appropriate mix of debt and equity, and a stringent termination rule. The equity requirement plays a very different role than in standard model of banking regulation, for example. It is not there to absorb losses but instead to discipline the firm and to trigger “prompt corrective action” from the regulator. Preventing regulatory forbearance is thus of primary importance for risky utilities.

## A Technical background

In the main text we set aside some technicalities. Underlying the choice of whether to speculate is an action:  $a \in \{0, 1\}$  that alters the drift  $\mu(a)dt$  and introduces the Poisson process  $dN_t$  of losses  $K$ . That action generates a probability distribution over the paths of both  $\mu(a)$  and  $N$ ; so (implicitly) all expectations are taken with respect to that distribution. Correspondingly, a contract involves a  $\mathcal{F}_t^N$ -adapted cumulative payment  $L_t$  to the shareholder and a  $\mathcal{F}_t^N$ -stopping time  $\tau_R$ .

To write Proposition 1 we need the dynamics of the agent's continuation value  $w_t$ . When she has a history of reports  $\tilde{x} = x$  (and so does not divert cash) up to time  $t$  and does not speculate at  $t$ , her value reads

$$\Psi_t = \int_0^t e^{-rs} dL_s(x) + e^{-rt} w_t(x)$$

and is clearly a martingale. Hence there exists a process  $\beta_t(x)$  such that  $d\Psi_t = e^{-rt} \frac{\beta_t(x)}{\sigma} [dx_t - \mu dt]$ . Differentiate the first expression and re-arrange these two expressions to obtain  $dw_t = rw_t - dL_t + \beta_t dZ_t$ . If instead the agent speculates and is subject to the penalty  $P_t$ ,

$$\widehat{\Psi}_t = \int_0^t e^{-rs} dL_s(x) + \eta \int_0^t e^{-rs} \Delta\mu ds + e^{-rt} w_t(x) - \int_0^t P_s dN_s$$

and is also a martingale with respect to the filtration  $\mathcal{F}_t^N$  given the action  $a = 1$ . The auxiliary process becomes

$$d\widehat{\Psi}_t = e^{-rt} \frac{\beta_t(x)}{\sigma} [dx_t - (\mu + \Delta\mu)dt] = e^{-rt} \beta_t(x) dZ_t$$

Differentiate  $\widehat{\Psi}_t$ :

$$d\widehat{\Psi}_t = e^{-rt} dL_t(x) + \eta e^{-rt} \Delta\mu - r e^{-rt} w_t(x) dt + e^{-rt} dw_t(x) - P_t dN_t,$$

so when she speculates the agent's continuation utility follows the dynamics

$$d\widehat{w}_t = rw_t + \eta \Delta\mu dt - dL_t + \beta_t dZ_t - P_t dN_t,$$

which is a jump-diffusion process.

## B Proofs

**Proof of Proposition 1:** Condition (3.3) expresses that what the agent earns from an additional \$ of profit ( $\beta_t$ ) is at least equal to what she could earned by secretly diverting this \$. It mirrors DeMarzo and Sannikov (2006) and follows from the derivations above. Note that because  $d\Psi_t = d\widehat{\Psi}_t$ , the sensitivity  $\beta_t$  is the same regardless of whether the agent speculates. To deter her from engaging in (excessively) speculative activities, one needs the penalty to be large enough so that

$$\mathbb{E}[dw_t] \geq \mathbb{E}[d\widehat{w}_t]$$

that is

$$P_t \geq \eta \frac{\Delta\mu}{\Delta\lambda}$$

Combining with (3.3) binding one has  $P_t \geq \eta \frac{\Delta\mu}{\Delta\lambda} \equiv w_m$ , which is feasible only when  $w_t \geq w_m$  by limited liability. ■

**Proof of Proposition 2:** Setting  $P_t > w_m$  is neutral on the firm's incentives whether to engage in speculation. However recall the recursive formulation of  $C(v)$  and that

$$\tau = \tau_L \wedge \tau_R = \tau_L \wedge \inf\{t|w_t = P_t\}$$

for any  $P_t$ , and where the equality owes to the limited liability constraint  $P_t \leq w_t$ . Clearly  $\tau_R$  is decreasing in  $P_t$  so that  $\tau$  is at least weakly decreasing. Therefore

$$C(v) = [\gamma + C(v_0)]\mathbb{E}[e^{-r\tau}|v]$$

is increasing in  $P_t$ . The lowest penalty  $P_t$  that is compatible with incentive compatibility is  $w_m$ . Because regulator and shareholder discount the future at the same rate  $r$ , there is no cost in substituting payments for an increase in the continuation value  $w_t$ . There is a strict benefit to doing so since  $\tau_R = \inf\{t|w_t = P_t\}$ . From the dynamics of the continuation value under an incentive compatible contract

$$dw_t = rw_t - dL_t + \beta_t dZ_t, \quad w_t \geq w_m$$

one sees that decreasing  $dL_t$  correspondingly shifts the trajectory of  $w_t$ . So it is in the regulator's interest to set  $dL_t \equiv 0$ . Then under an incentive compatible contract the agent's utility is

$$dw_t = rw_t + \beta_t dZ_t, \quad w_t \geq w_m.$$

Given this, the regulator's value function must satisfy the HJB equation of the form

$$(r + \delta)C(w) = \min_{\beta_t \geq \beta} rwC'(w) + \frac{\beta_t^2}{2}C''(w) + \text{constant}, \quad w_t \geq w_m, \quad (\text{B.1})$$

the maximum of which yields the differential equation (4.3) with the appropriate change of variable. Anticipating Proposition 3,  $C' < 0$  and  $C'' > 0$  so it is easy to see that (B.1) is a sub-martingale except when  $\beta_t = \beta$ . ■

**Proof of Lemma 1:** From (4.3)-(4.5), the function  $C$  takes the form

$$(r + \delta)C(v) = \delta [\gamma + C(v_0)] + c_0H_0(v) + c_1H_1(v)$$

where  $(H_0, H_1)$  are basis of solutions for the homogenous equation

$$(r + \delta)H(v) = rvH'(v) + \frac{\sigma^2}{2}H''(v)$$

with

$$\begin{aligned} H_0(0) &= 1 = H_1'(0) \\ H_1(0) &= 0 = H_0'(0). \end{aligned}$$

By the Cauchy-Lipschitz theorem the functions  $H_0$  and  $H_1$  are uniquely defined. The parameters  $c_0, c_1$  are derived from the boundary conditions. From (4.5),  $\lim_{v \rightarrow \infty} C(v) = \delta[C(v_0) + \gamma]/(r + \delta)$  implies

$$\lim_{v \rightarrow \infty} \left[ H_0(v) + \frac{c_1}{c_0}H_1(v) \right] = 0 \quad (\text{B.2})$$

directly from the definition of  $C(v)$ . So one has

$$\frac{c_1}{c_0} = - \lim_{v \rightarrow \infty} \frac{H_1}{H_0},$$

and where  $H_0, H_1$  are uniquely defined. Condition (4.4) gives:

$$\frac{\delta}{r + \delta}[\gamma + C(v_0)] + c_0 \left[ H_0(v_m) + \frac{c_1}{c_0}H_1(v_m) \right] = \gamma + \frac{\delta}{r + \delta}[\gamma + C(v_0)] + c_0 \left[ H_0(v_0) + \frac{c_1}{c_0}H_1(v_0) \right],$$

which simplifies for  $c_0$  in terms of the functions  $H_0, H_1$  only. So the constants  $c_1, c_0$  are uniquely identified. ■

**Proof of Proposition 3:** We first explicit show how to compute the parameters  $c_0, c_1$ . Let  $c \equiv \frac{c_1}{c_0}$ ; with this, rewrite (B.2)

$$\begin{aligned} C(v) &= \frac{\delta}{r+\delta} [\gamma + C(v_0)] + c_0 [H_0(v) + cH_1(v)] \\ &= \frac{\delta}{r+\delta} [\gamma + C(v_0)] + c_0 A(v) \end{aligned} \quad (\text{B.3})$$

with  $c_0, C(v_0)$  are just numbers to be determined. The termination condition (4.4) then becomes

$$\frac{\delta}{r+\delta} [\gamma + C(v_0)] + c_0 A(v_m) = \gamma + \frac{\delta}{r+\delta} [\gamma + C(v_0)] + c_0 A(v_0)$$

so the condition for  $c_0$  is

$$c_0 \equiv \frac{\gamma}{A(v_m) - A(v_0)}. \quad (\text{B.4})$$

We need to sign  $c_0$ , for which we need to understand the behaviour of  $A(v)$ , and therefore the sign of the constant  $c$  buried in the definition (B.3) of  $C(v)$ . For this we are left identifying the functions  $H_0, H_1$ , which will determine  $c$  and  $c_0$  given the exogenous values  $v_m$  and  $v_0$ .

The confluent hypergeometric function of the first kind  $M(a, b; z)$  is the unique solution the confluent hypergeometric differential equation (also called Kummer's equation)

$$zM'(z) = (b-z)M'(z) + zM''(z); \quad M(0) = 1, \quad M'(0) = \frac{a}{b} \quad (\text{B.5})$$

In the next two Lemmata we construct the basis functions  $H_0$  and  $H_1$  and show each solves Kummer's equation. With this one can then compute  $c$ .

**Lemma 2**  $H_0(v) = M\left(-\frac{1}{2}\left(1 + \frac{\delta}{r}\right), \frac{1}{2}; -\frac{rv^2}{\beta^2}\right)$

**Proof:** Differentiate:

$$\begin{aligned} H_0'(v) &= -\frac{2rv}{\beta^2} M' \\ H_0''(v) &= -\frac{2r}{\beta^2} M' + \frac{4r^2 v^2}{\beta^4} M'' \end{aligned}$$

So

$$rvH_0' + \frac{\beta^2}{2} H_0'' = -\left(\frac{2r^2 v^2}{\beta^2} + r\right) M' + \frac{2r^2 v^2}{\beta^2} M'' \quad (\text{B.6})$$

and (B.5) becomes

$$-\left(\frac{1}{2} + \frac{\delta}{2r}\right) M = -\frac{rv^2}{\beta^2} M'' + \left(\frac{1}{2} + \frac{rv^2}{\beta^2}\right) M',$$

where  $a = -(1/2 + \delta/2r)$  and  $b = 1/2$ . Hence by (B.6),

$$\underbrace{rvH'_0 + \frac{\beta^2}{2}H''_0}_{=(r+\delta)H_0} = \underbrace{-2r \left[ \left( \frac{rv^2}{\beta^2} + \frac{1}{2} \right) M' - \frac{rv^2}{\beta^2} M'' \right]}_{=-2r \cdot a M_0}$$

The proof is complete once we have noted that  $H_0(0) = M(0) = 1$  and  $H'_0(0) = 0$ . ■

**Lemma 3**  $H_1(v) = v \cdot M \left( -\frac{\delta}{2r}, \frac{3}{2}; -\frac{rv^2}{\beta^2} \right)$

**Proof:** As in the proof of Lemma 2, differentiate

$$\begin{aligned} H'_1 &= M - \frac{2rv^2}{\beta^2} M' \\ H''_1 &= -\frac{6rv}{\beta^2} M' + \frac{4rv^3}{\beta^4} M'' \end{aligned}$$

So that the RHS of the elementary differential equation writes

$$rvH'_0 + \frac{\beta^2}{2}H''_0 = - \left( 3rv + \frac{2r^2v^3}{\beta^2} \right) M' + \frac{2r^2v^3}{\beta^2} M'' + rvM_1 \quad (\text{B.7})$$

and (B.5) reads

$$-\frac{\delta}{2r}M = -\frac{rv^2}{\beta^2}M'' + \left( \frac{3}{2} + \frac{rv^2}{\beta^2} \right) M',$$

where  $a = -\delta/2r$  and  $b = 3/2$ . Therefore by (B.7),

$$\underbrace{rvH'_1 + \frac{\beta^2}{2}H''_1}_{=(r+\delta)H_1} = \underbrace{-2rv \left[ \left( \frac{rv^2}{\beta^2} + \frac{3}{2} \right) M' - \frac{rv^2}{\beta^2} M'' - \frac{M}{2} \right]}_{=-2rv \cdot a M_1}$$

and we note that  $H_1(0) = 0, H'_1(0) = M(0) = 1$ . ■

The functions  $H_0, H_1$  give us a determination for  $c = -\lim_{v \rightarrow \infty} \frac{H_0(v)}{H_1(v)}$ . Indeed, any confluent hypergeometric function  $M(a, b; z)$  can be expressed as

$$M(a, b; z) = \frac{\Gamma(b)}{\Gamma(b-a)} (-z)^{-a} [1 + O(|z|^{-1})], \quad z \in \mathbb{R}_-$$

where  $\Gamma(\cdot)$  is the Gamma function (see Abramovitz and Stegun (1964), Chapter 13, Theorems 13.1.4 and 13.1.5). Forming the ratio of  $H_0$  and  $H_1$  and simplifying yields

$$c = -\frac{\Gamma(1/2) \Gamma(3/2 + \delta/2r) \sqrt{r}}{\Gamma(3/2) \Gamma(1 + \delta/2r) \beta}$$

and since  $\Gamma(1/2) = \sqrt{\pi}$  and  $\Gamma(3/2) = (1/2)\sqrt{\pi}$ ,

$$c = -2 \frac{\sqrt{r} \Gamma(3/2 + \delta/2r)}{\beta \Gamma(1 + \delta/2r)} < 0.$$

With this we can establish, first

**Lemma 4** *The function  $A(v)$  is the unique solution to the homogenous equation*

$$(r + \delta)A(v) = rvA'(v) + \frac{\sigma^2}{2}A''(v) \quad (\text{B.8})$$

with boundary condition  $A(0) = 1$  and  $\lim_{v \rightarrow \infty} A(v) = 0$ .

**Proof:** That  $A(v)$  solves (B.8) follows directly from its definition:  $A(v) = H_0(v) + cH_1(v)$ . Then immediately  $A(0) = 1$  and  $A(\infty) = 0$  by (4.5). ■

Second

**Lemma 5** *The function  $A : \mathbb{R}_+ \mapsto \mathbb{R}$  is decreasing convex.*

**Proof:** Since  $A'(v) = H'_0(v) + cH'_1(v)$ ,  $A'(0) = 0 + c < 0$ , so  $A(v)$  is indeed decreasing in  $v$  from 0. Furthermore, by (B.8), at  $v = 0$

$$(r + \delta) = 0 + \frac{\sigma^2}{2}A''(0) > 0$$

Suppose now that  $A(v)$  is not monotone. We rule out all cases in turns. First a local maximum with  $A(v_1) > 0$  is impossible for then we must have  $A(v_1) > 0$ ,  $A'(v_1) = 0$  and  $A''(v_1) < 0$ , which contradicts (B.8). Second, a local minimum  $v_2$  with  $A(v_2) > 0$  is also impossible: at  $v_2$ ,  $A''(v_2) > 0$  and so there must be a local maximiser  $v_3$  with  $A(v_3) > 0$ ; we just ruled that out. Third, there cannot be an inflexion point with  $A(v_1) > 0$  for then  $A''(v_1) = 0$ , which is again impossible by (B.8). Fourth, it cannot reach a local minimum  $v_3$  where  $A(v_3) < 0$ , for then we must have  $A(v_3) < 0$ ,  $A'(v_3) = 0$  and  $A''(v_3) > 0$ . Again this is impossible by (B.8). Fifth, an inflexion point below 0 is impossible for then  $A''(v_1) = 0$ . Last, a local maximum with  $A(v_4) < 0$  can also be ruled out: if so, there must be a local minimum with  $A(v_5) < 0$ , which was just shown to be impossible.

■

With (B.4) the social cost function reads

$$C(v) = \frac{\delta}{r + \delta}[\gamma + C(v_0)] + \frac{\gamma}{A(v_m) - A(v_0)}A(v).$$

where  $A(v)$  is decreasing convex and  $C(v_0)$  is a number. Therefore

$$c_0 = \frac{\gamma}{A(v_m) - A(v_0)} > 0$$

since  $v_0 > v_m$ , and it follows that  $C(v)$  is also decreasing convex. Finally together both this definition and the boundary condition (4.4) tell us that

$$\gamma + C(v_0) = \frac{r + \delta}{r} \frac{\gamma}{A(v_m) - A(v_0)} A(v_m)$$

substituting in the definition of  $C(v)$  then yields

$$C(v) = \frac{\gamma}{A(v_m) - A(v_0)} \left[ \frac{\delta}{r} A(v_m) + A(v) \right]$$

as claimed. ■

**Proof of Proposition 4:** Items 4 and 5 are obvious from the definition of  $C(v)$ . To show item 1, rewrite the function  $C(v)$  as

$$\begin{aligned} C(v) &= \frac{\gamma}{r} \left[ \frac{\delta A(v_m) + r A(v)}{A(v_m) - A(v_0)} \right] \\ &= \frac{\gamma}{r} \left[ \delta + \frac{\delta A(v_0) + r A(v)}{A(v_m) - A(v_0)} \right] \end{aligned}$$

which is clearly increasing in  $v_m$  since  $A(v_m)$  is decreasing. Bearing this in mind, item 2 follows from the original definition of  $C(v)$ . Last, substitute  $v_t = w_t/\eta$  in  $C(v)$  and differentiate the above expression. It is sufficient to consider the numerator

$$\begin{aligned} &- \left[ \delta A' \left( \frac{\omega}{\eta} \right) \omega + A' \left( \frac{w}{\eta} \right) w \right] \eta^{-2} \left[ A \left( \frac{w_m}{\eta} \right) - A \left( \frac{\omega}{\eta} \right) \right] \\ &+ \left[ \delta A' \left( \frac{w_m}{\eta} \right) w_m - A' \left( \frac{\omega}{\eta} \right) \omega \right] \eta^{-2} \left[ A \left( \frac{\omega}{\eta} \right) + A \left( \frac{w}{\eta} \right) \right] > 0 \end{aligned}$$

since  $w_m \leq \omega$  and  $A(\cdot)$  is a decreasing function. ■

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