Restructuring Failure and Optimal Capital Structure

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Abstract

I build a dynamic capital structure model that allows the firm to renegotiate debt with its creditors. Debt forgiveness by some creditors creates a positive externality as it increases the value of other creditors' debt claims which can lead to bargaining breakdown and thus inefficient liquidation. The probability of successful renegotiations increases in the value of the firms assets at the time of restructuring, the concentration of the debt structure, and in the costs of liquidating the firm's assets. Anticipating the outcome of the debt restructuring I solve for the firm's optimal capital structure in a dynamic trade-off model. Contrasting the classical trade-off theory optimal leverage is non-monotonic in bankruptcy costs. For low bankruptcy costs renegotiations will fail and optimal leverage is decreasing in bankruptcy costs in line with the classic trade-off theory. High bankruptcy costs reduce the creditors' outside option, increase the probability that renegotiations succeed, and thus make debt more attractive resulting in higher optimal leverage. Firms might find it ex-ante optimal to write covenants to commit to enter renegotiations at an asset level where renegotiations can still succeed. Firms with low bankruptcy costs will optimally have a concentrated debt structure while firms with high bankruptcy costs maximize ex-ante firm value with dispersed debt.

Keywords: Renegotiations, Restructuring, Capital Structure, Bankruptcy, Covenants

1 Introduction

> *Ni iudicatum facit aut quis endo eo in iure vindicit, secum ducito, vincito aut nervo aut compedibus XV pondo.*
> Unless the debtor pays the amount of the judgment or somebody guarantees his debt the creditor shall take him home and fasten him in stocks or fetters. He shall fasten him with fifteen pounds of weight.
>
> The twelve tables - Roman code of law (451-450 B.C.)

When the bankruptcy process comes with dead weight bankruptcy costs both equityholders and creditors would be better off renegotiating out-of-court rather than going to bankruptcy court. However, not all firms are able to successfully restructure their debt and, as a result, have to enter an often costly bankruptcy process. This paper explores the conditions under which debt restructurings with multiple creditors fails and the implications on the firms optimal leverage and debt structure. I model bankruptcy as an endogenous outcome of a bargaining game where the creditors fail to reach an agreement. Bargaining frictions occur when creditors do not all simultaneously agree to forgive debt, such as in a debt exchange offer. Creditors that forgive debt generate an externality for other creditors as they put themselves at a disadvantage relative to other creditors, thus increasing the value of the other creditors' claims. Creditors that negotiate later can therefore demand more for surrendering their existing debt claim. If the externalities are too high the firm's assets may be insufficient to meet the creditors' demands and bargaining fails.

The efficiency of the bankruptcy process as the next best alternative to debt renegotiations is a key factor driving the likelihood of successful restructuring. When the dead weight costs of bankruptcy are low, creditors are able to obtain relatively high payoffs under court-supervised bankruptcy, especially when other creditors have already reduced their debt. Instead of participating in an out-of-court restructuring, they might therefore be better off taking the company to bankruptcy court. Anticipating that not all creditors are willing to participate in the restructuring, other creditors decline as well and the firm ends up in bankruptcy. In contrast, high bankruptcy costs reduce the creditors outside options and the prospect of small payouts in bankruptcy court facilitates out-of-court renegotiations.

A firm's chances of successful renegotiations also depend on its debt structure. Bargaining between one creditor and the equityholders is always successful, but a larger number of creditors increases the likelihood of bargaining failure. As each creditor successively agrees to a reduction in debt, subsequent creditors' existing debt claims become more valuable. Each subsequent creditor is able to extract a larger premium for surrendering its existing debt claim, making bargaining failure more likely. A continuum of infinitesimal small creditors has the smallest region of the parameter space within which renegotiations succeed.

Finally the value of the firm's assets at the time of restructuring contributes to the success of renegotiations. When there are more assets to share amongst creditors, renegotiations will be successful for a variety of bankruptcy costs and debt structures. When the firm's assets are worth little, renegotiations are more likely to fail; in these cases bankruptcy costs and the debt structure are more decisive in determining the success of renegotiations.

The firm's equityholders would thus like to commit ex-ante to start renegotiations at a point where there are still sufficient assets available for successful debt restructuring; however, the option value that the equityholders have due to limited liability creates an incentive to defer renegotiations. To solve for the optimal restructuring threshold at which equityholders initiate renegotiations with debtholders I embed the restructuring game in a standard continuous time capital structure model. Anticipating the outcome of the subsequent bargaining game, equityholders will initiate renegotiations when their marginal payoff under the restructuring plan equals the marginal payoff from keeping the firm alive.

The possibility of renegotiating debt thus determines not only the optimal restructuring threshold but also the firm's optimal ex-ante capital structure choice. Successful restructuring will affect the recovery rates of bondholders and thus the price at which the equityholders can sell bonds ex-ante. Successful renegotiations also eliminate dead weight bankruptcy costs that would be borne ex-ante by the equityholders, making it more attractive to issue debt.

In this paper I find that optimal leverage is non-monotonic in bankruptcy costs. Consistent with the trade-off theory, optimal leverage initially decreases in bankruptcy costs. However, once bankruptcy costs are sufficiently high, renegotiations are more likely to be successful which increases the attractiveness of debt. Optimal leverage thus increases with bankruptcy when higher bankruptcy costs allow firms to successfully renegotiate debt, such that the bankruptcy costs will never be realized in equilibrium. I also show that firms with high risk and low bankruptcy costs optimally seek a concentrated debt structure to facilitate renegotiations, while firms with low risk and high bankruptcy costs optimally choose dispersed debt.

Finally, I explore covenants as ex-ante commitment devices for equityholders to enter renegotiations with creditors at a point where the firm's outstanding debt can still be successfully restructured. Without any covenants, equityholders in firms with low bankruptcy costs optimally delay restructuring to a point where the value of the firm's assets is too small to ensure successful renegotiations. Once the assets of the firm have deteriorated, the call option of equity that is generated from limited liability is worth more to shareholders than the benefit of restructuring; without any ex-ante commitment, the firm gets liquidated in bankruptcy court. Since bankruptcy costs are borne ex-ante by equityholders, this outcome is inefficient even for them. Loan covenants commit equityholders to enter renegotiations once asset value hits a prespecified lower bound which can be optimally set to ensure successful restructuring. I derive an optimal covenant level, trading off the benefits of restructuring with the costs of a suboptimal restructuring threshold. Covenants are used by medium bankruptcy-cost firms, can overcome some of the costs of bargaining failure, and can increase ex-ante firm value.

This paper contributes to a large literature on debt renegotiations. Haugen and Senbet (1978) point out that bankruptcy costs should not influence capital structure decisions when debt can be renegotiated. Giammarino (1989) models debt restructuring with one creditor that can potentially fail in the presence of asymmetric information. In this paper, I analyze bargaining breakdown under multilateral bargaining with symmetric information. Bolton and Scharfstein (1996) examine in their seminal paper the optimal debt structure and renegotiations when an entrepreneur can default strategically. This paper abstracts from managerial agency problems and focuses on the implications of renegotiation failure on optimal capital structure. Gertner and Scharfstein (1991) examine renegotiations with one bank and with myopic dispersed debtholders. In this paper, I examine a multilateral bargaining game where each player behaves strategically and considers the impact of their actions on the overall outcome. More recently, Gennaioli and Rossi (2013) examine the optimal allocation of liquidation control rights and collateralization between one large and dispersed small bondholders. I focus more on the conditions under which successful renegotiations are possible and the implication for optimal capital structure.[1]

My research also ties into a large body of literature on optimal dynamic capital structure

[1]Choi, Hackbarth, and Zechner (2014) study the optimal maturity structure of debt, trading off issuance and rollover cost. In this paper firms issue perpetual debt that can be renegotiated or recalled. Morris and Shin (2002) examine coordination failure on loan rollovers using global games. In my paper coordination failure arises because of externalities that one cerditor's debt forgiveness creates for other players.

including, amongst others, Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Leland and Toft (1996). In all of these papers it is assumed that the firm gets liquidated when equity-holders walk away; this paper models how equity-holders optimally choose the point at which to initiate renegotiations with bondholders that may lead to positive payments to equityholders. This paper also relates to the literature on strategic debt service, e.g., Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Hege and Mella-Barral (2005), and Hackbarth, Hennessy, and Leland (2007). This stream of research focuses on a firm's ability to reduce debt service payments while in distress, while this paper models a permanent reduction of a firm's existing debt level. This paper is closely related to that of Christensen, Flor, Lando, and Miltersen (forthcoming), who model renegotiations with one creditor in a continuous time capital structure model. Similar to most papers in the literature, they focus on bargaining between one debtholder and one creditor, which is always successful.[2] Due to multilateral bargaining with externalities, renegotiations in this paper are not always successful, which has important implications for capital structure and the optimal number of creditors. David (2001) examines multilateral bargaining in the restructuring of putable bonds with an alternating offers bargaining game. He derives conditions under which bargaining succeeds and finds that in these cases the solution coincides with the Shapley value; in my paper bargaining breakdown will be an important driver of the optimal capital and debt structure decision. To my knowledge the only paper that considers multilateral bargaining breakdown under symmetric information is David and Lehar (2014), who examine multilateral renegotiations in interbank networks and their implications for systemic risk. They also find that bargaining is more likely to succeed when bankruptcy costs are high. The bargaining mechanism in this paper is related to the model proposed by Maskin (2003) who models multilateral bargaining with externalities in a cooperative model. In contrast I model bargaining as a non-cooperative game.

My findings are consistent with several empirical results. Roberts and Sufi (2009) document the importance of out-of-court restructurings. For a large sample of US firms they find that over 90% of long-term debt contracts are renegotiated at least once prior to their stated maturity. Brunner and Krahnen (2008) look at bank pools as vehicles of creditor coordination for German out-of-court restructurings. They show that restructuring is less likely to succeed and take longer if the number of creditors is large. The findings of my paper are also consistent with Jostarndt and Sautner (2010) who find that firms which have higher leverage, have more

[2]See also Fan and Sundaresan (2000).

concentrated debt structure, and exhibit higher going concern values are more likely to succeed in a workout. Gilson, John, and Lang (1990) document that firms that are more likely to restructure out-of-court if they have fewer lenders. Ongena, Tümer-Alkan, and Westernhagen (2012) find that firms with better re-deployable assets (and thus lower liquidation costs) have a more concentrated debtor structure. The finding in this paper that firms with high bankruptcy costs optimally choose more leverage because debt can always be renegotiated is consistent with the study of Hovakimian, Kayhan, and Titman (2011), who document that high bankruptcy cost firms chose capital structures with more bankruptcy risk. In a cross county study Claessens and Klapper (2005) find that higher judicial efficiency (and this presumably lower bankruptcy costs) are associated with higher bankruptcies. Glover (forthcoming) finds that firms with high bankruptcy costs take on less debt as equityholders have to pay ex-ante for the expected cost of default. Thus he finds that in any empirical sample of bankruptcies firms firms with high bankruptcies are underrepresented. My model has the same empirical implications through a different channel. Since high bankruptcy costs increase the likelihood of successful out of court renegotiations these firms will less likely end up in formal bankruptcy proceedings.

The rest of the paper is organized as follows: Section 2 describes the bargaining game, Section 3 describes the optimal capital structure choice, optimal covenant are derived in Section 4, the implications for the optimal debt structure are analyzed in Section 5, and Section 6 concludes.

2 Bargaining and debt renegotiations

The bargaining game starts once equityholders have decided to enter renegotiations with the creditors for a voluntary restructuring of the firm's debt. The payoffs for the equityholders from a restructuring will then determine the restructuring threshold at which equityholders optimally start renegotiations, which we will examine in detail in Section 3.

Denote the market value of the firms assets at this restructuring threshold by $\underline{v}$ and the face value of the outstanding debt D_f. Before exploring the bargaining process in detail we have to specify what happens when debtors and equityholders cannot reach an agreement and a voluntary restructuring fails.

2.1 The bankruptcy mechanism

Since out-of-court restructuring is voluntary every player has the option to go to bankruptcy court where a pre-specified bankruptcy mechanism gets implemented. I interpret this mechanism as liquidation but it can also be seen as the outcome of a chapter 11 reorganization which is handed down by the bankruptcy judge. In liquidation a fraction α of the assets gets destroyed and the value of the remaining assets get allocated proportionally to the creditors. We can interpret α either as direct bankruptcy costs that arise from going to court, e.g. for paying lawyers and accountants, or as indirect costs that arise when assets are liquidated.

Under the bankruptcy mechanism equityholders get zero.[3] Assume that there are n debtholders and denote the face value of creditor i at the time that the bankruptcy mechanism gets evoked with x_i. Then the payoff for creditor i in liquidation is

$$L_i = \frac{x_i}{\sum_{j=1}^{n} x_j}(1-\alpha)\underline{v} \tag{1}$$

The payoffs for equityholders and all debtholders under the bankruptcy mechanism are then

$$\pi_L^D = \sum_{i=1}^{n} L_i = (1-\alpha)\underline{v} \tag{2}$$

$$\pi_L^E = 0 \tag{3}$$

2.2 Bargaining protocol

In renegotiations equityholders bargain with n debtholders over a restructuring plan for the firm. I assume that the proposer cannot sign up all players to the restructuring plan simultaneously, like for example in an out-of-court debt exchange offer. To model sequential agreement to a restructuring plan I assume that nature selects a sequence according to which players arrive at the bargaining site. This sequential arrival order can also be motivated as the outcome of a timing game as it is modeled in greater detail in Pitchford and Wright (2012). In their model of sovereign debt restructuring creditors endogenously choose the time at which to approach

[3] I will later verify that equityholders optimally choose the restructuring threshold $\underline{v}$ so that debt is risky, i.e. $\underline{v} < D_f$, and therefore equityholders get no payout as the firm is liquidated.

the sovereign for bargaining. In equilibrium creditors randomize arrival time by drawing from an exponential distribution. The outcome is identical to the assumption of a sequential arrival order in this paper. In the main section of the paper I consider *pari-passu* offers under which the proposer gives the same terms to each creditor irrespective of their arrival time.[4]

The first player to arrive is the proposer who makes a take-it-or-leave-it offer to claim-holders to exchange their existing securities of the firm for new securities. Any claim-holder can refuse the proposer's offer and go to bankruptcy court, where the bankruptcy mechanism gets implemented. If, after the last player has arrived, the value of the firm's assets is at least has high as the aggregate value of the outstanding renegotiated debt the restructuring is successful, otherwise the firm goes to bankruptcy court and the bankruptcy mechanism gets implemented. The mechanism is in spirit very similar to a voluntary debt restructuring. Under the Trust Indenture Act of 1939 in the US any change in the interest rate, the principal amount, or the maturity of public debt in an out-of-court restructuring requires an unanimous vote, so in practice debt restructurings are often accomplished by exchanging existing for new claims.[5]

Assume that with probability γ the equityholders arrive first at the bargaining site and offer subsequently arriving debtholders to redeem their debt claim for a new claim on the firm and with probability $1-\gamma$ one of the debtholders arrives first and offers subsequent claimants new securities on the firm. In line with typical restructurings I assume that the proposer, wheter it is one of the debtholders or whether it is the equityholders, have to make *pari-passu* offers to all debtholders offering them the same amount of new claims per dollar of old debt.

The sequential arrival order generates a friction in the bargaining framework in which bondholders that agree to reduce their claim in a restructuring reduce the amount that they can claim in a subsequent bankruptcy compared to bondholders who hold out. In January 1990 bankruptcy Judge Burton Lifland ruled in the case of LTV Corp. that bondholders who participated in a voluntary restructuring before the bankruptcy proceedings could only claim a reduced value in bankruptcy compared to bondholders that held out.[6] Holdout problems are widely blamed for

[4]As an alternative one could assume that the proposer can sign individual binding agreements with each creditor at potentially different terms as they arrive at the bargaining site. Appendix C summarizes the outcome of the renegotiation process for this alternative bargaining protocol. The main findings of the paper are robust with respect to this alternative bargaining protocol.

[5]see e.g. Hotchkiss, John, Mooradian, and Thorburn (2008).

[6]Even though that ruling was ultimately overturned in April 1992 for this specific case, there is substantial concern amongst bankruptcy professionals regarding the legal uncertainty of restructured claims in subsequent bankruptcies (Betker (1995)).

the failure of out-of-court debt restructurings. Jensen (1991) argues that institutional changes have reduced the chances that private debt restructurings can succeed. Empirically Gilson, John, and Lang (1990) find that 53% of the firms in their sample fail to privately restructure their debt and subsequently file for bankruptcy.

I use the friction that is created by the sequential arrival order to model the holdout problem in a simple way. Creditors that make a concession to the company and reduce the face value of their debt claim decrease the fraction of the assets liquidation value that they are entitled to receive under the bankruptcy mechanism. At the same time the fraction that the other debtholders can claim increases. It is easy to see from equation (1) that the liquidation payoff of creditor i increases as other creditors forgive more debt (i.e. $x_{j,j\neq i}$ and thus the denominator decrease). Debt forgiveness by one creditor therefore generates a positive externality for the the other creditors by increasing the payoff that they can get when bargaining fails. This externality increases in the threatpoints of the other players, i.e. the value that players can extract by rejecting the proposer's offer and thus invoking the bankruptcy mechanism. Each creditor who arrives at the bargaining site will get a higher payoff from evoking the bankruptcy mechanism that the creditor before him. The proposer has to set the exchange rate of new debt to old debt high enough so that all creditors will agree to the exchange. This could lead to a situation where the value of the firm's assets is insufficient to meet the increasing demands of creditors. All creditors anticipate the demands of subsequent creditors and if assets are insufficient to meet aggregate creditor demand the first creditor will reject the proposer's offer and thus invoke the bankruptcy mechanism. In this case bargaining breaks down and the firm's debt cannot be successfully renegotiated. There are therefore only to possible outcomes for the bargaining game, either all creditors agree or the first creditor evoked the bankruptcy mechanism and there is no debt forgiveness.

The bargaining model in this paper differs in two important aspects from traditional bargaining problems that can be solves using a characteristic function and Shapley values: first, the payoff that one group of players can obtain is not independent of the other players' actions because of the positive externality that a player's concession creates for the other players' minimum payoff. Second, the value function can be non-monotonic in the size of the bailout coalition, i.e. a creditor can be better off by not joining a bailout coalition (evoking the bankruptcy mechanism) than by joining and thus agreeing to a restructuring. This possible non-monotonicity in the value function and the presence of externalities prevents me from applying standard solution concepts of multi-player bargaining theory like the Shapley value. Using the sequential arrival order imposes enough structure to ensure that a well defined equilibrium always exists.

2.3 Renegotiations with one creditor

With one creditor renegotiations of the firm's debt are fairly straightforward because there cannot be any externalities for other creditors. As the equityholders enter renegotiations the firms assets are worth $\underline{v}$. If the equityholders propose first, which happens with probability γ they will offer the debtholder a payment just above what the debtholder would get under the bankruptcy mechanism, i.e. L_1. The debtholder will accept as he cannot improve his payoff by rejecting the offer and thus invoking the bankruptcy mechanism. If the debtholder can propose first, he will offer the equityholders a payment just above zero, which the equityholders will accept as they get zero in bankruptcy court. Renegotiations will always be successful and the bankruptcy mechanism will never be evoked in equilibrium. The following proposition summarizes this finding (all proofs are in Appendix F)

Proposition 1 *Renegotiations with one creditor are always successful and the the bankruptcy mechanism will never be evoked in equilibrium. Given that the assets of the firm are worth $\underline{v}$ the expected payoffs for the debtholder and the equityholders given one creditor are $\pi_1^D = (1-\gamma\alpha)\underline{v}$ and $\pi_1^E = \gamma\alpha\underline{v}$, respectively.*

2.4 Renegotiations with many creditors

With many creditors bargaining might fail and the only equilibrium is the inefficient liquidation of the firm under the bankruptcy mechanism. Consider the following example:

Example 1 *Assume that upon entering renegotiations the firm has assets worth $\underline{v} = 100$. The firm has two creditors with claims of $d_1 = d_2 = 60$ each and that liquidation costs are $\alpha = 5\%$. Assume that the equityholders propose first.*

If the first creditor arriving at the bargaining site refuses the equityholders offer the company ends up in bankruptcy court at which point the first creditor will get a liquidation payoff as specified in Equation (1) of $L_1 = \frac{d_1}{d_1+d_2}(1-\alpha)\underline{v} = \frac{60}{60+60}(1-0.05)100 = 47.5$. The equityholders thus have to make a pari-passu offer to exchange each creditor's debt with a new claim worth at least $X \geq 47.5$.

The concession of the first creditor to reduce his claim to X, however, creates a positive externality for the second creditor. If he refuses the equityholders' offer he will get in liquidation $L_2 = \frac{d_2}{X+d_2}(1-\alpha)\underline{v} = \frac{60}{X+60}(1-0.05)100$. The equityholders thus have to set the exchange offer such that the second creditor is at least as well of by accepting the offer rather than going to bankruptcy court, that is

$$X \geq L_2 = \frac{60}{X+60}(1-0.05)100 \tag{4}$$

Solving for X we get that $X \geq X^* = 51.24$ in order to make both players accept the exchange offer. An offer of $X^* = 51.24$ is clearly better than player 1's liquidation value of $L_1 = 47.5$ and exactly equals player 2's liquidation value. Such an offer, however, is infeasible for the equityholders as the sum of the minimum acceptable offers for both creditors exceeds the firms resources, $2 \times 51.24 = 102.48 > 100$. Anticipating that the equityholders cannot make the second creditor an acceptable offer the first creditor refuses any offer from the equityholders and goes to bankruptcy court. The liquidation in court is clearly inefficient as a negotiated solution could have avoided the dead weight liquidation costs.

The example illustrates two necessary conditions for a breakdown of bargaining: first, the asset value can not be too high. In this particular example the two debtors can agree on a bargaining solution as long as the asset value upon entering renegotiations exceeds \$108. It is easy to verify that in this case the optimal strategy of the equityholders is to offer the creditors $X^* = 54$, respectively, and keep zero for themselves.

Second, bargaining will only break down when liquidation costs are low. Low liquidation costs increase players' payoff under the bankruptcy mechanism and thus decrease their willingness to accept the proposers offer. In the extreme case of 100% liquidation costs creditors' outside option is zero, they will therefore accept any offer that leaves them with a weakly positive payoff and renegotiations always succeed. In example 1 creditors can find a bargaining solution as long as liquidation costs $\alpha \geq 1/12$. The intuition for the symmetric bargaining case with three players is summarized in the following proposition:

Proposition 2 *Assume that upon entering renegotiations the firm has assets worth $\underline{v}$ and two* *outstanding debt claims with face value $D/2$ each. Renegotiations will only fail when liquidation costs $\alpha < \frac{1}{2}$ and $\underline{v} < v_3^* = D(1-2\alpha)$. If renegotiations are successful the expected payoff for equityholders is*

$$\pi_3^E = \frac{\gamma}{2}\left(2\underline{v} - \sqrt{D}\sqrt{D - 8(\alpha-1)\underline{v}} + D\right) \tag{5}$$

and for each debtholder is

$$\pi_3^D = \underline{v} - \frac{\gamma}{2}\left(2\underline{v} - \sqrt{D}\sqrt{D - 8(\alpha-1)\underline{v}} + D\right) \tag{6}$$

From Proposition 2 we can see that a seemingly more efficient bankruptcy mechanism with lower liquidation costs α can lead to a more inefficient outcome where a voluntary restructuring of the firms debt is impossible and dead weight losses are realized under the bankruptcy mechanism. The critical asset value v_3^* under which renegotiations fail increases as liquidation costs fall, making it even harder for firms to restructure.

These findings have also important empirical implications for estimating bankruptcy costs. Firms with higher bankruptcy costs will be able to restructure out-of-court and thus not show up in a sample of bankruptcy filings. This possible selection bias might lead to an underestimation of actual bankruptcy costs.[7] Estimating bankruptcy costs from observed bankruptcy filings may thus lead to an underestimation of average bankruptcy costs as firms with high bankruptcy costs will never realize them in equilibrium.

Proposition 2 also has important implication for the point at which equityholders optimally choose to restructure the firm. In the classic capital structure literature equityholders support the firm's debt as long as the call option value of keeping the firm alive is greater than the cost of the coupon payments. With renegotiations equityholders might collect a positive payoff in restructuring which will change the point at which they optimally decide to enter renegotiations. I will endogenize this lower restructuring threshold in Section 3 of this paper.

While I assume that players arrive sequentially at the bargaining site the actual arrival order does not matter in the case of symmetric creditors. Since all creditors get the same offer a

[7] Glover (forthcoming) finds a similar implication through a different channel. He argues that firms with high bankruptcy costs take on less debt and are thus underrepresented in a sample of bankruptcies. In this paper it will turn out that firms with high bankruptcy costs take on more debt as debt can always be renegotiated out of court.

specific creditor cannot be better or worse off by taking a specific spot in the line. The rate at which creditors be able to exchange the old debt for new claims is set to make the last creditor indifferent between accepting the offer or going to bankruptcy court.

When creditors are asymmetric, however, the arrival order becomes important. Creditors can maximize the share that they can extract from the company by letting the smallest creditor negotiate last. The more previous creditors have conceded in renegotiations, the stronger is the last creditor's outside option. By letting the smallest creditor go last, creditors can maximize the last creditor's bargaining power and since the last creditor's outside option will determine the offer that all creditors get through the pari-passu nature of the exchange offer, the amount that creditors can extract in renegotiations is maximized. The equityholders can maximize their share by negotiating with the largest creditor last. This minimizes the concessions that previous creditors have made and thus lowers the last creditors threat-point. Equithyholders can make a lower overall offer for the exchange offer which will reduce the payments to all creditors.

The exact outcome of the bargaining game with asymmetric creditors will therefore depend on the arrival order and on the assumptions whether and how creditors or equityholders can influence the bargaining sequence. While the bargaining order might change payoffs or the chance of bargaining failure for specific parameter values the general findings of the paper will be unaffected.

It is fairly easy to generalize the results of the two player game for the general case of n players. We have to solve for the equilibrium numerically using the following procedure:

Proposition 3 *Let $< d_i >$ be the sequence of face values of the n creditors' debt claims indexed in size where d_1 is the smallest claim and denote by $D = \sum_{i=1}^{n} d_i$ the firms total debt. Then renegotiations are successful as long as*

$$xD \leq \underline{v}, \tag{7}$$

where $x \in [\underline{x}, \overline{x}]$ is the rate exchange of old debt to new claims on the firm, which depends on *the arrival order of creditors. The bounds for x are defined as*

$$\underline{x} = \frac{\sqrt{d_n^2 + 4(D-d_n)\underline{v}(1-\alpha)} - d_n}{2(D-d_n)} \text{ and} \tag{8}$$

$$\overline{x} = \frac{\sqrt{d_1^2 + 4(D-d_1)\underline{v}(1-\alpha)} - d_1}{2(D-d_1)}, \tag{9}$$

respectively. If renegotiations succeed, the expected payoff for equityholders is

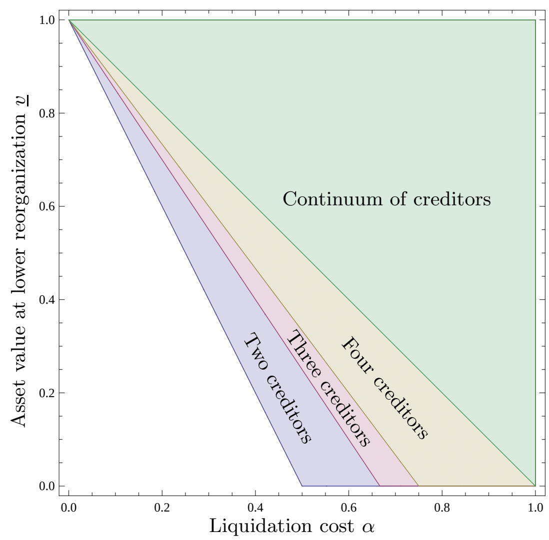
$$\pi_n^E = \gamma\left(\underline{v} - xD\right) \tag{10}$$

The case of dispersed debt can be approximated by assuming a continuum of infinitesimal small debtholders. Even in this case successful renegotiations are possible when we assume that debtholders consider the impact of their decision on the overall outcome of the restructuring.

Proposition 4 *Assume that there exists a continuum of infinitesimal creditors with an aggregate claim of D. Renegotiations of the firms debt is successful if $\frac{D-\underline{v}}{D} \leq \alpha$.*

Figure 1 illustrates the outcome of the renegotiation game graphically. When bankruptcy costs are low and the value of the firms assets is small compared to the outstanding debt then renegotiations will fail as soon as there is more than one creditor. Renegotiations are always successful when bankruptcy costs are high and the firm's assets are valuable. For intermediate regions of bankruptcy costs the debt structure and the the asset value are important. A more concentrated debt structure allows renegotiations to succeed. The success of renegotiations is also driven by the value of the assets that the firm still has. The decision to initiate renegotiations is endogenous and the result of an optimal decision by the firm's equityholders. They must decide whether it is more advantageous for them to keep the firm going and risk inefficient liquidation in case that the assets deteriorate further in value or if they renegotiate with creditors given the expected outcome of the bargaining process. I model the equityholders decision and its implications on renegotiation outcomes and optimal capital structure in the next section.

Figure 1. Region of successful renegotiations in the case of two, three, and four equally sized creditors as well as a continuum of infinitesimal small creditors. The graph shows the region of successful renegotiations for different values of the liquidation costs α on the x-axis and the asset value $\underline{v}$ on the y-axis. Debt is assumed to be 1. The largest area represents the case of two equally sized creditors and the smallest area is for the case of a continuum of infinitesimal small creditors.



3 Optimal capital structure

The solution to the renegotiations game defines payoffs conditional on the equityholders entering renegotiations but it does not define the point at which equityholders optimally start renegotiations with debtholders. As we saw from Section 2 the success of the bargaining process as well as its payoffs will depend on the value of the assets $\underline{v}$ that can be shared in the renogotiation process. The point at which shareholders open renegotiations is endogenous and optimally chosen by equityholders to maximize their payoff. Depending on the number of creditors, firms bankruptcy costs, and their bargaining power, the equityholders will compare the expected payoff from the bargaining game with the value of their claim when the firm is kept alive with the existing debt structure. The choice of the optimal threshold at which to start renegotiations gets further complicated as the value of the firm's assets is partly driven by the possibility to renegotiate claims again in the future. I address this problem by embedding the bargaining model into a classical EBIT based continuous time model in the spirit of Goldstein, Ju, and Leland (2001) to solve for the optimal reorganization threshold.

The firm's EBIT, denoted by ξ, is exogenously created by the unique technology of the firm and is assumed to follow a geometric Brownian motion under the pricing measure

$$d\xi_t = \xi_t \mu dt + \xi_t \sigma dW_t \tag{11}$$

with initial value ξ_0, constant drift μ and volatility σ.

The firm is controlled by the equityholders who issue perpetual, callable debt against the firm's EBIT. The debt pays an instantaneous coupon of c and can be called at any time at a proportional premium λ of the face value. Issuing debt incurs a proportional transaction cost of k. In line with the previous literature I assume that interest expenses are tax deductible resulting in a tax advantage of debt. Denote by $\hat{r}$ the constant risk free interest rate and the tax rates for interest income and dividend payments by τ_i and τ_e, respectively. The effective interest rate at which an investor can borrow to replicate contingent claims on the firm's EBIT is then $r = \hat{r}(1 - \tau_i)$.[8] As usual in these models a positive difference in tax rates $\tau_e > \tau_i$ generates a tax advantage of debt as a reason to issue debt.

When the firm's EBIT is either too high or too low, equity holders can adjust the firm's capital structure. Denote by ξ_0 the initial EBIT level, i.e. when debt is first issued. As the firm's EBIT grows the equityholders will find that the tax benefit at the current debt level is low enough to justify the transaction cost for re-levering the firm to the optimal debt level. At the upper restructuring threshold $u\xi_0$, for some constant u, equityholders call the outstanding debt and issue new debt. The upper restructuring threshold in the model is in line with the standard literature and the details of its derivation are therefore put in Appendix B.

As the firm's EBIT deteriorates the firms cash flow is insufficient to maintain the coupon payments and equityholders have to inject funds to meet the debt obligations. At the lower restructuring threshold, denoted by $l\xi_0$, for some constant l, equityholders will find it optimal to walk away from the firm or initiate renegotiations on the firm's debt. I will determine the optimal location of the lower restructuring threshold in Section 3.1

The value of the firm's debt and equity can be derived as contingent claims on the firm's EBIT. I follow closely the notation and the very clear derivation of Christensen, Flor, Lando,

[8] In line with the previous literature I have to assume that $\mu < r$ to ensure that the value of equity is finite.

and Miltersen (forthcoming) and similar to them I show that the time t market value of debt $D(\xi_t, \xi_s)$ and equity $E(\xi_t, \xi_s)$ can be written as functions of the current EBIT level ξ_t and the EBIT level ξ_s at which the claims were issued given that the EBIT process has neither hit the upper nor the lower restructuring threshold. The pricing function for debt and equity will be derived in detail in Appendix A. It is also noteworthy that debt and equity are homogeneous of degree one in EBIT, e.g. $D(\kappa\xi_t, \kappa\xi_s) = \kappa D(\xi_t, \xi_s)$, which allows us to simplify notation such that the value of debt an equity at the time of issuance can be written as the product of a constant and the EBIT level at the time of issuance:

$$
\begin{aligned}
D(\xi_s, \xi_s) = \xi_s D(1,1) &= D\xi_s \\
E(\xi_s, \xi_s) = \xi_s E(1,1) &= E\xi_s
\end{aligned}
$$

Because of the homogeneity of degree one the firm can be scaled up or down when the EBIT hits either the upper or the lower restructuring threshold.

Finally denote by $A\xi$ the total value of the firm to the equityholders just before new debt is being issued. In addition to the equity value $E\xi$, the owners will issue debt with proceeds of $(1-k)D\xi$ after issuance costs.

$$
A\xi = E\xi + (1-k)D\xi \tag{12}
$$

The total face value of debt is $D\xi_0$ as I follow the standard assumption that debt is issued at par.

3.1 Lower Restructuring Threshold and the equityholders' commitment problem

At the lower restructuring threshold the equityholders enter renegotiations with the debtholders to restructure the firm's debt. I assume that the decision to initiate renegotiations is irreversible and that upon entering renegotiations a certain fraction ϕ of the assets gets destroyed, e.g. because of advisory or legal fees, accounting costs, or managerial attention devoted to the renegotiation process. Once the equityholders have started bargaining the outcome and the payoffs are determined according to the bargaining game as detailed in Section 2.

To determine bargaining payoffs we have to specify the value of the assets $\underline{v}$ upon entering renegotiations. Any potential new buyers of these assets will maximize their after tax value and lever them up to the optimal leverage and thus generating extra value from the tax shield. At the lower restructuring boundary EBIT is $l\xi_0$ and the assets can be sold for the value of an optimally levered firm at that EBIT level. After subtracting the costs for entering renegotiation ϕ the value of the firms assets are thus

$$\underline{v} = (1 - \phi)Al\xi_0. \tag{13}$$

If this asset value at the lower restructuring threshold is greater than the face value of debt then debt is risk free as the are enough funds available at the restructuring threshold to pay off the firm's debt in full. Any remaining value would then go to the equityholders. In this paper I want to focus on the more interesting case when debt is risky and the asset value at the lower restructuring threshold is insufficient to satisfy the debtholders outstanding claims in full.[9]

For a given lower restructuring threshold the contingent claims of equity and debt are determined by the usual boundary (value matching) conditions. At the lower restructuring threshold the expected payoffs from the bargaining game for the equity and debtholders have to equal the value of the debt and equity claims, respectively. For the quityholders the condition is

$$E(l\xi_0) = \pi^E(l\xi_0) \tag{14}$$

The equityholders will determine the optimal lower restructuring threshold according to the usual smooth pasting condition such that at the lower restructuring threshold $l\xi_0$ the first derivative of the equity value, E, with respect to EBIT equals the equityholders' marginal payoff from renegotiations, π^E,

$$\left.\frac{\partial E(\xi)}{\partial \xi}\right|_{\xi=l\xi_0} = \left.\frac{\partial \pi^E(\xi)}{\partial \xi}\right|_{\xi=l\xi_0} \tag{15}$$

Without renegotiations equityholders always get zero at the lower restructuring threshold, $\pi^E = \pi^E_L = 0$ from Equation (3), and we get the standard result of the previous literature. With renegotiations we have to first determine whether or not renegotiations are successful as specified in

[9] I verify that debt is indeed risky for all the comparative statics results in the paper.

Condition (7). If renegotiations fail, the firm gets liquidated and the payoff for equityholders is again zero. If renegotiations are successful then equityholders can obtain a positive payoff and $\pi^E = \pi_n^E$ as specified in Equation (10).

Renegotiations have a profound impact on the lower restructuring threshold in three ways. First, when renegotiations are successful equityholders will be able to extract a certain fraction of the firm's value in bargaining. The equityholders payout is increasing in EBIT resulting in a positive value on the right hand side of the smooth pasting condition, Equation (15). Equityholders do not "walk away" at the lower restructuring threshold as it is assumed in the standard literature but they optimally choose the restructuring threshold based on what they can extract from creditors in renegotiations.

Second, the equityholders' choice of the restructuring threshold can affect the success or failure of renegotiations. When bankruptcy costs are low and with more than two creditors renegotiation success will depend upon the level of the restructuring threshold: A high restructuring threshold ensures successful renegotiations. Equityholders benefit from extracting a payment in bargaining and this outcome can also be more efficient as the firm can save the liquidation costs that arise under the bankruptcy mechanism. A low restructuring threshold will cause renegotiations to fail but equityholders may find it optimal defer restructuring to exploit the call option of equity accepting that once the lower restructuring threshold is reached, renegotiations will fail and the bankruptcy mechanism will be evoked. In this case the optimal lower restructuring threshold will similar to the case without renegotiations. This solution can be efficient when bankruptcy costs are low. Lowering the threshold EBIT level at which restructuring will occur results in less frequent renegotiations which saves the cost ϕ of entering renegotiations and the issuance cost of the new debt.

Finally, once EBIT hits the lower restructuring threshold equityholders must find it ex-post still optimal to enter renegotiations. Ex-ante equityholders might want to initiate renegotiations at an EBIT level that ensures successful renegotiations. However, once the EBIT hits that lower restructuring threshold, $l\xi_0$, they may find that the payoff that they get from renegotiations, $\pi^E(l\xi_0)$ is below the value continuation value of the firm that comes from the limited liability option. Immediately before entering renegotiations the equityholders thus have to solve for the optimal lower and upper restructuring thresholds given the firms existing coupon and assuming that they would not renegotiate the firm's debt. Technically equityholders take the current coupon C as given, set $\pi^E(\xi) = \pi_L^E(\xi) = 0 \forall \xi$ and solve the value matching conditions and smooth pasting conditions to obtain a new upper restructuring threshold u^N (Equations (24), and (25)) and a new lower restructuring threshold l^N (Equations (14), and (15)), respectively. Continuation only makes sense when $l^N < l$, i.e. without renegotiations the equityholders would keep the firm alive and not walk away, and when the continuation value exceeds the payoff from renegotiations. The continuation value is then the value of equity at the point where equityholders should enter renegotiations given the new restructuring thresholds u^N and l^N that are chosen without renegotiations. A lower restructuring threshold is therefore only ex-post optimal when

$$\pi^E(l\xi_0, l, u) \geq E(l\xi_0, l^N, u^N, \text{no renegotiations}) \tag{16}$$

The ex-post incentive problem of the equityholders will be analyzed in greater detail in Section 3.3.

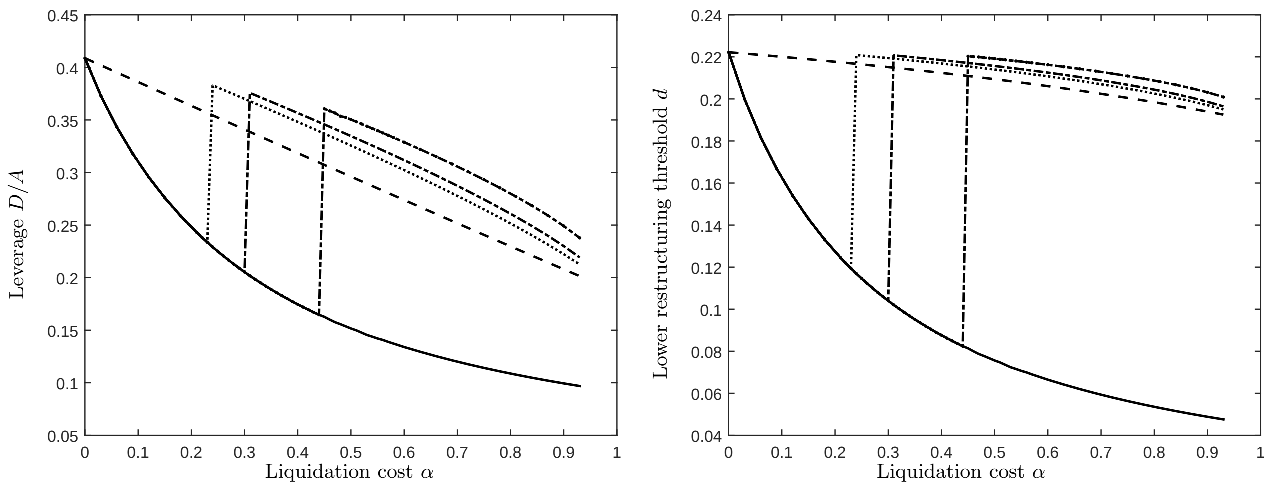
3.2 Comparative statics

To get some intuition for our model and show some comparative statics I solve the model for the following parameters which I refer to as the base case: $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, \alpha = 0.4, k = 0.03, \phi = 0.05, \gamma = 0.5$.

The left graph in Figure 2 plots the optimal initial leverage of the firm as a function of the liquidation cost α. Without renegotiations (solid line) we get the textbook result of the tradeoff theory that optimal leverage is declining in bankruptcy costs. With one creditor we still see that firms will take on more leverage. As shown in Proposition 1 renegotiations with one creditor are always successful and dead weight liquidations costs can be avoided thus making debt more attractive and firms have therefore higher leverage.

The most interesting case occurs with multiple creditors. With two (dashed line) or more creditors the optimal leverage is non-monotonic in bankruptcy costs. From Proposition 2 we know that renegotiations will not always be successful. Specifically for low bankruptcy costs renegotiations fail and the firm will get liquidated. The optimal leverage then coincides with the case of no renegotiations. If bankruptcy costs are high enough renegotiations will succeed and optimal leverage will jump to a much higher level. With multiple creditors an increase in

Figure 2. Optimal leverage (left) and lower restructuring threshold (right) as a function of liquidation costs α without renegotiations, and with renegotiations for several debt structures, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5$.



bankruptcy costs can ensure that debt will be successfully renegotiated and thus make debt financing more attractive, which contradicts the classic tradeoff theory. According to this model firms with low and high bankruptcy costs should choose high leverage, the former because bankruptcy is not very costly and the latter because bankruptcy can be avoided through renegotiations. Firms with intermediate bankruptcy costs will optimally choose a less debt as renegotiations are likely to fail and bankruptcy is somewhat costly. Renegotiation failure can thus have a profound impact on capital structure and lead to a non-monotonic relationship between liquidation cost and a firm's optimal capital structure.

As a second order effect we see that optimal leverage is declining in liquidation costs even when renegotiations are always successful, no firm gets liquidated, and liquidation costs never occur. Increasing liquidation costs lowers the amount that debtholders obtain in bankruptcy, i.e. their threatpoint for bargaining. Whenever the equityholders get to propose they offer the debtholders the value they would receive in bankruptcy court which is decreasing in liquidation cost. Rationally anticipating that they will obtain less in renegotiations debtholders are willing to pay less for the bonds when the firm first issues its debt, making it less attractive for the initial owners to issue debt. This logic is also the reason why optimal leverage is higher with two debtholders than one conditional that renegotiations are successful. Two debtholders can in total extract more from a firm than one debtholder and thus are, certis paribus, willing to bid more for the firms newly issued debt. When initially the firms owners can sell debt at a higher price they will optimally issue a larger amount resulting in higher optimal leverage.[10]

The optimal lower restructuring threshold is depicted in the right graph of Figure 2. After successful renegotiations the firm is re-levered to the optimal capital structure which increases firm value. When some of that value creation is shared with shareholders in a successful renegotiation of the firms debt, equityholders have an incentive to enter renegotiations early. We can see that equityholders enter renegotiations much earlier when they anticipate their success. For the region of the parameter space where they anticipate that renegotiations will be unsuccessful equityholders take full advantage of their call option and wait longer to start restructuring the firm. We can also see that the lower restructuring threshold is declining in liquidation costs which is a direct result of the lower leverage that is associated with higher liquidation costs. With lower debt obligations equityholders find it worthwhile to support the firm for a longer time.

Bargaining and the debtor structure have an important implication on the location of the default barrier. A traditional structural model would set the default barrier much lower than it is with bargaining. By allowing the equityholders to participate in the restructuring gains they have an incentive to open renegotiations much earlier. The default barrier is thus much higher than traditional models would suggest. The structure of the firms debt, whether it is concentrated like in a European style bank based financial system or weather it is more dispersed as for the typical North American firm can also influence the optimal EBIT level at which firms enter renegotiations.

In the basic version of the model rengeotiations either succeed or fail and therefore optimal capital structure shows a discrete jump. Appendix E summarizes results from an extended model where bankruptcy costs are stochastic and only become known after equityholders have entered renegoriations. The results of the model are very similar but the graphs look smoother

[10]Higher bankruptcy costs increase the share of the firm value that the equityholders can extract in renegotiations, and therefor also change slope of π^E. Because of the smooth pasting condition in Equation (15), higher liquidation costs therefore decrease the lower restructuring threshold, which leads to less frequent capital adjustments, result in lower firm value, and also have an influence on leverage.

than in the main model of the paper.

3.3 Equityholders ex-post incentive to renegotiate

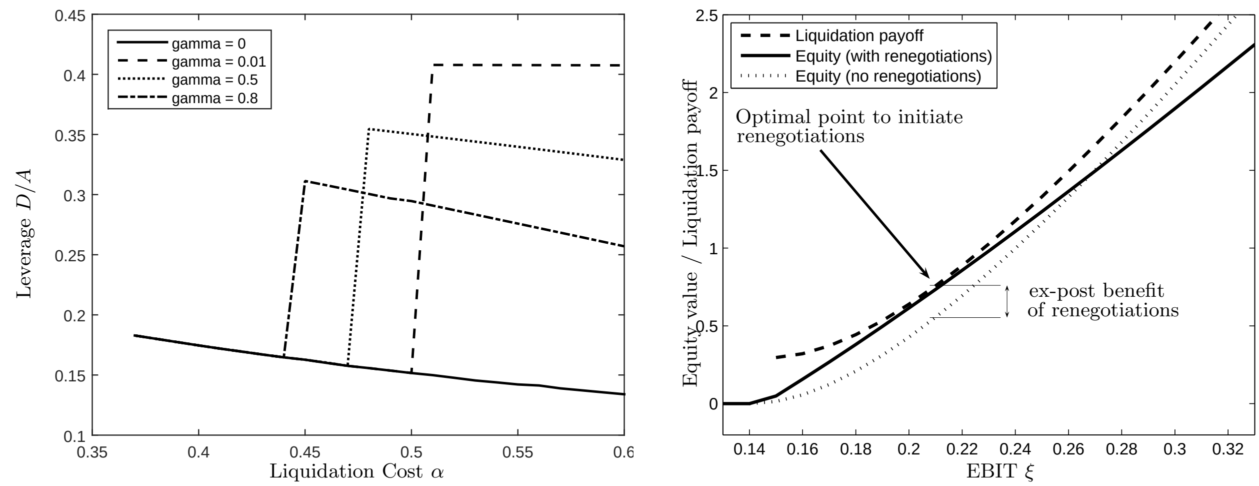
How credible is the equityholders promise to enter renegotiations on the firm's outstanding debt? Ex-ante, at the EBIT level ξ_0 as the unlevered firm issues debt to its creditors, equityholders may want to promise that they will enter renegotiations at a high enough EBIT level such that they will succeed and costly liquidation can be avoided. Since all liquidation costs are priced in by debtholders, the promise of efficient renegotiations allows the firm to sell its debt at a higher price. Ex-post however, once the debt has been issued and when the firm's EBIT comes close to the lower restructuring threshold, equityholders might find it optimal not to follow through with their promise and keep the firm alive to exploit the limited liability option.

Since keeping the firm alive is a call option with non-negative value, the equityholders will only initiate renegotiations when their expected payoff from renegotiations keeps them at least as well off as the option to continue running the firm. Awarding equityholders a positive payoff from renegotiations is therefore essential to get renegotiations started and to avoid dead weight bankruptcy costs. Leaving equityholders with a positive payoff is a clear violation of absolute priority, since the firm's asset at the time of renegotiations are always below its face value of debt. Nevertheless creditors are better off by essentially paying the equityholders to enter renegotiations at a point when the firm's debt can still be successfully restructured.

The right panel in Figure 3 illustrates the ex-post incentives. Equityholders optimally enter renegotiations when the equity value function (dashed line) smooth pastes to the payoff from renegotiations (solid line). Entering renegotiations is only optimal when their payoff exceeds the value of keeping the firm alive (dotted line). The graph illustrates two important conditions for successful renegotiations: first equityholders have to get at least as much from renegotiations as from keeping the firm alive, i.e. equity holder have to be left with a strictly positive payoff in the bargaining game. Second, equity holder bargaining power, the slope of the solid line in the graph, will influence the restructuring outcome. When equityholders get too little in restructuring, which corresponds to a flat slope the equity function might smooth paste in the region where restructurings fail, i.e. the payoff from restructuring is constant zero.

The left panel in Figure 3 shows the optimal leverage for a firm with dispersed debt as a func-

Figure 3. Optimal leverage as a function of bankruptcy costs for dispersed debt and different levels of bargaining power γ (left panel) and the Equityholders' ex post incentive to enter renegotiations once EBIT has reached the lower restructuring threshold (right panel). All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05$



tion of bankruptcy costs for different levels of the equityholders' bargaining power γ. When all the bargaining power resides with creditors, i.e. $\gamma = 0$, then equityholders will never renegotiate and optimal leverage is declining in bankruptcy cost in line with the standard tradeoff theory. When equityholders only get a small share of bargaining proceeds ($\gamma = 0.01$) they start initiate renegotiations only for relatively higher bankruptcy costs ($\alpha > 0.5$). Higher bankruptcy costs mean that the creditors can extract less in renegotiations and therefore increase the expected surplus for the proposer. Since equityholders in this case only seldom get the chance to propose they only negotiate when bankruptcy costs are high and thus the proposer's surplus is high. Once bankruptcy costs are above the threshold where equityholders negotiate, however, they will optimally take on a large amount of debt. Because creditors get most of the surplus of renegotiations, the recovery rate of creditors is high and debt sells at a high price (or low coupon). To get the desired amount of tax-deductible coupon payments equityholders sell a larger face value of debt. The upside of higher bargaining power for equityholders ($\gamma \geq 0.5$) is that they optimally enter renegotiations at lower bankruptcy costs ($\alpha > 0.42$) because their payoff from renegotiations is higher than the option of keeping the firm alive. The downside is that recovery for creditors in renegotiations is lower, which decreases the price (increases the coupon) at which debt can be sold in the market. A higher coupon means that the firm achieves the target coupon payments with a smaller amount of issued debt, i.e. a lower leverage.

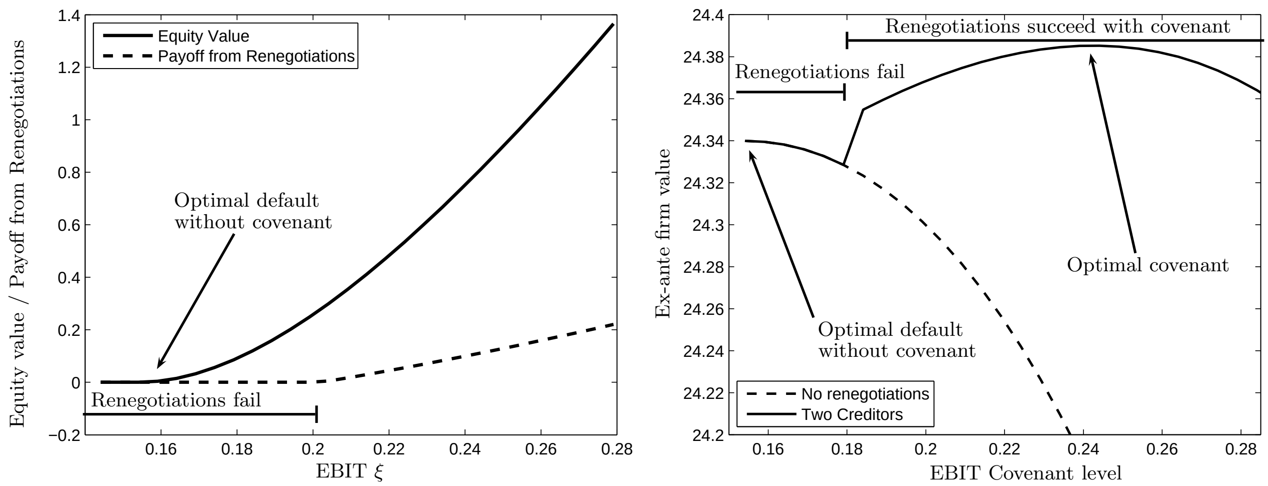
From a policy perspective the analysis in this paper shows that deviations from absolute priority are welfare increasing. By sharing part of the restructuring proceeds with equityholders the firm's creditor's can bring equityholders to the bargaining table and thus avoid dead weight bankruptcy costs. The share that equityholders should get in bankruptcy, however should be small to ensure high recovery for creditors in case of restructuring which allows the firm to sell its debt ex-ante at a higher price.

4 Optimal Covenants

One of the important determinants of renegotiations success is the value of the firm's assets when renegotiations are initiated. As demonstrated in Section 2 for a given debt structure and given bankruptcy costs renegotiations are more likely to succeed if a firm with declining EBIT starts renegotiations early while the EBIT and thus the asset value are high. Ex-ante dead weight bankruptcy costs can be avoided and a more efficient outcome can be achieved if equityholders start renegotiations early. Once debt is issued, however, equityholders will choose to default whenever the value of keeping the firm alive is less then the payments to the debtholders as defined in the smooth pasting condition in Equation (15). The left panel of Figure 4 illustrates the equityholders' decision problem. Equityholders will default whenever the equity value is tangential to the payoff that equityholders get in liquidation. In this particular example equityholders optimally default in a region where renegotiations fail and the firm gets liquidated.

One way in which the equityholders can credibly commit ex-ante to start renegotiations early is an EBIT covenant. Once EBIT hits the covenant level the firms is in teachnical default and equityholders have to initiate renegotiations with the debtholders. The right panel of Figure 4 shows the effect of covenants on the ex-ante value of the firm. Covenants below the EBIT level at which the equityholders voluntarily default are not binding and have no effect. Without renegotiations covenants clearly have no benefit for equityholders. The ex ante firm value is declining in the covenant level as equityholders are forced to default a suboptimal point where they would like to keep the firm alive and continue to serve the firm's debt.

Figure 4. Equityholders' optimal choice of the lower restructuring threshold where equity value smoothly intersects the payoff from renegotiations (left panel) and ex-ante firm value as a function of an EBIT covenant that fixes the lower restructuring threshold. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$ except for the bankruptcy cost parameter which is set at $\alpha = 0.12$.

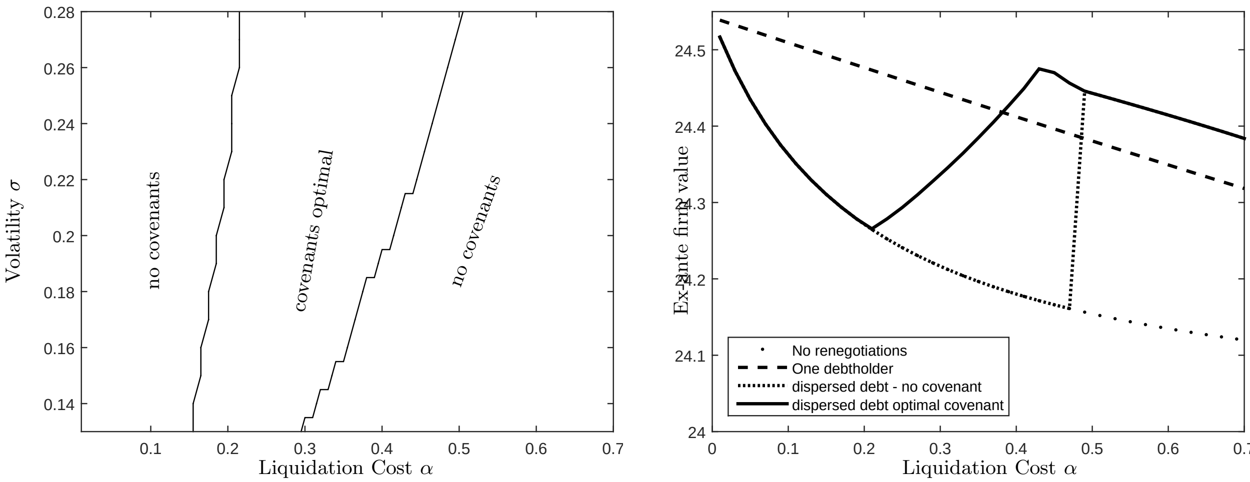


A covenant can increase ex-ante firm value when it moves the lower restructuring threshold into a region where renegotiations will be successful and dead weight liquidation costs are avoided. Since liquidation costs are ex-ante borne by the equityholders the benefit of avoiding the liquidation cost can more than offset the cost of having to default at a suboptimal point. As illustrated in the right panel of Figure 4 the ex-ante firm value jumps once the covenant is high enough to ensure successful renegotiations.[11] Once renegotiations are successful the firm can sell its debt at a different initial coupon and thus adjusts its ex-ante optimal leverage. Firm value increases up to an ex-ante optimal covenant level before it drops as the loss for equityholders from a suboptimal lower restructuring threshold outweighs the benefits of successful renegotiations.

Covenants can increase ex-ante firm value ensuring that ex-post renegotiations of the firms

[11]Note that the region where negotiations fail in the right panel of Figure 4 is different than in the left panel. In the left panel I keep the amount of debt, its coupon, and the restructuring thresholds fixed to highlight the optimal choice of the lower restructuring threshold for a given capital structure. In the right panel I compute for every covenant level the optimal capital structure. As the covenant level increases, leverage, coupon, the price of debt at issuance, the restructuring thresholds, and thus the region where renegotiations fail change.

Figure 5. Parameter region for which covenants can increase ex-ante firm value (left panel) and ex-ante firm value as a function of bankruptcy costs with and without optimal covenants. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$ except for the bankruptcy cost parameter which is set at $\alpha = 0.12$.

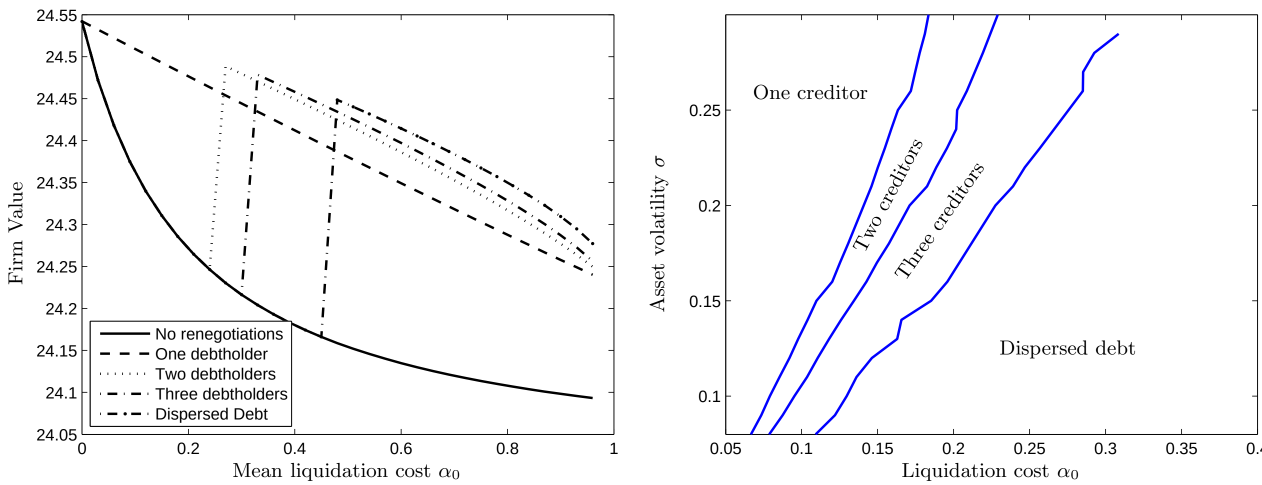


are initiated early enough to be successful. Covenants can thus prove to be valuable in the absence of asymmetric information, moral hazard, or lack of enforceability of debt contracts. In the context of this model covenants are only valuable whenever renegotiations would fail without the covenant, that is for firms with fairly low bankruptcy costs. Firms with high bankruptcy costs can always successfully renegotiate their debt and covenants therefore only force the equityholders to choose an inefficient restructuring threshold and reduce the ex-ante firm value. The left panel of Figure 5 illustrates the region of the parameter space where covenants can increase the ex-ante firm value.[12] We can see that covenants are optimal for firms with intermediate bankruptcy costs. Saving the liquidation cost does not outweigh the costs of choosing a suboptimal lower restructuring boundary through a covenant for firms with a low liquidation cost parameter α. Firms with high liquidation costs do not need covenants as they can always successfully renegotiate their debt.

Even though covenants can mitigate some of the frictions that are associated with bargaining breakdown and inefficient liquidation for medium bankruptcy firms they are not able to prevent

[12] The quality of the graph will be improved in future versions of the paper. The zig-zag is due to computational limitations.

Figure 6. The left panel shows ex-ante firm value (before the first debt is issued) under the optimal capital structure as a function of liquidation cost without renegotiations, and with one and two creditors, respectively. The right panel shows the optimal number of creditors that maximize ex-ante firm value for a range of liquidation costs and asset volatilities. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5$



liquidations entirely. The right panel of Figure 5 shows the optimal leverage as a function of bankruptcy costs with and without optimal covenants. The main insight that leverage is non-monotonic in bankruptcy cost is preserved.

5 Debt structure

Since the number of creditors will affect the chances off success and the payoffs for claimants in renegotiations debt structure will have an impact on firm value. Thus the model offers a novel explanation for the heterogeneity in firms' debtor structure. The left panel in Figure 6 plots the firm value under the optimal capital structure for different values of liquidation costs. Firms with low bankruptcy costs optimally seek concentrated debt, with only one creditor for very low bankruptcy costs as renegotiations will always be successful under this debtor structure. Firms with higher bankruptcy costs have a higher chance of renegotiating debt and will optimally choose more dispersed debt.

The right panel in Figure 6 shows the optimal debt structure as a function of liquidation costs and asset volatility. Concentrated debt is optimal for low bankruptcy cost high volatility firms while more dispersed debt is optimal for firms with low asset risk and higher liquidation costs.

6 Conclusion

In this paper I examine the conditions under which a firm is able to successfully renegotiate its debt with multiple creditors. The model offers an explanation why firms are sometimes unsuccessful at restructuring their claims out-of-court even though all parties would be better off avoiding dead weight bankruptcy costs. The possibility to renegotiate debt also impacts the optimal capital structure of the firm, the firms debt structure, and the optimal use of loan covenants.

The paper has four empirical predictions. First, leverage is non-monotonic in bankruptcy costs. Firms with high bankruptcy costs are willing to take on more debt because they are sure that that they can successfully renegotiate their debt and firms with low bankruptcy costs take on more debt because bankruptcy is not very costly. Firms with intermediate bankruptcy costs are reluctant to take on much debt because they will have a hard time to renegotiate with their creditors.

Second, the distribution of actual bankruptcy costs differs systematically from the distribution of observed bankruptcy losses. According to this model firms with high bankruptcy costs will always be able to successfully restructure their debt because creditors know that they will not be able to recover much in bankruptcy while firms with low bankruptcy costs are more likely to be be liquidated. In a sample of observed bankruptcies low bankruptcy firms will therefore be overrepresented relative to firms with high bankruptcy costs. Empirically estimating bankruptcy costs should therefore consider this selection bias.

Third, covenants are most useful for firms with medium bankruptcy costs, where renegotiations would otherwise fail. Those firms find it optimal to use covenants that commit the firm to enter renegotiations at an ex-ante optimal EBIT level. Finally, the model derives predictions on optimal debt structure. Riskier firms with low bankruptcy costs will choose a more concentrated debt structure while safer firms with high bankruptcy costs will optimally choose dispersed debt.

The paper also has some implications on the optimal design of the bankruptcy process. Increasing the efficiency of bankruptcy courts might not be welfare improving. While efficient bankruptcy resolution saves dead weight losses it might create bargaining failure so that creditors do not restructure out-of-court. Higher bankruptcy costs can serve as credible threat to encourage out-of-court renegotiations.

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A Pricing Functions for D and E

The continuous time capital structure model in this paper follows the specification of Christensen, Flor, Lando, and Miltersen (forthcoming) except for the lower restructuring threshold. Most of the proofs for the contingent claims are therefore identical to their paper. As standard in the literature consider the time t value of a contingent claim $F(\xi, \delta, b, \underline{\xi}, \underline{F}, \overline{\xi}, \overline{F})$ on the current level of EBIT ξ that pays a dividend stream of $\delta\xi_s + b$ at any time $s \geq t$ until EBIT hits either an upper bound $\overline{\xi}$ or a lower bound $\underline{\xi}$ in which case the claim pays $\overline{F}$ or $\underline{F}$, respectively. Such a claim follows the ODE

$$\frac{1}{2}\sigma^2\xi^2\underline{F}''(\xi) + \hat{(\mu)}\xi\underline{F}'(\xi) - r\underline{F}(\xi) + \delta\xi + b = 0 \tag{17}$$

with boundary conditions $F(\underline{\xi}) = \underline{F}$ and $F(\overline{\xi}) = \overline{F}$ that has the solution

$$F(\xi, \delta, b, \underline{\xi}, \underline{F}, \overline{\xi}, \overline{F}) = k_1\xi^{x_1} + k_2\xi^{x_2} + \frac{\delta\xi}{r-\mu} + \frac{b}{r} \tag{18}$$

with

$$\begin{aligned}
k_1 &= \frac{1}{\Sigma}\left((\overline{F} - -\Delta\overline{\xi} - B)\underline{\xi}^{x_2} - (\underline{F} - \Delta\underline{\xi} - B)\overline{\xi}^{x_2}\right) \\
k_2 &= \frac{1}{\Sigma}\left((\underline{F} - \Delta\underline{\xi} - B)\overline{\xi}^{x_1} - (\overline{F} - \Delta\overline{\xi} - B)\underline{\xi}^{x_1}\right) \\
x_1 &= \frac{(\frac{1}{2}\sigma^2 - \hat{\mu}) + \sqrt{(\hat{\mu} - \frac{1}{2}\sigma^2)^2 + 2(1-\tau_i)r\sigma^2}}{\sigma^2} \\
x_2 &= \frac{(\frac{1}{2}\sigma^2 - \hat{\mu}) - \sqrt{(\hat{\mu} - \frac{1}{2}\sigma^2)^2 + 2(1-\tau_i)r\sigma^2}}{\sigma^2} \\
\Sigma &= \overline{\xi}^{x_1}\underline{\xi}^{x_2} + \underline{\xi}^{x_1}\overline{\xi}^{x_2}
\end{aligned}$$

and $\Delta\xi_t + B$ where $\Delta = \delta/(r-\mu)$ and $B = b/r$.

As always in these type of models note that F is homogeneous of degree one:

$$F(h\xi, \delta, hb, h\underline{\xi}, h\underline{F}, h\overline{\xi}, h\overline{F}) = hF(\xi, \delta, b, \underline{\xi}, \underline{F}, \overline{\xi}, \overline{F})$$

Debt and Equity in the model can be represented as specific cases of the general claim F. Debtholders and equityholders share the after tax cash flow that is generated by the EBIT process. Debtholders pay a tax on interest income τ_i on their promised coupon C such that

$$\delta\xi + b = (1-\tau_i)C \tag{19}$$

Equityholders can claim the residual cash flow after paying corporate tax τ_c and personal tax on dividend income τ_d.

$$\delta\xi + b = (1-\tau_c)(1-\tau_d)(\xi - C) = (1-\tau_e)(\xi - C) \tag{20}$$

where the effective tax rate τ_e is

$$\tau_e = \tau_c + (1-\tau_c)\tau_d \tag{21}$$

In line with most papers in the previous literature I assume that the firm can realize a tax benefit for debt even when EBIT is less than the outstanding coupon payment, i.e. $\xi < C$. In most jurisdictions tax losses can be carried back and forward for several years so that tax losses can be offset with past or future tax liabilities.

B Upper restructuring threshold

As noted before the equityholders want to increase the firms leverage to take advantage of the tax shield when the EBIT process hits the upper restructuring threshold $u\xi_0$. At the upper restructuring threshold repayment of the firm's debt is imminent. When calling the issue the firm will pay debtholders a premium of λ over the face value $D\xi_0$ of the outstanding debt. The debt value has to satisfy the value matching condition

$$D(u\xi_0, \xi_0) = (1+\lambda)D\xi_0, \tag{22}$$

which ensures that when the EBIT reaches the upper threshold $u\xi_0$ the value of debt that was issued at EBIT level ξ_0 equals the call value, which is $(1+\lambda)$ times the face value of the debt, $D\xi_0$.

Similarly at the upper threshold $u\xi_0$ is the value of the equity $E(u\xi_0, \xi_0)$ under the old debt, which was issued at ξ_0, has to equal the value of equity $E(u\xi_0, u\xi_0)$ after the issuance of new debt at $u\xi_0$ plus the proceeds from the new debt issuance after transactions costs minus the repayment of the old debt including the call premium.

$$\begin{aligned} E(u\xi_0, \xi_0) &= E(u\xi_0, u\xi_0) + (1-k)D(u\xi_0, u\xi_0) - (1+\lambda)D\xi_0 && (23)\\ &= (Au - (1+\lambda)D)\xi_0 && (24) \end{aligned}$$

The pricing functions of debt and equity have to satisfy these two value matching conditions. The associated smooth pasting condition defines the optimal upper restructuring threshold. At the restructuring threshold the first derivative of equity with respect to EBIT immediately before issuance must equal the first derivative at issuance, i.e.

$$E^1(u\xi_0, \xi_0) = \frac{\partial A\xi}{\partial \xi} = A \tag{25}$$

where $E^1(x, \xi_0) = \left.\frac{\partial E(\xi, \xi_0)}{\partial \xi}\right|_{\xi=x}$ is the first partial derivative of the equity pricing function with respect to ξ evaluated at x.

C Alternative bargaining protocol

As an alternative to *pari passu* offers consider a framework in which the proposer can contract exchange rates with creditors individually. Then the first creditor will be in a weak position as no other creditors have made any concessions yet. As bargaining progresses each creditor will be in a stronger position than the previous one, benefiting from the concessions previous creditors make. This section briefly summarizes the outcomes from such an alternative bargaining framework. The main results of the paper are robust with respect to this alternative specification.

Assume the same structure as in Section 2.2 except for the fact that the proposer makes an offer to each individual creditor that arrives at the bargaining site. Bargaining with one creditor is unaffected by the change in bargaining protocol. With many creditors the bargaining solution changes. He is the modified solution of example 1 in the main text:

If the first creditor arriving at the bargaining site refuses the equityholders offer the company ends up in bankruptcy court at which point the first creditor will get a liquidation payoff as specified in Equation (1) of $L_1 = \frac{d_1}{d_1+d_2}(1-\alpha)\underline{v} = \frac{60}{60+60}(1-0.05)100 = 47.5$. The equityholders thus have to offer the first creditor at least $x_1 = 47.5$. The concession of the first creditor, however, creates a positive externality for the second creditor as he arrives at the bargaining site. If he refuses the equityholders' offer he will get in liquidation $L_2 = \frac{d_2}{x_1+d_2}(1-\alpha)\underline{v} = \frac{60}{47.5+60}(1-0.05)100 = 53.02$. The second creditor will therefore not accept any offer that is below \$53.02. Such an offer, however, is infeasible for the equityholders as the sum of the minimum acceptable offers for both creditors exceeds the firms resources, $53.02 + 47.5 = 100.52 > 100$. Anticipating that the equityholders cannot make the second creditor an acceptable offer the first creditor refuses any offer from the equityholders and goes to bankruptcy court.

The corresponding proposition to Proposition 5 with an identical interpretation is:

Proposition 5 *Assume that upon entering renegotiations the firm has assets worth $\underline{v}$ and two outstanding debt claims with face value $D/2$ each. Renegotiations will only fail when liquidation costs $\alpha < \frac{1}{3}$ and $\underline{v} < v_3^* = \frac{D(1-3\alpha)}{1-\alpha^2}$. If renegotiations are successful the expected payoff for equityholders is*

$$\pi_3^E = \frac{1}{2}\gamma\underline{v}\left(\alpha + \frac{2(\alpha-1)D}{D-\alpha\underline{v}+\underline{v}} + 1\right) \tag{26}$$

and for each debtholder is

$$\pi_3^D = \underline{v} - \frac{(\alpha - 1)\gamma D\underline{v}}{D - \alpha\underline{v} + \underline{v}} - \frac{1}{2}(\alpha + 1)\gamma\underline{v} \tag{27}$$

It is fairly easy to generalize the results for the case of n players for a given arrival order. We have to solve for the equilibrium numerically using the following procedure:

Proposition 6 *Let $< d_i >$ be the sequence of face values of the n creditors' debt claims. Then renegotiations are successful as long as*

$$\sum_{i=1}^{n} x_i \leq \underline{v}, \tag{28}$$

where

$$x_i = \frac{d_i}{\sum_{k=1}^{i-1} x_k + \sum_{l=i}^{n} d_l}(1 - \alpha)\underline{v} \tag{29}$$

In case of success the aggregate payoffs for equityholders and bondholders are

$$\pi_n^E = \gamma(\underline{v} - \sum_{i=1}^{n} x_i) \tag{30}$$

$$\pi_n^D = \underline{v} - \gamma(\underline{v} - \sum_{i=1}^{n} x_i), \tag{31}$$

respectively.

The case of diapered debt can be approximated by assuming a continuum of infinitesimal small debtholders. Even in this case successful renegotiations are possible when we assume that debtholders consider the impact of their decision on the overall outcome of the restructuring.

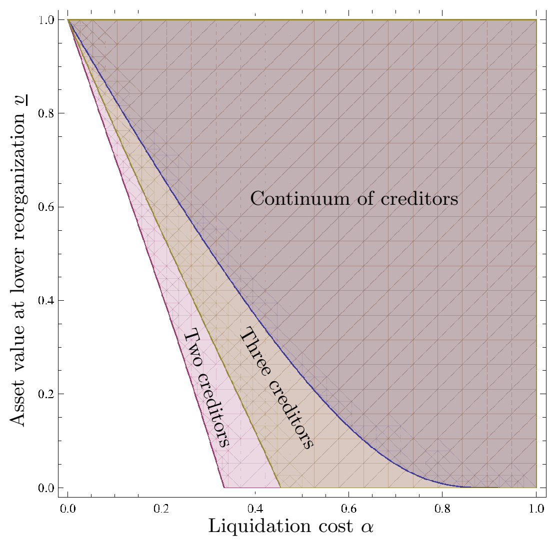
Proposition 7 *Assume that there exists a continuum of infinitesimal creditors with an aggregate claim of D. Renegotiations of the firms debt is successful if $c(1) > 1 - \underline{v}/D$ where $c(.)$ is the solution to the following differential equation*

$$c'(n) = 1 - \frac{1}{1 - c(n)}(1 - \alpha)\underline{v} \tag{32}$$

with the initial condition $c(0) = 0$.

Figure 7 illustrates the outcome of the renegotiation game under the modified protocol

Figure 7. Region of successful renegotiations in the case of two, three and a continuum of infinitesimal small creditors. The graph shows the region of successful renegotiations for different values of the liquidation costs α on the x-axis and the asset value $\underline{v}$ on the y-axis. Debt is assumed to be 1. The largest area represents the case of two equally sized creditors, the middle sized area is for the case of three equally sized creditors and the smallest area is for the case of a continuum of infinitesimal small creditors.



graphically. The outcome is more or less similar to the case of *pari-passu* offers and has the same implications for optimal capital structure choice.

D Collective action clauses

In the US bondholders cannot be forced to agree to a change in lending terms outside bankruptcy. Eichengreen and Mody (2004) note that "[t]he prohibition on majority voting in debt securities issued in the United States dates from the U.S. Trust Indenture Act of 1939. Section 316(b) of that act, which applies to the publicly-traded bonds of corporate issuers, prohibits any reduction in the amounts due a bondholder without that bondholders consent. This regulation was adopted in response to the belief that corporate insiders had taken advantage of other creditors in the financial crisis of the 1930s by forcing through restructurings that enriched shareholders at the expense of bondholders." They also note that in other jurisdictions, most notably the UK, bondholders can change terms of the bonds for all outstanding bonds given that a significant majority of outstanding principal agrees. A mechanism where only a fraction of votes is required to institute a binding of change contract terms for all bondholders is often referred to as collective action clause (CAC).

Collective action clauses are straightforward to incorporate in my bargaining model. Assume that a fraction ψ of the outstanding principal is required to get approval of the restructuring. The creditor that brings the aggregate notional value above the fraction ψ when he arrives at the bargaining site is pivotal. The proposer must make him indifferent between accepting the offer or going to bankruptcy court.

The main analysis in the paper looks at the special case of $\psi = 1$ in which the pivotal creditor is the last creditor. For any $\psi < 1$ the pivotal creditor is in a weaker bargaining position since the number of creditors that have made concessions before him is smaller and thus the externality that is generated by the other creditors' agreement to forgive debt. To formalize this intuition consider the case of dispersed debt:

Proposition 8 *Assume that upon entering renegotiations the firm has assets worth $\underline{v}$ and dispersed debt D with a collective action clause that requires a fraction ψ of the outstanding principal to agree in order to impose a restructuring for all creditors. Renegotiations are successful as long as*

$$xD \leq \underline{v} \tag{33}$$

where

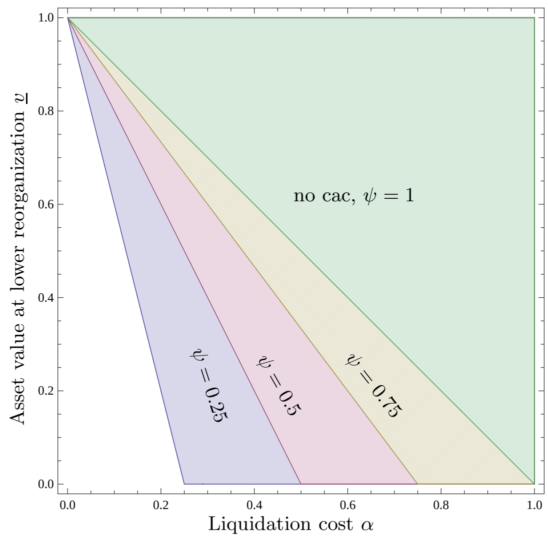
$$x = \frac{-D(1-\psi) - \sqrt{D}\sqrt{D(1-\psi)^2 + 4\psi\underline{v}(1-\alpha)}}{2D\psi} \tag{34}$$

Figure 8 illustrates the outcome of the renegotiation game for dispersed debt with different CAC thresholds. As expected a CAC overcomes some creditor coordination problems and the region for which renegotiations succeed increases as the CAC threshold decreases. While the overall payoffs for players change with the adoption of a CAC there is still the possibility that renegotiations break down and the main intuition of the paper still holds.

E Stochastic bankruptcy costs

So far equityholders can fully anticipate the outcome of the renegotiations with the firm's debtholders and optimally set the default boundary, the point at which they start renegotiations, to avoid liquidation of the firm if they find this to be optimal. In reality the outcome of renegotiations might not be known ex-ante and equityholders might have to initiate bargaining without certainty of success and their payoff. If the firm's assets are industry specific it might for example be unclear at what price assets can be sold for or if the firm's assets consist mainly of human capital it is unclear how many key employees will be lost in the restructuring process. One way to analyze this uncertainty about bargaining outcome is to assume that liquidation

Figure 8. Region of successful renegotiations for dispersed debt with different collective action clause (CAC) thresholds ψ. The graph shows the region of successful renegotiations for different values of the liquidation costs α on the x-axis and the asset value $\underline{v}$ on the y-axis. Aggregate debt is assumed to be 1.



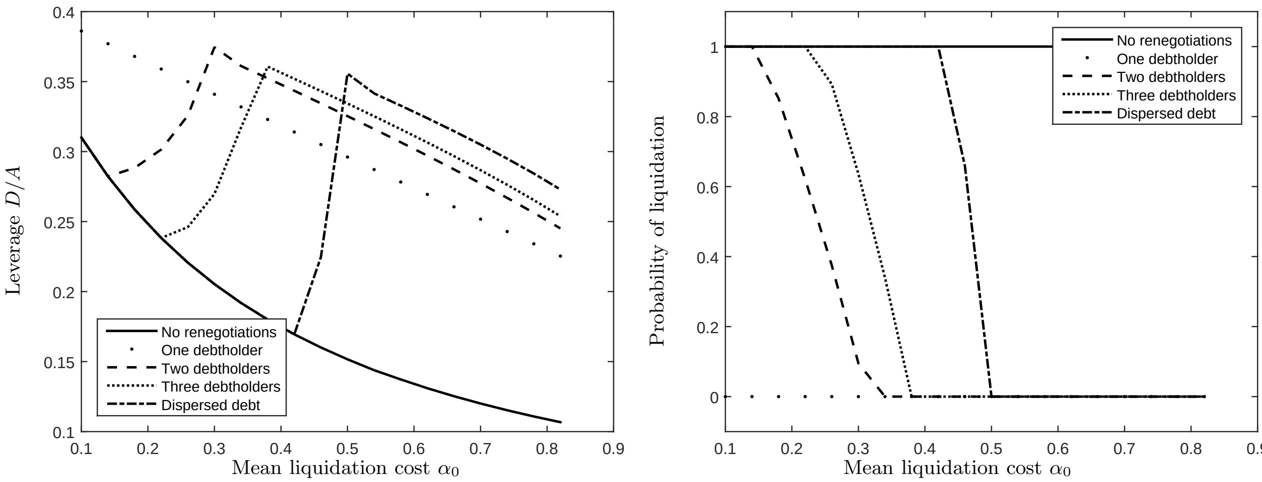
costs are stochastic. Specifically I assume that liquidation costs α are drawn from a uniform distribution with mean α_0 once the firm enters the bargaining game:

$$\alpha \sim U\left[\alpha_0 - d, \alpha_0 + d\right]. \tag{35}$$

The left panel in Figure 9 shows the optimal initial leverage as a function of average liquidation costs. Again the intuition from the model with fixed bankruptcy costs applies. Optimal leverage is a non-monotonic function in bankruptcy costs as firms with higher bankruptcy costs might find it optimal to take on more leverage as they will always be able to successfully renegotiate their debt and never realize the bankruptcy costs.

The right panel in Figure 9 shows the firms probability or getting liquidated under the optimal capital structure. Without renegotiations the firm gets always liquidated (solid line) and with one creditor renegotiations always succeed and the firm survives. For both, two and three debtors, the liquidation probability is decreasing in liquidation costs and in the number of debtors. Following the intuition of the previous section firms with higher expected liquidation costs are more likely to successfully restructure their debt and avoid liquidation.

Figure 9. Optimal leverage (left) and probability of liquidation (right) as a function of mean liquidation cost without renegotiations, and with one and two creditors, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$



F Proofs

Proof of Proposition 1. If the equityholders arrive first, which happens with probability γ, they offer the debtholder just as much as he would get in bankruptcy, which is $\underline{v}(1-\alpha)$ and keep the rest, i.e. $\underline{v}\alpha$. If the debtholder arrives first he will offer zero to the equityholders and keep the value of the assets $\underline{v}$. The expected payoffs are $\pi_1^D = \gamma(1-\alpha)\underline{v} + (1-\gamma)\underline{v} = (1-\gamma\alpha)\underline{v}$ and $\pi_1^E = \gamma\alpha\underline{v}$, respectively. ■

Proof of Proposition 2. Since we are considering pari-passu offers the proposer has to offer all the debtholders the same exchange offer X which has to better than their liquidation value under bankruptcy. Since the liquidation payoff is increasing in the number of creditors that already have accepted the exchange offer only the last creditor is important. After the first creditor has reduced its debt to X the second creditor's payoff in bankruptcy court is $L_2 = \frac{D/2}{X+D/2}\underline{v}(1-\alpha)$ and making the second creditor in the exchange offer weakly better off than in bankruptcy requires that

$$\begin{aligned} X &\geq L_2 \\ X &\geq \frac{D/2}{X + D/2}\underline{v}(1-\alpha) \\ X^2 + \frac{D}{2}X &\geq \frac{D}{2}\underline{v}(1-\alpha) \end{aligned}$$

which has the positive root:

$$X \geq X_3^* := \frac{-D + \sqrt{D^2 + 8Dv(1-\alpha)}}{4} \tag{36}$$

Renegotiations fail if the firm's assets are insufficient to cover the minimum offer that has to be made to the firm's debtholders, i.e. $\underline{v} < 2X$. Substituting from equation 36 we get

$$\begin{aligned} \underline{v} &< 2X && (37) \\ 2\underline{v} + D &< \sqrt{D^2 + 8D\underline{v}(1-\alpha} && (38) \\ 2D(1-\alpha) &< \underline{v} + D && (39) \\ v &< v_3^* := D(1-2\alpha) && (40) \end{aligned}$$

Renegotiations will thus only fail when $\alpha < \frac{1}{2}$ and $\underline{v} < v_3^*$.

When the equityholders arrive first, which occurs with probability γ, they offer the debtholders their liquidation values for a total of $2X_3^*$. When a debtholder arrives first they will offer the equityholders their outside value which is zero. Their expected payoff is thus $\pi_3^E = \gamma(\underline{v} - 2X_3^*) = \frac{\gamma}{2}\left(2\underline{v} - \sqrt{D}\sqrt{D - 8(\alpha-1)\underline{v}} + D\right)$.

We do not have to worry about the specific arrival order of the debtholders since they will in expectation be identical. Debtholders get their minimum offer value, $2X_3^*$, when the equityholders arrive first and keep all the assets when they arrive first. Their expected payoff is $\pi_3^D = \gamma(2X_3^*) + (1-\gamma)\underline{v} = \underline{v} - \frac{\gamma}{2}\left(2\underline{v} - \sqrt{D}\sqrt{D - 8(\alpha-1)\underline{v}} + D\right)$ ■

Proof of Proposition 3. Again each creditor's debt forgiveness increases the liquidation values for the next creditors that approach the bargaining site, so the proposers offer has to make sure that the last creditor is indifferent between accepting the restructuring proposal or going to bankruptcy court. index the last creditor with k and denote the aggregate debt of all creditors before the exchange as $D = \sum_{i=1}^{n} d_i$. The last creditor would get in bankruptcy court $L_k = \frac{d_k}{x(D-d_k)+d_k}\underline{v}(1-\alpha)$. Then to make the last creditor indifferent between the exchange offer with exchange rate x and going to bankruptcy court it has to hold that

$$\begin{aligned} xd_k &\geq L_k \\ x &\geq \frac{1}{x(D-d_k)+d_k}\underline{v}(1-\alpha) \end{aligned} \tag{41}$$

which has the (positive) solution:

$$x \geq x_n^* := \frac{-d_k + \sqrt{d_k^2 + 4(D-d_k)\underline{v}(1-\alpha)}}{2(D-d_k)} \tag{42}$$

Since $x \leq 1$ the right hand side of Inequality 41 is decreasing in d_k, relaxing the inequality and allowing for a lower exchange rate x when the last creditor is larger. Substituting for d_k the largest creditor d_n yields the lower bound $\underline{x}$ in Equation 8 and substituting for d_k the smallest creditor d_1 yields the upper bound $\overline{x}$ in Equation 9. ■

Proof of Proposition 4. Let d_k go to zero in Equation (42) and get

$$x^{\infty} := \lim_{d_k \to 0} x^* = \frac{\sqrt{D\underline{v}(1-\alpha)}}{D} \tag{43}$$

Renegotiations succeed creditor's aggregate demand can be met with the firm's assets

$$\begin{aligned} x^{\infty}D &\leq \underline{v} \\ D\underline{v}(1-\alpha) &\leq \underline{v}^2 \\ D-\underline{v} &\leq D\alpha \\ \frac{D-\underline{v}}{D} &\leq \alpha \end{aligned}$$

■

Proof of Proposition 5. The proposer has to offer the first debtholder the liquidation value under bankruptcy, which is $L_1 = \frac{1}{2}(1-\alpha)\underline{v}$. If the first creditor accepts the offer the second creditor's outside option, i.e. his liquidation value under the bankruptcy mechanism is $L_2 =$

$\frac{D/2}{D/2+L_1}(1-\alpha)\underline{v}$. Renegotiations fail if

$$\underline{v} \leq L_1 + L_2 \tag{44}$$

$$\underline{v} \leq (1-\alpha)\underline{v}\left(\frac{1}{2} + \frac{D}{D+(1-\alpha)\underline{v}}\right) \tag{45}$$

$$1 \leq (1-\alpha)\left(\frac{3D+(1-\alpha)\underline{v}}{2D+2(1-\alpha)\underline{v}}\right) \tag{46}$$

$$\frac{2D}{1-\alpha} + 2\underline{v} \leq 3D + (1-\alpha)\underline{v} \tag{47}$$

$$\underline{v} \leq v_3^* = \frac{D(1-3\alpha)}{1-\alpha^2} \tag{48}$$

The expression v_3^* is only positive when $\alpha > \frac{1}{3}$. Renegotiations will thus only fail when $\alpha < \frac{1}{3}$ and $\underline{v} < V_3^*$.

When the equityholders arrive first, which occurs with probability γ, they offer the equityholders their liquidation values for a total of $L_1 + L_2$. When a debtholder arrives first they will offer the equityholders their outside value which is zero. Their expected payoff is thus $\pi_3^E = \gamma[\underline{v} - (L_1 + L_2)] = \gamma\left[\underline{v} - (1-\alpha)\underline{v}\left(\frac{1}{2} + \frac{D}{D+(1-\alpha)\underline{v}}\right)\right] = \frac{1}{2}\gamma\underline{v}\left(\alpha + \frac{2(\alpha-1)D}{D-\alpha\underline{v}+\underline{v}} + 1\right)$.

We do not have to worry about the specific arrival order of the debtholders since they will in expectation be identical. Debtholders get their liquidation value, $L_1 + L_2$, when the equityholders arrive first and keep all the assets when they arrive first. Their expected payoff is $\pi_3^D = \gamma(L_1 + L_2) + (1-\gamma)\underline{v} = \underline{v} - \frac{(\alpha-1)\gamma D\underline{v}}{D-\alpha\underline{v}+\underline{v}} - \frac{1}{2}(\alpha+1)\gamma\underline{v}$ ■

Proof of Proposition 6. Equation (7) just states that the sum of the new claims that the players agree to cannot exceed the firm's assets in order for renegotiations to be successful. Whatever the new creditors will agree to can be solved by an iterative procedure as outlined in Equation (31). Each creditor i must be given at least as much as he could obtain in bankruptcy court given that all previous creditors have agreed to reduce their claim to their respective liquidation values.

The equityholders only get a positive payoff when they arrive first, which happens with probability γ. In this case they have to offer the debtholders the sum of their respective liquidation values $\sum_{i=1}^{n} x_i$ and keep the assets of the firm $\underline{v}$. The debtholders get in expectation the rest of the firm's assets that does not go to the equityholders. ■

Proof of Proposition 7. Scale the assets and outstanding debt by D so that the firm's assets are $\underline{v}/D$ and the aggregate face value of debt equals 1 and assume that the continuum of creditors is located in the unit interval. Let $c(n)$ be the aggregate concessions that the creditors in $[0, n]$ have made to the firm. The firm will survive if the face value of debt minus the aggregate concenssion of all debtholders, $1 - c(1)$, is less or equal to the firm's assets, $\underline{v}/D$, or $c(1) > 1 - v/D$.

The marginal concession that the proposer can extract from each debtholder is the difference between the face value, 1, and the liquidation value of that creditor given that the creditors before him have conceded an aggregate of $c(n)$, which is $\frac{1}{1-c(n)}(1-\alpha)\underline{v}$.

The initial condition $c(0) = 0$ comes from the fact that zero creditors have made zero aggregate concessions to the proposer. ■

Proof of Proposition 8. Assume that the pivotal debtholder who arrives at the bargaining site such that the aggregate debt of all players that have arrived equals the CAC threshold ψD has debt with face value ϵD. The outstanding debt of the firm is then the sum of the fraction ψ of creditors that have participated in the exchange and the fraction $(1-\psi)$ that have not yet participated for a total of $D\psi x + D(1-\psi)$. The pivotal creditor has to be indifferent between liquidation or participating in the exchange and getting $x\epsilon D$, i.e.

$$\begin{aligned}
\frac{\epsilon D}{D\psi x + D(1-\psi)}\underline{v}(1-\alpha) &= x\epsilon D \\
\underline{v}(1-\alpha) &= x(D\psi x + D(1-\psi)) \\
x^2 D\psi + xD(1-\psi) - \underline{v}(1-\alpha) &= 0
\end{aligned}$$

which has the positive solution

$$x = \frac{-D(1-\psi) + \sqrt{D^2(1-\psi)^2 - 4D\psi\underline{v}(1-\alpha)}}{2D\psi} \tag{49}$$

■