

# Asymmetric information and the pecking (dis)order\*

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## Abstract

This paper revisits the pecking-order theory of Myers and Majluf (1984) in a real options framework, where asymmetric information is the only contracting friction. We show that when insiders are relatively better informed on the assets in place, rather than on new growth opportunities, equity can dominate debt, reversing the pecking order. The preference of debt versus equity financing is not driven by the absolute amount of asymmetric information, but rather by the composition of a firm's assets and their exposure to this asymmetric information. Finally, we provide conditions under which convertible debt and warrants emerge as optimal financing instruments.

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# 1 Introduction

Raising capital under asymmetric information exposes firms to potential value dilution. When insiders have better information than investors on firm value, firms of better-than-average quality will find that the market prices their securities below their fundamental value. Under these circumstances, Myers and Majluf (1984) suggest that firms can reduce dilution (i.e., mispricing) by issuing debt rather than equity, an intuition known as the pecking order theory. The rationale behind the pecking order, as argued in Myers (1984), is that the value of debt, by virtue of being a senior security, is less sensitive to private information.<sup>1</sup>

Important deviations from the pecking-order theory emerged in several recent empirical studies. For example, Frank and Goyal (2003) and Fama and French (2005) document that small, high-growth firms, a class of firms which is presumably more exposed to the effects of asymmetric information, typically rely heavily on financing through outside equity, rather than debt. Leary and Roberts (2010) conclude that “the pecking order is never able to accurately classify more than half of the observed financing decisions.”<sup>2</sup> This evidence has led researchers to conclude that asymmetric information is not a first-order determinant of corporate capital structures.<sup>3</sup>

Failure of the pecking order theory in empirical tests may be due to the fact that asymmetric information is not a first-order driver of capital structure choices, but it may also be a sign that the circumstances under which the pecking order preference arises are not met. The precise conditions under which the pecking theory holds have been object of considerable research. In a seminal paper, Nachman and Noe (1994) show that the original Myers and Majluf’s result obtains only under very special conditions about how the insiders’ private information affects firm-value distributions.<sup>4</sup> Interestingly, an important class of distributions

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<sup>1</sup>Myers and Majluf (1984) also suggest that internal financing dominates external financing, where external financing can either be in the form of debt or equity. In this paper, we do not consider internal financing because it would dominate external financing, just as in Myers and Majluf (1984). In addition, by design, we do not consider the possibility of (partially) separating equilibria, and thus, the possibility of “announcement effects” also discussed in Myers and Majluf (1984).

<sup>2</sup>Leary and Roberts (2010) also note that most of the empirical evidence is inconclusive, and write: “Shyam-Sunder and Myers (1999) conclude that the pecking order is a good descriptor of broad financing patterns; Frank and Goyal (2003) conclude the opposite. Lemmon and Zender (2010) conclude that a ‘modified’ pecking order—which takes into account financial distress costs—is a good descriptor of financing behavior; Fama and French (2005) conclude the opposite. Frank and Goyal (2003) conclude that the pecking order better describes the behavior of large firms, as opposed to small firms; Fama and French (2005) conclude the opposite. Finally, Bharath, Pasquariello, and Wu (2010) argue that firms facing low information asymmetry account for the bulk of the pecking order’s failings; Jung, Kim, and Stulz (1996) conclude the opposite.”

<sup>3</sup>For example, Fama and French (2005) suggest that violations of the pecking order theory imply that “asymmetric information problems are not the sole (or perhaps even an important) determinant of capital structures.”

<sup>4</sup>In particular, they show that debt emerges as the solution of an optimal security design problem if and only

that satisfies these conditions is the lognormal distribution (a Geometric Brownian Motion), where the mean of the distribution (or the drift) is private information. While the circumstances under which the pecking order theory holds are now well understood, considerably less is known with respect to those cases where such conditions are not met. These cases will be the focus of our paper.

In this paper, we address the fundamental question that is at the very heart of Myers and Majluf (1984): what is the relative mispricing of debt and equity under asymmetric information? In other words, if firms of heterogeneous quality raise capital by issuing the same security in a pooling equilibrium, are firms of better-than-average quality less exposed to value dilution with a debt or an equity issue?

In our basic model, we consider a firm that must raise funds to finance an investment in capital markets characterized by asymmetric information.<sup>5</sup> The value of both assets in place and the assets after the exercise of the growth opportunity are characterized by lognormal distributions, where the growth opportunity is riskier than the assets in place.<sup>6</sup> We model asymmetric information by assuming that the firm insiders have private information on the means (or drifts) of the distributions, while their second moments are common knowledge.

We show that equity financing can dominate debt financing when insiders are relatively better informed than investors on the firm's assets in place, rather than on its (riskier) growth opportunities. In other words, the pecking order can be reversed when a firm's assets in place are more exposed to asymmetric information relative to its new investments. We show that equity is more likely to dominate debt for young firms that have greater investment needs, and that have access to riskier and more valuable growth options. Thus, our model can explain why young firms may initially prefer equity over debt, and then switch to debt financing as they mature.

Greater information asymmetry on a firm's assets in place relative to its growth opportunities may emerge in cases where a firm is exposed to substantial "learning-by-doing." Consider a firm whose assets in place have been obtained by the exploitation of past investment opportunities, while the firm still has untapped growth options. In this situation it is plausible that the firm has accumulated relatively more accurate information on its assets in place relative to the still undeveloped growth opportunities. This is because more information

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if the private information held by firm insiders orders the distribution of firm value by Conditional Stochastic Dominance (CSD). The Statistics and Economics literature also often uses the term Hazard Rate Ordering to refer to CSD, and we will use both terms interchangeably.

<sup>5</sup>Note that, in the spirit of Myers and Majluf (1984), we rule out of the possibility that firms finances their growth opportunities separately from the assets in place, i.e., by "project financing."

<sup>6</sup>The idea of the firm as a collection of assets is a common one in the literature, see Berk, Green, and Naik (1999) for a recent example.

has become privately available over time (for example, as the result of past R&D activities), rather than on the new potential investments, where critical information still has yet to be revealed. If the new growth opportunities have greater volatility (as one would expect), our model shows that the original Myers and Majluf’s result may not hold.<sup>7</sup>

More generally, we show that the pecking order theory can be violated in the case of firms endowed with multiple asset classes, such as multidivisional firms. If the riskier division is less exposed to asymmetric information, a reversal of the pecking order arises. Thus, our model generates new predictions on the cross-sectional variation of firm capital structures of multidivisional firms.

We consider next the case when the firm has pre-existing debt in its capital structure. We show that firms that already have debt outstanding are, all else equal, relatively more likely to prefer equity over debt financing, for reasons solely driven by information asymmetry considerations. This feature of our model suggests that (pre-existing) high leverage may lead to more equity financing, and vice versa. Thus, asymmetric information may in fact lead to a “mean reversion” in leverage levels, as it is often documented in the empirical literature on capital structure (see Leary and Roberts, 2005). These predictions are novel within models based on informational frictions, and invite for further research.<sup>8</sup>

Intuitively, our results depend on the fact that the properties of the firm-value distribution for high realizations of firm value (that is, in the right-tail) are determined by the asset with higher volatility. When the asset that is relatively less mispriced (that is, less exposed to asymmetric information) also has greater volatility, issuing a security with exposure to payoffs in the right tail of the firm-value distribution, such as equity, can be less dilutive than a security which lacks such exposure, leading to a reversal of the pecking order. This means that, contrary to the common intuition, the preference of debt versus equity financing is not driven by the absolute amount of asymmetric information, but rather by the *composition* of a firm’s assets and their *relative exposure* to this asymmetric information.

Technically, our paper makes Myers and Majluf (1984) meet Black and Scholes (1973)/Merton (1974). A specific contribution of our paper is to provide a tractable lognormal model

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<sup>7</sup>Another case with a potential for a pecking order reversal is offered by a firm that has the option to acquire another firm. The firm wishes to raise capital to pay for the acquisition. The acquiring firm’s insiders have private information on both its assets in place and the value of the target firm’s assets. In this setting, it is again plausible to expect that the acquiring firm has relatively better information on its assets in place, that are already under the firm’s control, than on the new assets that still have to be acquired. If the volatility of the acquirer’s assets in place is lower than the volatility of the target firm’s assets, we have again a situation that can generate a reversal of the pecking order.

<sup>8</sup>Our model features a static capital structure choice, but it lends itself to a dynamic specification (in a similar framework, also Leland, 1994, allows for a static financing decision). Further research focusing on dynamic capital structure choices is suggested by the fact that the existing set of securities in a firm’s balance sheet affects the optimal financing choice at later dates (see Section 5.1).

where private information orders firm-value distributions by first-order stochastic dominance, but where monotone-likelihood ratio properties and/or hazard rate orderings may not hold. Since the standard lognormal model implies the monotone likelihood ratio order, there is no room in a lognormal “single-asset” setting to violate the pecking order theory. By introducing a second source of uncertainty in a multiple-asset model, we are able to separate first-order stochastic dominance from monotone likelihood ratio properties of the firm-value distribution which generates robust deviations from the pecking order.<sup>9</sup>

We conclude the paper with an explicit optimal security design problem, where the firm can issue other securities than equity and debt. Feasible securities include convertible bonds, warrants, as well as equity and debt, among others. Our main conclusions extend to the more general security design problem: we show that when a certain “low-information-cost-in-the-right-tail” condition holds, straight (but risky) debt is optimal when the firm needs to raise low levels of capital, but equity-like securities — such as convertible debt — emerge as the optimal securities when the firm must raise larger amounts of capital. Furthermore, we find that warrants can be optimal securities in the presence of pre-existing debt.

Our paper contributes to the ongoing research on the pecking order and, more generally, the security design literature. In addition to Nachman and Noe (1994), subsequent research has focused on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the *ex-ante* security design problem faced by a firm before learning its private information, rather than the *interim* security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). DeMarzo (2005) considers both the *ex-ante* and the *interim* security design problems, and examines the question of whether to keep multiple assets in a single firm (pooling), and the priority structure of the securities issued by the firm (tranching). DeMarzo, Kremer, and Skrzypacz (2005) examine the security design problem in the context of auctions. Chakraborty and Yılmaz (2009) show that when investors have access to noisy public information on the firm’s private value, the dilution problem can be costlessly avoided by issuing securities having the structure of callable, convertible bonds. Chemmanur and Fulghieri (1997) and Chakraborty, Gervais, and Yılmaz (2011) argue that warrants may be part of the optimal security structure. Finally, a growing literature considers dynamic capital structure choice (Fischer, Heinkel, and Zechner, 1989; Hennessy and Whited, 2005; Strebulaev, 2007; Morellec and Schürhoff, 2011). We conjecture that the economic forces of our static framework will play a first-order role in a dynamic version of the model.

There are several other papers that challenge Myers and Majluf (1984) and Myers (1984)

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<sup>9</sup>As we will show below, our results technically reflect the rather limited closure properties of the conditional stochastic dominance order (see section 1.B.3 of Shaked and Shanthikumar (2007)).

by extending their framework in different ways. These papers show that a wider range of financing choices, which allow for signaling with costless separation, can invalidate the pecking order (see, e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989). However, Admati and Pfleiderer (1994) point out that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive. Cooney and Kalay (1993) relax the assumption that projects have a positive net present value. Fulghieri and Lukin (2001) relax the assumption that the informational asymmetry between a firm’s insiders and outside investors is exogenous, and allow for endogenous information production. Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, and Edmans and Mann (2012) look at the possibility of asset sales for financing purposes. Hennesy, Livdan, and Miranda (2010) consider a dynamic model with asymmetric information and bankruptcy costs, with endogenous investment, dividends and share repurchases, where the choice of leverage generates separating equilibria. Bond and Zhong (2014) show that stock issues and repurchases are part of an equilibrium in a dynamic setting. In contrast to these papers, but in the spirit of Myers and Majluf (1984), we consider a pooling equilibrium of a static model where the only friction is asymmetric information between insiders and outsiders.

The remainder of the paper is organized as follows. Section 2 presents a simple example that illustrates the basic results and intuition of our paper. Section 3 outlines the basic model and provides some general results regarding the debt-equity choice. Section 4 considers the real options model that illustrates the main results. Section 5 considers several extensions to our basic model: we extend the analysis to allow for pre-existing debt, we study the security design problem, where we provide conditions under which convertible debt and warrants are the optimal securities, and we examine the robustness of our results to alternative model specifications. All the proofs are in the Appendix.

## 2 A simple example

The essence of the pecking order theory is typically illustrated via a pooling equilibrium with two types and a discrete state space. The basic results of our paper, and their intuition, can be shown with a simple numerical example, summarized in Table 1.<sup>10</sup>

We consider two types of firms: good type,  $\theta = G$ , and bad type,  $\theta = B$ , where a firm’s type is private information to its insiders. We assume that the two types of firms are equally likely in the eyes of investors. At the beginning of the period, firms wish to raise capital  $I$ .

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<sup>10</sup>The numerical example presented in this section builds on the discussion in Nachman and Noe (1994), Section 4.3.

When raising capital, the two types of firms pool and issue the same security, so that investors do not change their priors on the firms' type when seeing the security issuance decision.

For reasons that will become apparent below, we will assume that a firm's end-of-period firm value,  $Z$ , is characterized by a *trinomial* distribution with three possible outcomes  $Z \in \{z_1, z_2, z_3\}$ . To fix ideas, we assume that the states  $z_1$  and  $z_2$  are relevant for the value of assets in place, while state  $z_3$  is relevant for the growth opportunity. In particular, we assume that the end-of-period value of the assets in place is given by  $z_1 = 10$ ,  $z_2 = z_3 = 100$ . If the growth opportunity is exercised, firm value becomes  $z_1 = 10$ ,  $z_2 = 100$ ,  $z_3 = 300$ . Thus, exploitation of the growth opportunity adds value to the firm only in state  $z_3$ , increasing the end-of-period firm value in that state from 100 to 300. The firm's capital requirements are set to be equal to  $I = 60$ .

The probability of the three possible outcomes of  $Z$  depends on private information held by the firm's insiders, and is given by  $f_\theta \equiv \{f_{\theta 1}, f_{\theta 2}, f_{\theta 3}\}$  for a firm of type  $\theta$ , with  $\theta \in \{G, B\}$ . In our examples below, we will assume that  $f_G = \{0.2, 0.4, 0.4\}$  and  $f_B = \{0.3, 0.4 - x, 0.3 + x\}$ , and we will focus in the cases  $x = 0.02$  and  $x = 0.08$  in the discussion.<sup>11</sup> Note that the presence of the growth opportunity has the effect of changing the distribution of firm value in its right tail, and that the parameter  $x$  affects the probability on the high state  $z_3$  relative to the middle state  $z_2$  for the type- $B$  firm.

Consider first the case where  $x = 0.08$ . The values of the firm for the good and bad types are given by  $\mathbb{E}[Z_G] = 162$  and  $\mathbb{E}[Z_B] = 149$ , with a pooled value equal to 155.5. Firms can raise the investment of 60 to finance the growth opportunity by issuing a fraction of equity equal to  $\lambda = 0.386 = 60/155.5$ . This means that under equity financing the initial shareholders of a firm of type- $G$  retain a residual equity value equal to  $(1 - 0.386)162 = 99.5$ . The firm could also raise the required capital by issuing debt, with face value equal to  $K = 76.7$ . In this case debt is risky, with payoffs equal to  $\{10, 76.7, 76.7\}$ , and it will default only in state  $z_1$ . The value of the debt security when issued by a type- $G$  firm is  $D_G = 63.3$ , and when issued by a type- $B$  firm is  $D_B = 56.7$ , with a pooled value of 60, since the two types are equally likely. This implies that under debt financing the shareholders of a type- $G$  firm will retain a residual equity value equal to  $\mathbb{E}[Z_G] - D_G = 98.7 < 99.5$ , and equity is *less dilutive* than debt. Thus, the pecking order preference is reversed.

The role of the growth opportunity in reversing the pecking order can be seen by considering the following perturbation of the basic example. Set now  $x = 0.02$ , so that  $f_B = \{0.30, 0.38, 0.32\}$ . In the new example the growth opportunity is relatively less important for a type- $B$  firm than in the base case. Note that this change does not affect debt financing,

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<sup>11</sup>Table 1 considers all cases  $x \in (0, 0.1)$ . We remark that  $x \leq 0.1$  is necessary to maintain first-order stochastic dominance.

because debt is in default only in state  $z_1$ . Therefore the change in  $x$  affects equity dilution but not debt dilution. In the new case,  $\mathbb{E}[Z_B] = 137$ , lowering the pooled value to 149.5. This means that now the firm must issue a larger equity stake,  $\lambda = 0.401 = 60/149.5$ , and thus existing shareholders' value is now equal to  $(1 - 0.401)162 = 97.0 < 98.7$ . Thus, equity financing now is more dilutive than debt financing, restoring the pecking order.

The reason for the change in the relative dilution of debt and equity rests on the impact of asymmetric information on the right-tail of the firm-value distribution. In the base case, for  $x = 0.08$ , asymmetric information has a modest impact on the growth opportunity (since  $f_{G3} - f_{B3} = 0.02$ ) relative to the “middle” of the distribution (since  $f_{G2} - f_{B2} = 0.08$ ), which is determined by the exposure of the assets in place to asymmetric information. Thus, firms of type- $G$  can reduce dilution by issuing a security that has greater exposure to the right-tail of the firm-value distribution, such as equity, rather than debt, which lacks such exposure. In contrast, in the case of  $x = 0.02$ , asymmetric information has a more substantial impact on the growth opportunity, and thus on the right tail relative to the middle of the distribution, since now we have  $f_{G3} - f_{B3} = 0.08$  and  $f_{G2} - f_{B2} = 0.02$ , making equity more mispriced.

A second key ingredient of our example is that the firm is issuing (sufficiently) risky debt to make dilution a concern. If debt is (nearly) riskless, the pecking order would hold. We obtain this in our example by assuming  $z_1 = 10$  and by setting  $I = 60$ . If the level of investment is reduced to  $I = 10$ , then the firm could issue riskless debt and avoid any dilution altogether. Similarly, for investment needs sufficiently close to  $I = 10$  debt has little default risk and the potential mispricing will still be small. In contrast, for sufficiently large investment needs the firm will need to issue debt with non-trivial default risk, creating the potential for a reversal of the pecking order.

Finally note that in the special case in which  $f_B \equiv \{0.3, 0.3, 0.4\}$  there is no asymmetric information at all in the right-tail (that is, for  $z_3 = 300$ ). In this case, type- $G$  firms would in fact be able to avoid dilution altogether by issuing securities that load only on cash flows in the right tail, such as warrants. We will exploit this feature in Section 5.2, where we study the security design problem, proving the optimality of securities with equity-like features.

In the rest of the paper we build models that generate a reversal of the pecking order, and we show that a reversal can emerge in many economically relevant situations. In Section 3 we introduce a condition, which we refer to as “low-information-costs-in-the-right-tail,” that generalizes the parametric assumptions in the previous example. This condition is novel in the literature and it is critical to generate reversals of the pecking order. The decomposition of the firm-value distribution into three regions in Section 3.3 establishes formally that the trinomial structure of our example is necessary for our results, and it provides its key drivers. Section 4 considers a simple real options model which generates new cross-sectional predictions that

can be used to test asymmetric information theories, with and without a pecking order.

### 3 The basic model

#### 3.1 The capital raising game

An all equity-financed firm with no cash has a one-period investment project. The project requires a capital outlay  $I$  at the beginning of the period. Conditional on making the investment, the firm's value at the end of the period is given by a random variable  $Z_\theta$ . There are two types of firms: "good" firms,  $\theta = G$ , and "bad" firms,  $\theta = B$ , which are present in the economy with probabilities  $p$  and  $1 - p$ , respectively. A firm of type  $\theta$  is characterized by its density function  $f_\theta(z)$  and by the corresponding cumulative distribution function  $F_\theta(z)$ , with  $\theta \in \{G, B\}$ . Because of limited liability, we assume that  $Z_\theta$  takes values of the positive real line. For ease of exposition, we will also assume that the density function of  $Z_\theta$  satisfies  $f_\theta(z) > 0$  for all  $z \in \mathbb{R}_+$ . In addition, we assume type  $G$  firms dominate type  $B$  ones by first-order stochastic dominance, defined as follows.

**Definition 1 (FOSD).** *We will say that the distribution  $F_G$  dominates the distribution  $F_B$  by (strong) first-order stochastic dominance if  $F_G(z) \leq (<)F_B(z)$  for all  $z \in \mathbb{R}_{++}$ .*

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).

**Definition 2 (CSD).** *We will say that the distribution  $F_G$  dominates the distribution  $F_B$  by conditional stochastic dominance if  $F_G(z|z') \leq F_B(z|z')$  for all  $z' \in \mathbb{R}_+$ , where*

$$F_\theta(z|z') \equiv \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')}.$$

By setting  $z' = 0$ , we see that CSD implies FOSD. We note that CSD can equivalently be defined by requiring that the truncated random variables  $[Z_\theta|Z_\theta \geq \bar{z}]$ , with distribution functions  $(F_\theta(z) - F_\theta(\bar{z})) / (1 - F_\theta(\bar{z}))$ , satisfy FOSD for all  $\bar{z}$ .<sup>12</sup> In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio  $(1 - F_G(z)) / (1 - F_B(z))$  is non-decreasing in  $z$  for all  $z \in \mathbb{R}_+$  (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are always more

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<sup>12</sup>We remark that the CSD (hazard-rate) ordering is weaker than the Monotone Likelihood Ratio order, which requires  $[Z_G|Z_G \in (\underline{z}, \bar{z})] \stackrel{\text{fosd}}{\geq} [Z_B|Z_B \in (\underline{z}, \bar{z})]$  for all  $\underline{z}$  and  $\bar{z}$ ; see equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).

likely to occur for a type- $G$  firm relatively to a type- $B$  firm.<sup>13</sup>

Firms raise the amount  $I$  to fund the investment project by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm's type  $\theta \in \{G, B\}$  is private information to its insiders. We also assume that the NPV of the project is sufficiently large that firms will always find it optimal to issue securities and invest, rather than not issuing any security and abandon the project.<sup>14</sup>

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let  $\mathbb{S}$  be the set of admissible securities that the firm can issue to raise the required capital  $I$ . As is common in this literature (see, for example, Nachman and Noe (1994)), we let the set  $\mathbb{S}$  be the set of functions satisfying the following conditions:

$$0 \leq s(z) \leq z, \quad \text{for all } z \geq 0, \quad (1)$$

$$s(z) \text{ is non-decreasing in } z \quad \text{for all } z \geq 0, \quad (2)$$

$$z - s(z) \text{ is non-decreasing in } z \quad \text{for all } z \geq 0. \quad (3)$$

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are monotonicity conditions that ensure absence of risk-less arbitrage.<sup>15</sup> We define  $\mathbb{S} \equiv \{s(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ : s(z) \text{ satisfies (1), (2), and (3)}\}$  as the set of *admissible securities*.

We consider the following capital raising game. The firm moves first, and chooses a security  $s(z)$  from the set of admissible securities  $\mathbb{S}$ . After observing the security  $s(z)$  issued by the firm, investors update their beliefs on firm type  $\theta$ , and form posterior beliefs  $p(s) : \mathbb{S} \rightarrow [0, 1]$ . Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price  $V(s)$ . The value  $V(s)$  that investors are willing to pay for the security  $s(z)$  issued by the firm is equal to the expected value of the security, conditional on the posterior beliefs  $p(s)$ , that is

$$V(s) = p(s)\mathbb{E}[s(Z_G)] + (1 - p(s))\mathbb{E}[s(Z_B)]. \quad (4)$$

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<sup>13</sup>Referring back to the example in Section 2, it is easy to verify that if  $x \leq 0.05$  the type- $G$  distribution not only dominates the type- $B$  in the first-order sense, but *also* in the CSD sense.

<sup>14</sup>Note that this assumption is made to rule out the possibility of separating equilibria where type- $B$  firms raise capital and invest, while type- $G$  firms separate by not investing in the project.

<sup>15</sup>See, for example, the discussion in Innes (1990). Note that, as pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may have a "do or die" component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

Condition (4) implies that securities are fairly priced, given investors' beliefs. If security  $s$  is issued, capital  $V(s)$  is raised, and the investment project is undertaken, the payoff to the initial shareholders for a firm of a type  $\theta$  is given by

$$W(\theta, s, V(s)) \equiv \mathbb{E}[Z_\theta - s(Z_\theta)] + V(s) - I. \quad (5)$$

The firm will choose the security issued to finance the investment project by maximizing its payoff (5), subject to the constraint that the security is admissible and that it raises at least the required funds  $I$ . Let  $s_\theta(z) \in \mathbb{S}$  be the security issued by a firm of type  $\theta$ .

### 3.2 Equilibria

Following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as follows.

**Definition 3 (Equilibrium).** *A PBE equilibrium of the capital raising game is a collection  $\{s_G^*(z), s_B^*(z), p^*(s), V^*(s)\}$  such that: (i)  $s_\theta^*(z)$  maximizes  $W(\theta, s, V^*(s))$  subject to the constraint that  $s \in \mathbb{S}$  and  $V^*(s) \geq I$ , for  $\theta \in \{G, B\}$ , (ii) securities are fairly priced, that is  $V^*(s) = p^*(s)\mathbb{E}[s(Z_G)] + (1 - p^*(s))\mathbb{E}[s(Z_B)]$  for all  $s \in \mathbb{S}$ , and (iii) posterior beliefs  $p^*(s)$  satisfy Bayes rule whenever possible.*

We start with a characterization of the possible equilibria in our capital raising game.<sup>16</sup>

**Proposition 1.** *(Nachman and Noe, 1994) Let  $F_\theta$  satisfy strict FOSD. No separating equilibrium exists in the capital raising game. In addition, in any pooling equilibrium, with  $s_G^* = s_B^* = s^*$ , the capital raising game is uninformative,  $p(s^*) = p$ , and the financing constraint is met with equality:*

$$I = p\mathbb{E}[s(Z_G)] + (1 - p)\mathbb{E}[s(Z_B)]. \quad (6)$$

Proposition 1 follows from the fact that, with two types of firms only, a type- $B$  firm has always the incentive to mimic the behavior of a type- $G$  firm (i.e., to issue the same security). This happens because (2) and strict FOSD together imply that securities issued by a type- $G$  firm are always priced better by investors than those issued by a type- $B$  firm, and type- $B$  firm is always better-off by mimicking a type- $G$  one. This also implies that, in equilibrium, the type- $G$  firm is exposed to dilution due to the pooling with a type- $B$  firm,

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<sup>16</sup>We note that the strong form of FOSD is only necessary for Proposition 1. Our main results go through assuming only FOSD.

and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay  $I$ .

Proposition 1 allows us to simplify the exposition as follows. Since both types of firms pool and issue the same security  $s$  and the capital constraint is met as equality, (5) and (6) imply that the payoff to the original shareholders of firm of type  $G$  becomes

$$W(G, s, V(s)) = \mathbb{E}[Z_G] - I - (1 - p)\mathcal{D}_s,$$

where the term

$$\mathcal{D}_s \equiv \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)] \tag{7}$$

represents the mispricing when security  $s \in S$  is used, which is the cause of the dilution suffered by a firm of type  $G$ .

Under these circumstances, firms of type  $G$  will find it optimal to finance the project by issuing a security that minimizes dilution  $\mathcal{D}_s$ , that is

$$\min_{s \in \mathbb{S}} \mathcal{D}_s \tag{8}$$

subject to the financing constraint (6). Defining the function  $c(z) \equiv f_G(z) - f_B(z)$ , the dilution costs of security  $s(z)$  can be expressed as:

$$\mathcal{D}_s = \int_0^\infty s(z)c(z)dz. \tag{9}$$

Note that the density function  $f_\theta(z)$  measures, loosely speaking, the (implicit) private valuation of a \$1 claim made by the insiders of a firm of type  $\theta \in \{G, B\}$  if the final payoff of the firm is  $z$ . Thus, we can interpret the term  $c(z)$  as representing the private “asymmetric information cost” for a firm of type  $G$ , relative to a firm of type  $B$ , of issuing a security that has a payoff of \$1 if the final firm value is  $z$ . In particular, if  $c(z) > 0$  we will say that the information costs for a type  $G$  are “positive,” and that these costs are “negative” if  $c(z) < 0$ . More formally, the asymmetric information costs of a security that pays \$1 if and only if the final payoff is in the interval  $z \in [z_L, z_H]$  is equal to  $\int_{z_L}^{z_H} c(z)dz$ .

In what follows we will be concerned on the asymmetric information costs in the right tail of the value distribution  $F_\theta(z)$  for a firm of type  $G$  relative to a firm of type  $B$ . These asymmetric information costs are related to the function  $H(z)$  defined as:

$$H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)}, \tag{10}$$

where  $F(z)$  denotes the mixture of the distributions of the good and bad types, that is,

$$F(z) = pF_G(z) + (1 - p)F_B(z). \quad (11)$$

The function  $H(z)$  plays a critical role in our analysis, as it is a measure of the informational costs faced by firms. First note that FOSD implies that  $H(z) > 0$  for all  $z \in \mathbb{R}_{++}$ . In addition, and more importantly, monotonicity of  $H(z)$  is equivalent to CSD, as it is established in the following proposition.<sup>17</sup>

**Proposition 2.** *The distribution  $F_G$  dominates  $F_B$  by (strong) conditional stochastic dominance if and only if the function  $H(z)$  is (strictly) increasing in  $z$  for all  $z \in \mathbb{R}_+$ . This is equivalent to requiring that the hazard rates  $h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z))$  satisfy  $h_G(z) \leq (<)h_B(z)$  for all  $z \in \mathbb{R}_+$ .*

The function  $H(z)$  provides a measure of the extent of asymmetries of information, and for monotonic securities it is closely linked to the cost to a type- $G$  firm of promising to investors an extra dollar in state  $z$ .<sup>18</sup> In what follows, it will be important to characterize properties the right tail of the firm-value distribution that are stronger than FOSD, but at the same time weaker than CSD. Note first that  $H(0) = 0$  and that, from FOSD, we have  $H(z) > 0$  for  $z$  in a right neighborhood of  $z = 0$ , which together imply that  $H'(0) > 0$ . We note that, while the monotonicity properties of  $H(z)$  on the left-tail of the distribution of  $z$  are dictated by FOSD, this is not the case for the right-tail of the distribution.

The next Proposition gives one leading example of distributions satisfying the CSD condition, and thus monotonicity of  $H(z)$ , namely the lognormal model.

**Proposition 3.** *If  $Z_\theta$  is lognormal, with log-mean  $\mu_\theta$  and log-variance  $\sigma^2$ , with  $\mu_G > \mu_B$ , then the distribution  $F_G$  dominates the distribution  $F_B$  by conditional stochastic dominance.*

To characterize the behavior of the information costs in the right-tail of the distribution, we introduce the following definition, which will play a key role in our analysis.

**Definition 4 ( $h$ -ICRT).** *We will say that distribution  $F_G$  has information costs in the right tail of degree  $h$  ( $h$ -ICRT) over distribution  $F_B$  if  $\lim_{z \uparrow \infty} H(z) \leq h$ .*

We will use the term NICRT (no-information-costs-in-the-right-tail) to refer to the case  $h = 0$ . The relationship between FOSD, CSD and  $h$ -ICRT may be seen by noting that for two

<sup>17</sup>In the simple example of Section 2, the function  $H$  is increasing if  $x \leq 0.05$ . Thus a necessary condition for the distributions in the example to *not* satisfy CSD is that  $x > 0.05$ .

<sup>18</sup>This happens because, for monotonic securities, an extra dollar paid in state  $z$  means that investors will be paid an extra dollar also in all states  $z' > z$ . This interpretation will become apparent in Section 5.2 (see equation (30)).

distributions  $\{F_G, F_B\}$  that satisfy FOSD, there may exist a sufficiently low  $h \in \mathbb{R}_+$  such that the  $h$ -ICRT property holds, while conditional stochastic dominance fails. Thus, intuitively, distributions that satisfy the  $h$ -ICRT condition “fill” part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for  $h = 0$  (NICRT) will fail to satisfy the CSD condition.

We conclude this section by introducing an additional regularity condition that will simplify the analysis and greatly streamline the presentation of some of the results.

**Definition 5 (SCDP).** *The distributions  $F_\theta(z)$ , for  $\theta = G, B$ , satisfy the single-crossing density property (SCDP) if  $F_G$  strictly first-order stochastically dominates  $F_B$ , and there exists a unique  $\hat{z} \in \mathbb{R}_+$  such that  $f_G(\hat{z}) = f_B(\hat{z})$ .*

Note that the SCDP condition implies that for all  $z \leq \hat{z}$  we have  $f_B(z) \geq f_G(z)$ , and for all  $z \geq \hat{z}$  we have  $f_B(z) \leq f_G(z)$ . Intuitively, this means that cash flows above the critical cutoff  $\hat{z}$  have a positive information cost for type  $G$  firms,  $c(z) > 0$ , whereas cash flows below that cutoff have negative information costs,  $c(z) < 0$ . Note that FOSD alone only implies that there exists  $z_1$  and  $z_2$  such that  $c(z) < 0$  for all  $z < z_1$  and  $c(z) > 0$  for all  $z > z_2$ , but it does not rule out other interior crossings; in contrast, SCDP ensures that  $z_1 = z_2$ .<sup>19</sup>

### 3.3 The debt-equity choice

We start the analysis by restricting our attention to two classes of securities, debt and equity. From (7), the dilution costs associated with equity are given by

$$\mathcal{D}_E = \lambda (\mathbb{E}[Z_G] - \mathbb{E}[Z_B]), \quad (12)$$

with  $\lambda = I/\mathbb{E}[Z]$ , whereas those associated with debt

$$\mathcal{D}_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)], \quad (13)$$

where  $K$  represents the (smallest) face value that satisfies the financing constraint  $I = p\mathbb{E}[\min(Z_G, K)] + (1-p)\mathbb{E}[\min(Z_B, K)]$ . In what follows we will say the pecking order obtains if  $\mathcal{D}_E > \mathcal{D}_D$ , and the reverse pecking order holds if  $\mathcal{D}_D > \mathcal{D}_E$ .

The next proposition provides a simple necessary and sufficient condition for the reverse pecking order to hold. Furthermore, it shows how debt will always dominate equity under the CSD condition of Nachman and Noe (1994).

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<sup>19</sup>We will assume SCDP for ease of exposition. The discussion below could be adapted to take into account the presence of multiple crossings.

**Proposition 4.** *The dilution costs of equity will be strictly smaller than those of debt, i.e.,  $\mathcal{D}_E < \mathcal{D}_D$ , if and only if*

$$\frac{\mathbb{E}[Z_G]}{\mathbb{E}[Z_B]} < \frac{\mathbb{E}[\min(Z_G, K)]}{\mathbb{E}[\min(Z_B, K)]}. \quad (14)$$

*Condition (14) cannot hold if  $F_G$  dominates  $F_B$  in the conditional stochastic dominance sense.*

The proposition implies that when the security choice is restricted between equity and debt, the security that generates lowest dilution in dollar terms is the one with the lowest *relative* valuation between the good and bad type.<sup>20</sup> Furthermore, the proposition shows that under CSD, debt will always dominate equity as a financing instrument, as shown in Nachman and Noe (1994).

To obtain further insights on the factors that drive the relative dilution of debt and equity, note that the difference in the dilution costs of debt and equity can be written as:

$$\mathcal{D}_D - \mathcal{D}_E = \int_0^{\infty} (\min(z, K) - \lambda z) c(z) dz > 0. \quad (15)$$

This expression can be further decomposed as follows. Define  $\bar{z}(K, \lambda) \equiv K/\lambda$  and note that for  $z < \bar{z}(K, \lambda)$  we have that  $\min(z, K) > \lambda z$ , which implies that the payoffs to debtholders are greater than those to equity holders; the converse holds for  $z > \bar{z}(K, \lambda)$ .

Next, it is useful to compare the point where equity payouts are equal to debt payouts,  $\bar{z}(K, \lambda)$ , with respect to the critical point in the SCDP condition, given by  $\hat{z}$ . As we will show below, this condition is necessary to reverse the pecking order, that is for an “un-pecking” order to arise.

**Definition 6 (UNC).** *The “un-pecking” necessary condition (UNC) is satisfied if  $\bar{z} > \hat{z}$ .*

Under SCDP, the point  $\hat{z}$  divides the positive real line into two disjoint sets: a first set at the lower end of the positive real line,  $[0, \hat{z})$  where  $c(z) < 0$ , that is where a type- $G$  firm enjoys “negative information costs” (that is, effectively an information benefit), and a second set  $[\hat{z}, \infty]$  where  $c(z) \geq 0$ , that is where a type- $G$  firm faces “positive information costs.” The point  $\bar{z}(K, \lambda)$  divides the positive real line in two other subsets, depending on whether or not equity yield higher payoffs than debt to investors. Thus, SCDP and UNC together divide the positive real line into three regions: (i) a low-value region where  $z < \hat{z}$  and  $z \leq \bar{z}$ ; (ii) an

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<sup>20</sup>A similar condition was obtained, in the context of unit IPOs, in Chakraborty, Gervais, and Yilmaz (2011), see their Propositions 1 and 2.

intermediate region where  $[\hat{z}, \bar{z}]$ ; and (iii) a high-value region where  $z > \bar{z}(K, \lambda)$  (see Figure 1).

The relative dilution costs of equity and debt,  $\mathcal{D}_D - \mathcal{D}_E$ , depend on the comparison of the information costs and relative payoffs of debt and equity in each of these different regions, as formalized in the next Proposition.

**Proposition 5.** *Assume the SCDP holds. Then a necessary and sufficient condition for the reverse pecking order is that (i) UNC holds, and (ii)*

$$\mathcal{D}_D - \mathcal{D}_E = \int_{\hat{z}}^{\bar{z}} (\min(z, K) - \lambda z) c(z) dz - \int_0^{\hat{z}} (\lambda z - \min(z, K)) c(z) dz - \int_{\bar{z}}^{\infty} (\lambda z - K) c(z) dz > 0. \quad (16)$$

Under UNC and the maintained assumptions the three integrals in (16) are all positive. The first term of the r.h.s. of (16) measures the dilution cost of debt relative to equity in the *intermediate-value region*  $[\hat{z}, \bar{z}]$ , where debt has higher payouts than equity and type- $G$  firms suffer a positive information cost,  $c(z) > 0$ . In this region dilution costs of equity are lower than those of debt because equity has lower payoff than debt precisely in those states in which type- $G$  firms are exposed to positive information cost (since  $c(z) > 0$ ). Note that existence of this region is guaranteed by UNC. It is the presence of this term that makes equity potentially less dilutive than debt.

The second term of the r.h.s. of (16) measures the benefits of debt financing for low realizations of firm value (i.e., for  $z < \hat{z}$ ). In this *low-value region*, dilution costs are lower for debt than equity because debt gives a higher payoff than equity, but such payoff has negative information costs (i.e.,  $c(z) < 0$ ). The third and last term measures the dilution costs of equity relative to debt for high realizations of firm value (i.e., for  $z > \bar{z}$ ). In this *high-value region*, equity payoffs are greater than debt in those states that are more likely to occur to a type- $G$  firm, and thus carry positive information costs (i.e.,  $c(z) > 0$ ).

The relative importance of these three regions determines the optimality of debt versus equity choice. In particular, equity financing dominates debt financing when the advantages of equity financing originating from the intermediate region of firm value (for  $z \in [\hat{z}, \bar{z}]$ ), that is, the first term on the r.h.s. of (16) dominate the disadvantages in the low (for  $z < \hat{z}$ ) and the high (for  $z > \bar{z}$ ) regions of firm value, that is, the second and the third term on the r.h.s. of (16). Note that if UNC does not hold (so that  $\bar{z}(K, \lambda) < \hat{z}$ ), equity has negative information costs (that is,  $c(z) < 0$ ) precisely in the states where the payouts to equityholders are greater than those to debtholders, making it impossible for the inequality (16) to be satisfied. Thus, UNC is a necessary condition to reverse the pecking order.

## 4 A real options model

### 4.1 The model

In this section, we present a real options specification of our model that will serve as the basis for our main cross-sectional predictions. We adopt this approach for two main reasons. First, it draws on well established option pricing techniques that allow analytically tractable solutions. Second, it provides sufficient flexibility for modeling the source of asymmetric information that allows us to have both first-order stochastic dominance and the right-tail behavior of firm-value distribution that generates a reverse pecking order.

We model the real options problem as follows. By paying the investment cost  $I$  at the beginning of the period,  $t = 0$ , the firm generates a new growth opportunity that can be exercised at date  $T$ . We model the growth opportunity as an exchange (or “rainbow”) option. That is, the firm holds an option to exchange the existing assets in place, with a value of  $X_{\theta T}$  at a future date  $T$ , for the new assets with value  $Y_{\theta T}$ , for  $\theta \in \{G, B\}$ . We interpret the new assets  $Y_{\theta T}$  as embedding the incremental firm value of the new investment project; therefore, we refer to  $X_{\theta T}$  as the “assets in place” and to  $Y_{\theta T}$  as the “growth opportunity.” Formally, after the initial investment  $I$  is made, at the end of the period  $T$  the value of a firm of type  $\theta$  is given by  $Z_{\theta T} \equiv \max(X_{\theta T}, Y_{\theta T}) = X_{\theta T} + \max(Y_{\theta T} - X_{\theta T}, 0)$ , for  $\theta \in \{B, G\}$ .

We adopt the exchange option framework because it is quite common in the real options literature, it admits a closed form solution (Rubinstein, 1991), and it is a natural description of an investment decision in a firm (Stulz, 1982). In Section 5.3 we will examine the case in which the growth opportunity requires an additional investment at the time the option is exercised. This case will be discussed with numerical simulations, since this class of (“double strike”) options does not have a closed form solution.<sup>21</sup>

We assume that both  $X_{\theta T}$  and  $Y_{\theta T}$  follow a lognormal process, that is, both  $\log(X_{\theta T})$  and  $\log(Y_{\theta T})$  are normally distributed with means  $\mu_{\theta x}$  and  $\mu_{\theta y}$  and with variances  $\sigma_{\theta x}^2$  and  $\sigma_{\theta y}^2$ . Let  $\rho_{\theta}$  be the correlation coefficient between  $\log(X_{\theta T})$  and  $\log(Y_{\theta T})$ . Thus, the real option specification is isomorphic to a model where time flows continuously, that is  $t \in [0, T]$ , and where asset values  $X_{\theta T}$  and  $Y_{\theta T}$  follow two geometric Brownian motions with drifts  $\mu_{\theta x}$  and  $\mu_{\theta y}$ , variances  $\sigma_x^2$  and  $\sigma_y^2$ , respectively, and correlation coefficient  $\rho$ .

In the spirit of Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while

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<sup>21</sup>In addition, in Section 5.3, we will argue that our model is robust to other model specifications, such as — for example — one in which the value of the growth option is additive. In this case, total firm value would be equal to the value of assets in place plus the value of a call option. While this specification has the advantage of being closer to others in the literature, it would come at the cost of losing analytical tractability (since no closed form solution is available).

their variances are common knowledge. We let  $\mathbb{E}[X_{\theta T}] = X_{\theta}$  and  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ , and we assume  $X_G \geq X_B$  and  $Y_G \geq Y_B$ , with at least one strict inequality. We define the average value of the assets in place and of the growth opportunity as  $\bar{X} = pX_G + (1-p)X_B$  and  $\bar{Y} = pY_G + (1-p)Y_B$ , respectively, and let  $c_x = X_G - X_B$  and  $c_y = Y_G - Y_B$ . Thus,  $c_x$  and  $c_y$  measure the exposure to asymmetric information of the assets in place and the growth opportunity. Finally, to ensure FOSD we assume that  $\sigma_{Gx} = \sigma_{Bx} = \sigma_x$ ,  $\sigma_{Gy} = \sigma_{By} = \sigma_y$ ,  $\rho_G = \rho_B = \rho$ , and, without loss of generality, that  $\sigma_y \geq \sigma_x$ .<sup>22</sup>

The value of a firm of type  $\theta$  is given by the value of the exchange option, denoted by  $\mathcal{A}_{\theta} \equiv \mathbb{E}[Z_{\theta T}]$ . If the firm raises the required capital by issuing equity, existing shareholders will have to sell to outside investors a fraction  $\lambda$  of the firm to satisfy the financing constraint, that is

$$\lambda = \frac{I}{p\mathcal{A}_G + (1-p)\mathcal{A}_B}. \quad (17)$$

Denote by  $\mathcal{V}_{\theta}(K)$  the value of risky debt with face value of  $K$  when issued by a firm of type  $\theta$ . Note that the debt value  $\mathcal{V}_{\theta}$  can be written as  $\mathcal{V}_{\theta}(K) = \mathbb{E}[\min(Z_{\theta T}, K)] = K - \mathcal{P}_{\theta}$ , that is, as the value of the default-free debt,  $K$ , minus the value of the option to default, which is equal to  $\mathcal{P}_{\theta} = \mathbb{E}[\max(K - Z_{\theta T}, 0)]$ . The option to default for a firm of type  $\theta$  is given by the compound put option with payoffs  $\max(K - Z_{\theta T}, 0)$ , where in turn  $Z_{\theta T}$  is given by the exchange option  $\max(X_{\theta T}, Y_{\theta T})$ . If the firm raises the required capital by issuing debt, the face value offered by the firm must satisfy the participation constraint:

$$K = I + (p\mathcal{P}_G + (1-p)\mathcal{P}_B). \quad (18)$$

We can now proceed to explicitly characterize the choice of financing in our real options model.

**Proposition 6.** *The firm should raise capital issuing an equity security if and only if*

$$\lambda(\mathcal{A}_G - \mathcal{A}_B) < \mathcal{P}_B - \mathcal{P}_G, \quad (19)$$

where  $\mathcal{A}_{\theta}$  and  $\mathcal{P}_{\theta}$  are given by

$$\mathcal{A}_{\theta} = X_{\theta}\hat{\Delta}_{x\theta} + Y_{\theta}\hat{\Delta}_{y\theta}, \quad (20)$$

$$\mathcal{P}_{\theta} = K\Gamma_{\theta} - X_{\theta}\Delta_{x\theta}^* - Y_{\theta}\Delta_{y\theta}^*, \quad (21)$$

and the terms  $\hat{\Delta}_{x\theta}$ ,  $\hat{\Delta}_{y\theta}$ ,  $\Delta_{x\theta}^*$  and  $\Delta_{y\theta}^*$  are defined in the proof.

<sup>22</sup>Recall that we assume that the project's NPV is sufficiently large for investment to be optimal. Thus, the assumption  $\sigma_y \geq \sigma_x$  is without loss of generality.

Condition (19) is a necessary and sufficient condition for generating a reversal of the pecking order. It states that the difference in value of the equity issued by each type of firm must be smaller than the difference in the corresponding default premium of the bonds. Note that (20) is equal to the value of the replicating portfolio of the exchange option, and the terms  $\hat{\Delta}_{x\theta}$  and  $\hat{\Delta}_{y\theta}$  represent the deltas of the option, that is, the sensitivity of the value of the exchange option with respect to the value of the underlying assets,  $X_\theta$  and  $Y_\theta$ , respectively. The expression in (21) has a similar interpretation in terms of the value of the replicating portfolio of the put option, where the terms  $\Delta_{x\theta}^*$  and  $\Delta_{y\theta}^*$  in (21) measure the exposure of the debt security to the value of underlying assets  $X_\theta$  and  $Y_\theta$ , respectively, and  $K\Gamma_\theta$  represents the investment in the riskless asset in the replicating portfolio.

It is important to note that while condition (19) gives a closed-form solution for the preference of equity over debt financing, its tractability is limited by the fact that the left hand side of (19) includes the term  $\lambda$ , which depends on the model's primitives via the financing constraint (17), and the right hand side depends on  $K$  which is determined by the financing constraint (18).

## 4.2 Comparative statics

We start the analysis by considering a perturbation of the parameter values around the case without asymmetric information, i.e., when  $Y_G = Y_B$  and  $X_G = X_B$ . In the perturbation, only the assets in place are exposed to (a small amount of) asymmetric information:  $X_G = \bar{X} + \epsilon$  and  $X_B = \bar{X} - \epsilon$ . For  $\epsilon$  sufficiently close to zero, it is easy to see that condition (19) reduces to

$$\lambda \hat{\Delta}_x < \Delta_x^*, \quad (22)$$

where we have dropped the type  $\theta$  subscript. This delta condition implies that equity is less dilutive than debt if the sensitivity to  $X$  of the value of the equity sold to outside investors (which depends on the delta of the exchange option,  $\hat{\Delta}_x$ ) is smaller than the corresponding sensitivity of debt (which depends on the corresponding delta of the compound put option,  $\Delta_x^*$ ). The deltas of the two options measure the sensitivity of the value equity and debt to the value of the underlying asset(s) and, therefore, their exposure to asymmetric information and potential mispricing.

Condition (22) can be further simplified in terms of univariate cumulative normal distributions when  $\sigma_x = \rho\sigma_y$ . We note that as long as  $\sigma_x < \sigma_y$  and the assets are positively correlated, such a condition will arise. The next Proposition characterizes the reverse pecking order under these parametric assumptions.<sup>23</sup>

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<sup>23</sup>We conjecture that the statements in the following Proposition are more general, as we verify in the

**Proposition 7.** *Consider the case where there is no informational asymmetry on  $Y$ ,  $Y_G = Y_B = \bar{Y}$ , but there is on  $X$ , namely,  $X_G = \bar{X} + \epsilon$  and  $X_B = \bar{X} - \epsilon$ , with  $\bar{X} > 0$ . Further assume that  $\rho\sigma_y = \sigma_x$ . Then, as we let  $\epsilon \downarrow 0$ , we have that (19) reduces to*

$$\lambda = \frac{I}{\bar{X}\hat{\Delta}_x + \bar{Y}\hat{\Delta}_y} < \Delta_x, \quad (23)$$

where  $\hat{\Delta}_x$  and  $\hat{\Delta}_y$  are the deltas of the exchange option in (20), and  $\Delta_x$  is the delta of a plain vanilla put option written on the assets in place only, given in the proof. Furthermore, we have that: (i) condition (23) holds for sufficiently large values of  $\bar{Y}$ , where it can never hold for small values of  $\bar{Y}$ ; (ii) as  $\sigma_x \downarrow 0$ , condition (23) holds if  $\bar{X} < K$ , but cannot hold if  $\bar{X} > K$ .

Proposition 7 establishes analytically two of the main results of our paper. Part (i) of Proposition 7 states that a reverse pecking order obtains if the value of the growth opportunity,  $\bar{Y}$ , is sufficiently large, while the pecking order prevails when  $\bar{Y}$  is sufficiently small. This can be seen as follows. Note that (23) is more likely to be satisfied when  $\bar{Y}$  is large compared to  $\bar{X}$ . This happens because, in this case, the exchange option is sufficiently in-the-money with respect to  $Y$  to make  $\hat{\Delta}_y$  relatively large. This feature, combined with a large value of the growth opportunity itself,  $\bar{Y}$ , leads to a low value of  $\lambda$  on the l.h.s. of (23), while the r.h.s. is independent of  $\bar{Y}$ . This implies the firm must issue to outside investors a relatively small equity share  $\lambda$ , while the sensitivity of the option to default with respect to  $X$  may still be rather significant.

Part (ii) of Proposition 7 stresses the role of the option to default and of the initial investment  $I$  (since under debt financing, the face value of the debt  $K$  is an increasing function of the required investment  $I$ ) to generate reversals of the pecking order. When the volatility of the assets in place,  $\sigma_x$ , is sufficiently small, the variables  $\bar{X}$  and  $K$  identify two separate regions. The first region occurs for  $K < \bar{X}$  (that is, for low levels of the initial investment) and is a “safety region” where the debt is in default with very low probability. In this case, the value of delta of the put option,  $\Delta_x$ , is very small, and (23) cannot be verified. Thus, a reversal of the pecking order does not arise.

The second region occurs for  $K > \bar{X}$  (that is for large levels of the initial investment) and is a “bankruptcy region” where debt has a non-trivial chance of default. This implies that the value of the debt is highly sensitive to changes in value for the assets in place  $X$ . Thus, the value of delta of the put option,  $\Delta_x$ , in (23) is large (close to one). At the same time,

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subsequent numerical analysis. Analytical proofs in the general case are substantially hindered by the presence of the bivariate normal cumulative distribution function  $\Gamma$  in the valuation equation (21) for the put option.

the exchange option still gets a significant value from the growth opportunity component  $Y$ . This implies that the l.h.s. of (23),  $\lambda$ , is small (i.e., smaller one) and that (23) is always verified, generating a reverse pecking order.

So far we have considered perturbations where only the assets in place (i.e., the assets with the lower volatility) are exposed to a small amount of asymmetric information. The symmetric case occurs when there is no asymmetric information on the assets in place,  $X$ , but the growth opportunity  $Y$  is exposed to a small amount of asymmetric information. This corresponds to the case where  $X_G = X_B = \bar{X}$  and  $Y_G = \bar{Y} + \epsilon$  with  $Y_B = \bar{Y} - \epsilon$ , for  $\epsilon > 0$  arbitrarily small.<sup>24</sup> We will show in Section 5.2 that in the case where the asymmetric information loads only on the growth-option  $Y$ , debt financing is always optimal, and the standard pecking order holds. More generally, we will show that a reverse pecking order will occur when the assets in place are more exposed to asymmetric information than the growth opportunity. This means that a key feature to generate the reversal of the pecking order is that asymmetric information characterizes the assets with lower volatility.

### 4.3 Numerical results

We now conduct a series of numerical examples to complement the previous analytical results. We focus on the general condition (19), and we center our examples on the base case reported in Table 2. In order to keep the parameters as parsimonious as possible, we will assume that  $Y_G = Y_B = 175$ ,  $\sigma_x = 0.3$ ,  $\sigma_y = 0.6$ ,  $T = 10$  and  $\rho = 0$ . The asymmetric information corresponds to the assets in place, namely  $X_G = 125$  and  $X_B = 75$ . We let both types be equally likely,  $p = 0.5$ . The value of the firm post-investment for the two types is given by  $\mathbb{E}[Z_{GT}] = 257.6$  and  $\mathbb{E}[Z_{BT}] = 218.3$ , so that  $p\mathbb{E}[Z_{GT}] + (1-p)\mathbb{E}[Z_{BT}] = 237.9$ . In the base case specification, we let the investment amount  $I = 110$ .

Without the project, the status-quo firm value is the value of asset  $X$ , which is equal to  $\bar{X} = 100$ . Since the value of the firm post investment is 237.9, and the investment is  $I = 110$ , the project has an (unconditional) positive NPV of 27.9. Note also that the efficient outcome is for *both* types of firms would be to finance the project, since for a type- $G$  we have that  $\mathbb{E}[Z_{GT}] - I = 257.6 - 110 = 147.6 > 125 = X_G$ , and for a type- $B$  we have that  $\mathbb{E}[Z_{BT}] - I = 218.3 - 110 = 108.3 > 75 = X_B$ .

It is easy to verify that issuing equity will require that the equity holders give up a stake of  $\lambda = 0.462 = 110/237.9$ . In order to finance the project with debt, the firm needs to promise

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<sup>24</sup>Note that (22) does *not* simplify as in the case with  $X$ , since the correlation term  $\rho_y = (\sigma_y - \rho\sigma_x)/\Sigma$  defined after (21) satisfies  $\rho_y > 0$  when  $\sigma_y > \sigma_x$ . Its implicit term, the face value of the debt  $K$ , makes it analytically challenging. We note how none of the volatility limits in Proposition 7 apply under our stated condition  $\rho\sigma_y = \sigma_x$ .

bondholders a face value of  $K = 198.3$  at maturity. Using (21), one can readily check that the values of debt for the good type and the bad type are  $\mathcal{V}_G(K) = 120.6$  and  $\mathcal{V}_B(K) = 99.4$ , respectively.<sup>25</sup> The dilution costs of equity are  $\mathcal{D}_E = 0.462 \times (257.8 - 218.4) = 18.2$ , whereas those of debt are  $\mathcal{D}_D = 120.6 - 99.4 = 21.2$ . Thus, the type- $G$  firm is exposed to lower dilution by raising capital with equity rather than debt.

For the parameter values in Table 2, Figure 1 displays the plots of the function  $c(z)$  (top panel, solid line) and of the densities of firm value for both type of firms and their average,  $\{f_G(z), f_B(z), f(z)\}$  (bottom panel). Note that in this numerical example the region in which debt has a disadvantage over equity, the intermediate region of (16)) is large, the interval  $[79, 429]$ . In addition, the bottom panel of Figure 1 plots the distributions of  $Z_{\theta T} \equiv \max(X_{\theta T}, Y_{\theta T})$  for  $\theta \in \{B, G\}$ . By direct inspection, it is easy to verify that the distribution of firm value  $Z_{\theta T}$  closely resembles a lognormal distribution, with the important difference that the asymmetric information loads in the “middle” of the distribution, and to a lesser extent in its right tail.

Figures 2 and 3 present more general comparative static exercises based on the example from Table 2. The top graph in Figure 2 displays indifference lines of  $\mathcal{D}_D = \mathcal{D}_E$ , as a function of the exposure to asymmetric information of the assets in place,  $c_x$ , and the growth opportunity,  $c_y$ , for three levels of the volatility of the growth opportunity,  $\sigma_y \in \{0.6, 0.7, 0.8\}$ . In the region above the lines, we have that  $\mathcal{D}_D > \mathcal{D}_E$  and hence equity is less dilutive than debt and the reverse pecking order obtains. In the region below the lines, we have that  $\mathcal{D}_D < \mathcal{D}_E$  and hence equity is more dilutive than debt, and the usual pecking order obtains. Note that the slope of the indifference lines declines when the volatility of the growth opportunity rises. These graphs reveal that equity is more likely to be less dilutive than debt when the exposure to asymmetric information on the less volatile assets in place,  $c_x$ , is larger, when the exposure to asymmetric information of the more volatile growth opportunities,  $c_y$ , is smaller. In addition, the parameter region where equity dominates debt becomes larger when the volatility of the growth opportunity increases.

The bottom graph in Figure 2 sets the exposure to asymmetric information back to the values of the base case of Table 2 (i.e.  $c_x = 25$  and  $c_y = 0$ ) and charts indifference lines of  $\mathcal{D}_D = \mathcal{D}_E$ , as a function of the time horizon,  $T$ , and the investment cost,  $I$ , for three levels of the average value of assets in place  $\bar{X} \in \{95, 100, 105\}$ . For pairs of  $(I, T)$  below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt for higher investment costs  $I$ , and longer time horizons

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<sup>25</sup>It is worthwhile to remark that the investment choices are individually rational when using either debt or equity. For example, the residual equity value for a type- $G$  type firm is equal to  $(1 - 0.462) \times 257.6 = 138.6 > 125 = X_G$ , and for a type- $B$  firm it is equal to  $(1 - 0.462) \times 218.3 = 117.4 > 75 = X_B$ .

$T$  (i.e., for younger firms). In addition, the parameter region where equity dominates debt becomes larger when the (average) values of assets in place,  $X$ , is lower (i.e., smaller firms).

The top graph of Figure 3 displays the pairs of the average value of assets in place and the average value of the growth option,  $(\bar{X}, \bar{Y})$ , for which the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ , for different level of asymmetric information on asset  $c_x \in \{10, 25, 40\}$ . For pairs of  $(\bar{X}, \bar{Y})$  below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the growth opportunities represent a larger component of firm value. In addition, the parameter region where equity dominates debt becomes larger when the exposure to asymmetric information of assets in place,  $c_x$ , increases.

Finally, the bottom graph of Figure 3 sets  $\bar{X} = 100$  and  $\bar{Y} = 175$ , as in the base case of Table 2, and plots the pairs of volatilities,  $(\sigma_x, \sigma_y)$ , such that the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ , for three levels of the investment cost  $I \in \{100, 110, 120\}$ . For pairs of volatilities,  $(\sigma_x, \sigma_y)$ , below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the volatility of assets in place is low, and when the volatility of growth opportunities is large. In addition, the parameter region where equity dominates debt becomes larger when the firm's investment need,  $I$ , increases.

In summary, the example in Table 2, as well as Figures 2 and 3, reveal a very consistent pattern: violations of the pecking order are likely to be optimal for young firms, endowed with valuable and risky growth opportunities and with large investment needs. In addition, equity is more likely to be less dilutive than debt when growth opportunities represent a greater proportion of firm value, when these growth opportunities are riskier, and when the firm has greater financing needs. Thus, our model can help explain the stylized fact that small and young firms with large financing needs tend often to prefer equity financing over debt financing, even in circumstances where asymmetric information is potentially severe.

## 5 Extensions and robustness

### 5.1 Optimal financing with existing debt

We have considered so far a firm that is all equity-financed ex-ante. In this section, we study the effect of prior financing on the debt-equity choice. In particular, we assume the firm has already issued straight debt with face value  $K_0$  prior to the beginning of the period,  $t = 0$ , which is due at the end of the period,  $T$ . In accordance to anti-dilutive “me-first” rules that may be included in the debt covenants, we assume that this pre-existing debt is senior

to all new debt that the firm may issue in order to finance the new project. We maintain the assumption that the new investment is sufficiently profitable that all firms want to raise external capital to finance it.<sup>26</sup>

As in the previous analysis, we restrict the choice of security to equity or (junior) debt. We assume that the firm can raise the necessary capital either by sale of junior debt with face value  $K$ , or by sale of a fraction  $\lambda$  of total (levered) equity of the firm to outside investors. Following an argument similar to the one in Section 3.3, the relative dilution of debt versus equity is now given by:

$$\mathcal{D}_D - \mathcal{D}_E = \int_{K_0}^{\infty} [(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0)] c(z) dz. \quad (24)$$

Note that the main difference of (24) relative to the corresponding expression in (15) is the fact that all payoffs below  $K_0$  are allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval  $[K_0, \infty)$  is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (16) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. Intuitively, the presence of pre-existing debt in a firm's capital structure, by reducing the importance of the left-tail region, makes equity more likely to be the less dilutive source of financing.

The presence of pre-existing debt affects the financing choice in the context of the real options model presented in Section 4 as follows. Similar to the previous case,  $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$  represents the exchange option between the assets in place and the growth opportunity. At the beginning of the period,  $t = 0$ , the value of levered equity for a firm of type  $\theta$  with a face value of debt  $K_0$  is given by

$$\mathcal{C}_{\theta}(K_0) \equiv \mathbb{E}[\max(Z_{\theta T} - K_0, 0)] \quad (25)$$

where  $\mathcal{C}_{\theta}(K_0)$  represents the value of a call option on the with strike price  $K_0$ , written on the exchange option  $Z_{\theta T}$ . Similarly, the value of the junior debt with face value  $K$ ,  $\mathcal{J}_{\theta}(K)$ , is given by

$$\mathcal{J}_{\theta}(K) = \mathcal{C}_{\theta}(K_0) - \mathcal{C}_{\theta}(K_0 + K), \quad (26)$$

From Stulz (1982) and Rubinstein (1991), we know that the value of these call options is

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<sup>26</sup>This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977), whereby the presence of pre-existing debt may induce a firm not to undertake a positive-NPV project.

given by

$$\mathcal{C}_\theta(\hat{K}) = X_\theta \Delta_{x\theta}^* + Y_\theta \Delta_{y\theta}^* - \hat{K}(1 - \Gamma_\theta), \quad (27)$$

where  $\hat{K} \in \{K, K_0 + K\}$  represents the investment in the risk-free asset in the corresponding replicating portfolios, and  $\Delta_{x\theta}^*$ ,  $\Delta_{y\theta}^*$ , and  $\Gamma_\theta$  are defined in the previous section.

The analysis of the relative dilution costs of debt versus equity follows an argument similar to the one developed in Section 4. Given an investment amount  $I$ , the new equity holders require a fraction of the outstanding equity  $\lambda$  such that the financing constraint  $I = \lambda[pC_G(K_0) + (1 - p)C_B(K_0)]$  holds. Similarly, the new debt holders will ask for a face value  $K$  such that the financing constraint  $I = p\mathcal{J}_G(K) + (1 - p)\mathcal{J}_B(K)$  holds. We have the following Proposition.

**Proposition 8.** *The capital raising game exhibits a reverse pecking order, in which equity is preferred to debt, if and only if*

$$\mathcal{C}_G(K_0 + K) - \mathcal{C}_B(K_0 + K) < (1 - \lambda)(\mathcal{C}_G(K_0) - \mathcal{C}_B(K_0)).$$

The financing choice with pre-existing debt is illustrated in the example in Table 3. For simplicity, we assume again there is information asymmetry on  $X$ , but not  $Y$ , namely let  $Y_G = Y_B = 120$ , and  $X_G = 110$  and  $X_B = 90$  and we set the volatilities at  $\sigma_x = 0.25$  and  $\sigma_y = 0.50$ . Furthermore, we assume that  $I = 50$ , and that the project's payoffs are realized at  $T = 10$ . Finally, we assume that type- $G$  and type- $B$  firms are again equally likely,  $p = 1/2$ . We consider the case where the firm has pre-existing debt outstanding maturing in  $T = 10$  with a face value of  $K_0 = 50$ .

Equity financing of the project requires setting  $\lambda = 0.38$ , with associated dilution costs of  $\mathcal{D}_E = 5.6$ . Junior debt financing requires a promised payment of  $K = 91.3$ , with has associated dilution of  $\mathcal{D}_D = 6$ . Thus, under these parameter values, equity is less dilutive than debt. The payoffs to debt and equity holders are presented in the top part of Figure 4 as dotted lines. The solid line represents again the information costs for a type- $G$  firm. Note that debt yields higher payoffs than equity as long as  $z \leq 295$ .

It is easy to verify that in the absence of pre-existing debt (i.e. for  $K_0 = 0$ ) the project can be financed by selling a fraction  $\lambda = 0.28$  of the firm, or promising bond holders a face value  $K = 53.7$ . It is straightforward to check that in this case the dilution of the new debt is  $D_D = 1.2$ , while the dilution of new equity is  $D_E = 4.4$ , which means that debt dominates equity. This happens because when  $K_0 = 0$  the firm can finance the project by issuing close to risk-free debt, which makes the dilution costs of debt very small. In contrast, the presence of pre-existing debt forces the firm to issue new debt that is riskier, and thus more

information sensitive, creating the potential for greater dilution. This means that, due to the existence of senior debt, equity becomes a better financing instrument than junior debt. This feature suggests that (pre-existing) high leverage may lead to more equity financing. Thus, asymmetric information may in fact lead to “mean reversion” in leverage levels, as it is often documented in the empirical literature on capital structure (see Frank and Goyal, 2003; Fama and French, 2005; Leary and Roberts, 2005).

## 5.2 Optimal security design

We now relax the assumption that the firm is constrained to issue only debt and equity and we allow any admissible security,  $s \in \mathcal{S}$ .<sup>27</sup> Following Nachman and Noe (1994), the optimal security design problem in (8) can be expressed as:

$$\min_{s \in \mathcal{S}} \int_0^\infty s'(z)(F_B(z) - F_G(z))dz, \quad (28)$$

subject to

$$\int_0^\infty s'(z)(1 - F(z))dz = I. \quad (29)$$

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^\infty s'(x)(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz = \quad (30)$$

$$\int_0^\infty \frac{s'(z)}{1 - F(z)}(H(z) - \gamma)dz, \quad (31)$$

where  $H(z)$  was defined in (10). Remember that the the function  $H(z)$  measures, for any value  $z$ , the extent of the asymmetric information costs in the right tail of the firm-value distribution. The following is an immediate consequence of the linearity of the security design problem.

**Proposition 9.** *(Nachman and Noe, 1994) A solution  $s^*$  must satisfy, for some  $\gamma \in \mathbb{R}_+$ ,*

$$(s^*)'(z) = \begin{cases} 1 & \text{if } H(z) < \gamma; \\ [0, 1] & \text{if } H(z) = \gamma; \\ 0 & \text{if } H(z) > \gamma. \end{cases} \quad (32)$$

Note that the value of the Lagrangian multiplier  $\gamma$  depends on the tightness of the financing

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<sup>27</sup>Remember that, in our setting, we only impose that the probability distributions of firm value satisfy FOSD; thus the only departure from Nachman and Noe (1994) is that we relax CSD.

constraint (29) and, thus, on the level of the required investment  $I$ , with  $\partial\gamma/\partial I > 0$ . From  $H(0) = 0$  and FOSD we have that  $H(z) < \gamma$ , which implies that the optimal security must satisfy  $(s^*)' = 1$  in a right neighborhood of  $z = 0$ . This means that an optimal security will always have a (possibly small) straight-debt component.<sup>28</sup> The importance of this straight-debt component (that is, the face value of the debt) will depend on the size of the investment  $I$  (since it affects the Lagrangian multiplier  $\gamma$ ), as well as on the particular functional form for  $H(z)$ . The shape of the optimal security for greater value  $z$  depends on monotonicity properties of the function  $H(z)$  and, thus, on the extent of asymmetric information in the right tail of the firm-value distribution, and it is characterized in the following proposition.

**Proposition 10.** *Consider the security design problem in (28)–(29).*

- (a) *(Nachman and Noe, 1994) If the distribution  $F_G$  conditionally stochastically dominates  $F_B$ , then straight debt is the optimal security.*
- (b) *If the problem satisfies the NICRT condition, and  $H'(z^*) = 0$  for a unique  $z^* \in \mathbb{R}_+$ , then convertible bonds are optimal for all investment levels  $I$ .*
- (c) *If  $\lim_{z \uparrow \infty} H(z) = \bar{h} > 0$  and there exists a unique  $z^* \in \mathbb{R}_+$  such that  $H'(z^*) = 0$ , then there exists  $\bar{I}$  such that for all  $I \leq \bar{I}$  straight debt is optimal, whereas for all  $I \geq \bar{I}$  convertible bonds are optimal.*

Part (a) of Proposition 10 assumes CSD. In this case, monotonicity of the function  $H'(z)$  implies that there is a  $z^*$  below which  $s^*(z) = 1$ , and then  $s^*(z) = 0$ , yielding straight debt as an optimal security (see Figure 5).<sup>29</sup> The intuition for the optimality of straight debt can be seen as follows. As discussed in Nachman and Noe (1994), CSD (and thus monotonicity of  $H(z)$ ) requires that the ratio of the measure of the right tails of the probability distribution for the two types,  $(1 - F_G(z))/(1 - F_B(z))$ , is monotonically increasing in  $z$ .<sup>30</sup> For monotonic securities, this ratio can be interpreted as measuring the marginal cost of increasing the payouts to investors by \$1 for a type- $G$  firm relative to a type- $B$  firm. When it becomes more expensive to increase a payouts to investors as the firm value  $z$  becomes larger the proposition shows that the optimal security is debt. In this case, firms of better types prefer to have the maximum payout to investors for low realizations of  $z$  and then to limit the payout

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<sup>28</sup>Note, however, that as Proposition 11 shows, this property hinges critically on the assumption that the firm has no pre-existing debt.

<sup>29</sup>Note that straight debt is the *unique* solution of the security design problem only under *strict* CSD.

<sup>30</sup>Equivalently, CSD requires that the hazard rate of the payoff distribution for a type- $G$  is smaller than that for a type- $B$  for all values of  $z$ .

to investors for high realizations of  $z$ . These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

The cases considered in parts (b) and (c) of Proposition 10 provide the conditions under which securities with equity-like components are optimal. Just as in the debt-equity case discussed in the earlier sections of our paper, the key driver of the optimal security choice is the size of the informational costs in the right tail of the payoff distribution, measured by  $H(z)$ . Under NICRT, we have that in the limit  $H(z) = 0$  and, thus, that the information costs suffered by a type- $G$  becomes progressively smaller as the firm-value  $z$  increases. Part (b) of Proposition (10) shows that in this case type- $G$  firms can reduce their overall dilution by maximizing the payout to investors also in the right tail of the distribution in addition to a neighborhood of  $z = 0$ . This happens because, by increasing the payoffs in the right tail, where information costs are now low because of NICRT, allows the firm to correspondingly reduce the (fixed) payout in the middle of the distribution, where the information costs are now relatively higher. This implies that the optimal security will initially have a unit slope, then a fixed payout, and then again a unit slope (see Figure 5). Thus the optimal security will have the shape of a convertible bond.

In part (c) of Proposition (10), neither CSD nor NICRT hold, since we have both a non-monotone function  $H$  and the  $\bar{h}$ -ICRT condition holds for  $\bar{h} > 0$ . The proposition shows that the size of a project affects the financing choices of a firm: straight debt is optimal for low levels of  $I$ , while convertible debt becomes optimal for large levels of the investment  $I$ . This happens because when investment needs are low, the firm can finance the project by issuing only straight debt, a security that loads only in the left tail of the distribution, where the information costs are the lowest. For greater investment needs, under  $\bar{h}$ -ICRT the firm finds it again optimal to maximize its payout to investor in the right tail of the distribution, as discussed for part (b) of the proposition, by issuing convertible debt.

We now turn to the optimal security design problem when the firm has already issued straight debt with face value  $K_0$  to outside investors (as discussed in Section 5.1). We assume again that pre-existing debt is senior with respect to any of the new securities that the firm may issue in order to finance the project. We also continue to assume that the project is sufficiently profitable, so the firm always seeks external finance to undertake the project (rather than not issuing any security and passing on the new investment opportunity).

The presence of pre-existing debt changes the structure of information costs in a non-trivial way, because cash flows in the left tail of the distribution cannot be pledged any longer to new investors. As we argued earlier, this makes equity-like securities relatively more attractive.

**Proposition 11.** *Consider the optimal security design problem when the firm has a senior debt security with face value  $K_0$  outstanding. Assume that  $F_\theta(z)$  satisfies the NICRT condition, and that there exists a unique  $z^*$  such that  $H'(z^*) = 0$ .*

(a) *If  $H'(K_0) > 0$ , then there exists  $\bar{I}$  such that: (i) warrants are optimal for  $I < \bar{I}$ , and (b) convertible bonds are optimal for  $I \geq \bar{I}$ .*

(b) *If  $H'(K_0) < 0$ , then the optimal securities are warrants.*

Proposition 11 provides conditions under which warrants arise as optimal financing instruments, in contrast to the case in which only straight debt or convertible bonds are solution to the optimal security design problem that we discussed in Proposition 10. Intuitively, warrants are optimal securities when pre-existing debt has absorbed the information benefits in the left tail (which generate the optimality of debt when  $K_0 = 0$ ). When  $K_0$  is moderate, so that  $H'(K_0) > 0$ , NICRT implies that the optimal security design is one that always loads in the right-tail, where information costs are the now the lowest (since now the left tail is already committed). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants; when the financing needs are high, the firm raises the additional capital by issuing also (junior) debt, that is by using convertible debt. When  $K_0$  is large, so that  $H'(K_0) < 0$ , the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs).

We conclude this section by illustrating the characterizations of the optimal securities in Propositions 10 and 11 in the real options model of Section 4. The next proposition gives an analytical condition under which the NICRT condition holds, which together with the previous two propositions generates simple predictions in the real options model.

**Proposition 12.** *The model satisfies the NICRT condition if there is no information asymmetry on  $y$ ,  $c_y = 0$ , and the volatility of the growth opportunity is higher than that of the assets in place,  $\sigma_y > \sigma_x$ .*

Since second moments dominate tail behavior for Gaussian random variables, the assumptions that  $Y$  has no information costs and  $\sigma_y > \sigma_x$  are sufficient to generate a non-monotonic  $H(z)$  function.

We consider three different parameter values to numerically illustrate the results in this section, specialized to the real options model of Section 4. The three difference scenarios are illustrated in Table 4. Figure 5 plots the  $H(z)$  function in the left panels, and the optimal security in the right panels, each row corresponding to each of the cases in Table 4. In all cases we assume that  $p = 1/2$ ,  $\sigma_x = 0.2$  and  $\sigma_y = 0.5$ .

The first scenario (Panel A) presents the case where the asymmetric information is concentrated in the high volatility asset, namely we let  $X_G = X_B = 120$ ,  $Y_G = 110$ ,  $Y_B = 90$  and we set the investment to be  $I = 100$ . In this case straight debt will be optimal, as the  $H(z)$  function is monotone over its whole domain (see the top left graph in Figure 5). In particular, a standard bond with a face value of  $K = 101.6$  is sufficient to finance the project and minimize information costs.

The second case (Panel B) is closer to the examples from Section 4, in that the asymmetric information is concentrated in the low-volatility asset. Namely, we set  $Y_G = Y_B = 175$ ,  $X_G = 120$ ,  $X_B = 100$ , and the investment amount to  $I = 90$ . The optimal security in this case is a convertible debt contract with  $K = 88.9$ , and conversion trigger at  $z_c = 309.4$ . As in the case of Proposition 12 securities load in the *lower* end of the payoffs, due to the usual Myers and Majluf (1984) intuition, *but also on upper end of the payoff distribution*.

The last figure (Panel C) provides an illustration of case (b) in Proposition 11. We use the same parameter values as in previous example, but we now assume that the firm has debt outstanding with  $K_0 = 100$ . Further, let the investment amount be  $I = 15$ . In this case, Proposition 11 shows that the optimal security is warrants, with an exercise price of  $\kappa = 167.9$ .

### 5.3 Robustness and simulations

In this section, we examine the robustness of our results to alternative model specifications and we discuss the empirical implications of our paper through simulated data. In previous sections of this paper, we characterized the optimal financing choices under asymmetric information for general distributions that satisfy FOSD but not necessarily CSD. The parametric examples that we use to generate the reverse pecking order have been based on an exchange option specification. In this section, we discuss another specification of firm value which also generates the reverse pecking order.

We start by considering a small extension of the basic model. In this perturbation of the model we assume that the exercise of the exchange option requires the firm to make at the end of the period  $T$  an additional investment  $I_T$ . This means that the end-of-period firm value is now described by  $Z_{\theta T} \equiv \max(X_{\theta T}, Y_{\theta T} - I_T)$ . Note that this characterization of firm value represents a dual-strike option (where the first asset,  $X_{\theta T}$ , has a zero strike price) and it does not have a closed-form solution. The results for are displayed in Tables 5. We consider similar parameter values as in Table 2, with the difference that now  $\sigma_y = 0.50$ ,  $I = 100$ , and  $Y_G = Y_B = 200$ . With this lower value of volatility of the growth opportunity, debt is less dilutive than equity and the pecking order holds if  $I_T = 0$  (see the first row in Table 5).

Increasing the level of investment  $I_T$  increases the dilution of both debt and equity, but the dilution of debt increases faster than the dilution of equity and a reversal of the pecking order obtains for  $I_T \geq 50$ . The intuition for these results is as follows. Increasing the investment level  $I_T$  makes the exchange option to be less in-the-money, which increases the leverage that is embedded in the option. Thus, a larger value of the investment  $I_T$  has an effect similar to a greater value of the volatility of the growth opportunity  $\sigma_y$ . The numerical results imply that equity financing is more likely to be less dilutive than debt if the exploitation of the growth opportunity requires a greater additional investment  $I_T$ .

We next consider a variation of the model where the firm is now endowed by two assets,  $X_{\theta T}$  and  $Y_{\theta T}$ . The random variables  $X_{\theta T}$  and  $Y_{\theta T}$  are again characterized by lognormal distribution (as in Section 4). Firm value is now additive and it is given by  $Z_{\theta T} = X_{\theta T} + Y_{\theta T}$ , where we can interpret  $X_{\theta T}$  and  $Y_{\theta T}$  as the value of the assets of the two divisions of a firm. Under the lognormality assumption, this model specification does not admit a closed-form solution, but it can be characterized numerically.<sup>31</sup>

In Table 6 we consider an example that uses parameter values close to those in Table 2. The firm can finance the new project, at a cost of  $I = 110$  by either selling a fraction  $\lambda = 0.40$  of the equity of the firm, or by issuing straight debt with a face value equal to  $K = 213.2$ , which carries a credit spread of 451 basis points. As in the base case, the parameter values are such that the NICRT condition is satisfied. It is easy to verify that the dilution costs associated with equity are  $\mathcal{D}_E = 20.1$ , whereas those associated with debt are  $\mathcal{D}_D = 21.2$ .

The comparative static results for the additive case mimic the results we obtained in the exchange-option case from Section 4. Equity financing is again more likely to dominate debt financing when the asset with less exposure to asymmetric information has also greater volatility, when the level of investment is greater, when the asset with greater volatility is a greater proportion of firm value and is relatively more volatile.

This specification of our model generates novel predictions on the cross-sectional variation of the capital structures of multidivisional firms. Specifically, we predict that firms are more likely to be equity financed when they consist of heterogeneous divisions, and when the relatively more transparent division — that is the one that is less likely to be afflicted by asymmetric information — is also riskier (i.e., it is characterized by greater volatility).<sup>32</sup>

Throughout the discussion we have assumed that the payoff to the firm before the investment is  $X_{\theta T}$ , so that the incremental cash flow from investment is the exchange option

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<sup>31</sup>In the analysis that follows we approximate the relevant integrals by simulations, with sample sizes that guarantee accuracy on the order of four significant digits.

<sup>32</sup>Note also that multidivisional firms may arise as the outcome of a merger. Thus, our model also generates new predictions on why in cash-financed mergers, the funds required for the acquisition are raised by either borrowing or issuing new equity.

$\max(X_{\theta T}, Y_{\theta T}) - X_{\theta T} = \max(Y_{\theta T} - X_{\theta T}, 0)$ . But the model is more general. The analysis, under the positive NPV assumption, goes through as stated if the payoff of the firm without the investment is  $w \max(X_{\theta T}, Y_{\theta T})$  for some  $w \in (0, 1)$ , so the incremental value of the investment at maturity is  $(1 - w) \max(X_{\theta T}, Y_{\theta T})$ .<sup>33</sup>

We conclude this section by characterizing the comparative statics that would be obtained in a economy generated by simulations of our basic model.<sup>34</sup> In particular, we generate a “panel dataset” randomly sampling from our model primitives.<sup>35</sup> We simulate the model one million times, solving it numerically, using the closed-form solutions from Section 5.1 at each iteration, and save only the results for which the relative dilution of equity is within 20% of that of debt.<sup>36</sup> We then run regressions of the form  $Y_i = \beta^\top X_i + \epsilon_i$ , where  $Y_i$  is either (i) the ratio of the dilution costs of equity over the dilution costs of debt,  $\mathcal{R}_i = \mathcal{D}_{Ei}/\mathcal{D}_{Di}$ , or (ii) a dummy that equals to 1 if the firm finds it optimal to issue debt, i.e.,  $\mathcal{D}_{Ei} > \mathcal{D}_{Di}$ . In the later case we estimate a logit model, whereas in the former case we report ordinary-least-squares (OLS) coefficients.

As the set of explanatory variables  $X_i$  we include: a constant; two metrics of the information asymmetry faced by investors,  $c_x = X_G - X_B$  and  $c_y = Y_G - Y_B$ ; the level of the payoffs  $\bar{X}$  and  $\bar{Y}$ ; the volatilities of each of the components of the assets,  $\sigma_x$  and  $\sigma_y$ , as well as the correlation  $\rho$ ; the probability of a good type  $p$ , and the face value of senior debt  $K_0$ , as well as the investment amount  $I$  and the time to maturity  $T$ . When giving point estimates of a regression, we normalize all independent variables to zero mean and unit variance, so the intercept of the OLS regression can be interpreted as an unconditional mean, and the OLS coefficients as the marginal effect of a one standard deviation change in the independent variables. In the logit results, where the point estimates do not have marginal interpretations, the relative size of the estimates give a sense of the importance of each of the explanatory variables.

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<sup>33</sup>This remark is important in order to formally nest the lognormal specification. In our framework we can take the limit  $Y_{\theta T} \downarrow 0$  (a.s.) but still have a non-trivial asymmetric information problem with a positive NPV if  $w < 1$  and we do not interpret  $Y_{\theta T}$  as the incremental cash flow.

<sup>34</sup>See Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2006) for similar approaches.

<sup>35</sup>In the Appendix we give the details on the sampling procedure we use. Note that the actual generation of the parameter values is rather irrelevant for our purposes, in the sense that we can condition on different subsets of the parameter space in the analysis that follows. In general, the simulation procedure will generate scenarios where (quasi) risk-less debt is feasible, and thus optimal. But it will also generate parameter values for which the trade-off at the financing choice satisfies conditions that are close to those associated with the  $h$ -ICRT for low values of  $h$ .

<sup>36</sup>Namely, if we let the relative dilution of equity (over debt) to be defined as  $\mathcal{R} = \mathcal{D}_E/\mathcal{D}_D$ , we only consider those cases where  $\mathcal{R} \in (0.8, 1.2)$ . About 46.1% of the simulated parameters satisfy this constraint. For 45.5% debt’s dilution is less than 20% that of equity, mostly when debt is (close to) riskfree. For 5.9% of the cases studied, equity’s dilution is less than 20% than that of debt. We focus on parameter values for which there is some tension in the debt-equity choice.

Panel A of Table 7 gives the estimates of the logit specification, whereas Panel B presents the results where the relative dilution  $\mathcal{R}_i$  is the dependent variable. Each set of pair of columns contains the estimated coefficients, and the related comparative static. For example, the point estimate on  $c_x$  of  $-4.8$  (Panel B, second column) means that a one standard deviation increase in  $c_x$ , decreases the relative dilution of equity, i.e., it makes debt relatively more expensive by 4.8%. The first column presents the estimates over all the cases that comprise the main sample. The rest of the columns present the results for different sub-samples, depending on whether the observations are in the top or bottom quintiles of the variables that measure existing debt,  $K_0$ , the information asymmetry on the assets in place,  $c_x$ , and the information asymmetry on the assets in place,  $c_y$ . Given the size of the sample, all point estimates are highly significant, so  $t$ -statistics are omitted.

The regression suggests that the relative dilution costs of equity increase as  $c_y$ ,  $\sigma_x$ , and  $\mu_x$  increase, but decrease if  $c_x$ ,  $\mu_y$ ,  $\sigma_y$ ,  $\rho$ ,  $K_0$ ,  $I$ ,  $p$  and  $T$  increase. It is remarkable that across all seven subsets of the parameter space considered, and both the logit and OLS specifications, the comparative statics with respect to ten primitives of the model, out of eleven, do not change signs. Only for the parameter  $I$  does the coefficient flip signs in the OLS specification, albeit with economically small magnitudes (0.0), which hints at the non-linearities of the model.

These comparative statics reinforce the intuition behind the reverse pecking order from the previous sections. The information asymmetry needs to be concentrated on the low-volatility asset: the higher the parameter  $c_x$  is, the more likely equity will be issued. The information asymmetry on the high-volatility asset,  $c_y$ , which governs the behavior of the information costs on the right-tail, has the opposite effect. Rather intuitively, if the right-tail cash flows become more expensive in terms of information costs, then the firm is more likely to issue straight debt. The volatility parameters play a dual role — amplifying/reducing the information asymmetry costs. The higher the existing assets volatility ( $\sigma_x$ ) is, the more likely straight debt is optimal, whereas higher volatility ( $\sigma_y$ ) for the new assets favors equity. More interestingly, the numerical analysis shows that higher correlation between the two assets makes equity more desirable, which suggests that firms with more similar assets are more likely to violate the pecking order.

Table 7 also shows how the size of the assets (existing and new) favor debt over equity. The mechanism is simple: the higher the asset value, relative to the investment needs  $I$ , the closer the debt contract is to be risk-free. Furthermore, the higher the probability of the good type  $p$ , the more likely equity becomes optimal. Table 7 further confirms that the presence of existing debt is an important determinant of the debt/equity choice. In particular, equity is more likely to be optimal if the firm already has some debt in its capital structure. Finally,

the larger the investment  $I$ , the less likely it is that the firm will issue debt.

The regression results for the simulated datasets confirm our early intuition that the critical drivers of the reverse pecking order are low information asymmetry on the right tail of the payoff distribution, large investment needs, and existing debt in the capital structure. Under such conditions, a debt security will be more sensitive to private information than an equity security. As such, equity financing can be less dilutive than debt financing under asymmetric information.

## 6 Conclusion

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984) in the context of a simple real options problem. We model firm value as an exchange option between two risky assets, and show that even if the distribution of each individual assets satisfies the conditional stochastic dominance condition, the distribution of the exchange option may not. This means that, contrary to common intuition, equity financing can dominate debt financing under asymmetric information, even in cases where individual assets would be financed by debt when taken in isolation. We also show that the presence of existing debt makes equity less dilutive than debt. Finally, our model also predicts the optimality of convertible debt and warrants. Taken together, these results suggest that the relationship between asymmetric information and choice of financing is more subtle than previously believed.

## Appendix

**Simulation details.** Let  $U_i$ ,  $i = 1, \dots, 10$ , denote a set of independent uniformly distributed random variables in  $[0, 1]$ . We set  $\sigma_x = \min(0.2 + 0.8U_1, 0.2 + 0.8U_2)$ , and  $\sigma_y = \max(0.2 + 0.8U_1, 0.2 + 0.8U_2)$ , so that  $Y$  maps into the higher volatility asset component, which we previously referred to as the firm's "growth opportunity." Note how the volatilities are bounded in the set  $[0.2, 0.8]$ . We let  $\rho = -0.5 + 1.5U_3$ , so that the correlation parameter is uniformly distributed in  $[-0.5, 1]$ . We set the time to maturity to be  $T = 5 + 25U_4$ , with support in  $[5, 30]$ . We further let  $\mu_x = U_5$  and  $\mu_y = U_6$ . We then set  $\mu_{xG} = \mu_x + kU_7$  and  $\mu_{xB} = \mu_x - kU_7$ , and similarly  $\mu_{yG} = \mu_y + kU_8$  and  $\mu_{yB} = \mu_y - kU_8$ , where we set arbitrarily  $k = 0.3$ . We let  $X_\theta = e^{\mu_x \theta}$  and  $Y_\theta = e^{\mu_y \theta}$ . Note how the information asymmetry is parametrized by a uniformly distributed random variable that spreads the means of the type- $G$  and type- $B$  by at most a log-return of 60%. We set  $p = 0.2 + 0.6U_7$  as the probability of the type- $G$  firm, with support in  $[0.2, 0.8]$ . We set the value of the existing senior debt at  $K_0 = (0.2 + 0.6U_9)\mathcal{A}$ , where  $\mathcal{A}$  denotes the value of the (total) assets post-investment. Thus the principal of the old debt will be between 20-80% of the total firm value. Finally, we let  $\bar{C} = p\mathcal{C}_G(K_0) + (1 - p)\mathcal{C}_B(K_0)$  denote the value of the equity of the firm (net of the senior debt), and set the investment amount at  $I = (0.3 + 0.5U_{10})\bar{C}$  (this guarantees that the problem has a solution).

**Proof of Proposition 1.** The claim follows from Proposition 1 and Lemma A.9 in Nachman and Noe (1994). ■

**Proof of Proposition 2.** From the definition of  $H(z)$  in (10), we have:

$$\begin{aligned} \frac{dH(z)}{dz} &= \frac{(f_B(z) - f_G(z))(1 - F(z)) + (pf_G(z) + (1 - p)f_B(z))(F_B(z) - F_G(z))}{(1 - F(z))^2} \\ &= \frac{f_B(z) - f_G(z) + F_B(z)f_G(z) - F_G(z)f_B(z)}{(1 - F(z))^2} \\ &= \frac{f_B(z)(1 - F_G(z)) - f_G(z)(1 - F_B(z))}{(1 - F(z))^2}. \end{aligned}$$

Thus  $H'(z) > 0$  if and only if  $f_B(z)(1 - F_G(z)) > f_G(z)(1 - F_B(z))$ , which reduces to the CSD condition. ■

**Proof of Proposition 3.** We argue that the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely  $f_G(z)/f_B(z)$  is monotonically

non-decreasing for all  $z \in \mathbb{R}_+$ . From basic principles we have:

$$\begin{aligned}
\frac{f_G(z)}{f_B(z)} &= \frac{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2}}{\frac{1}{z\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2}} \\
&= e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2} + \log(z)\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2}} z^{\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)};
\end{aligned}$$

which is monotonically increasing in  $z$  when  $\mu_G > \mu_B$ , as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), this concludes the proof.  $\blacksquare$

**Proof of Proposition 4.** From the budget constraint for equity and debt securities, one has that

$$\lambda = \frac{p\mathbb{E}[\min(Z_G, K)] + (1-p)\mathbb{E}[\min(Z_B, K)]}{p\mathbb{E}[Z_G] + (1-p)\mathbb{E}[Z_B]} \quad (33)$$

Using (33) in (12) and comparing this to (13) one easily arrives at (14).

The following result from Shaked and Shanthikumar (2007) is useful.

**Lemma 1** (Theorem 1.B.12 from Shaked and Shanthikumar (2007)). *Given two distribution functions  $F_G$  and  $F_B$ , the following two statements are equivalent: (a)  $F_G$  conditionally stochastically dominates  $F_B$ ; (b)  $\mathbb{E}[\alpha(X_B)]\mathbb{E}[\beta(X_G)] \leq \mathbb{E}[\alpha(X_G)]\mathbb{E}[\beta(X_B)]$ , for all functions  $\alpha$  and  $\beta$  such that  $\beta$  is non-negative and  $\alpha/\beta$  and  $\beta$  are non-decreasing.*

Let  $\alpha(z) = z$  and  $\beta(z) = \min(z, K)$  for some  $K \geq 0$ . Clearly  $\beta$  is non-decreasing and non-negative for  $x \geq 0$ . Furthermore,  $\alpha(z)/\beta(z) = z/\min(z, K)$  is non-decreasing. Thus, if  $F_G$  conditionally stochastically dominates  $F_B$  it must be that

$$\mathbb{E}[Z_B]\mathbb{E}[\min(Z_G, K)] \leq \mathbb{E}[Z_G]\mathbb{E}[\min(Z_B, K)]$$

which clearly rules out (14).  $\blacksquare$

**Proof of Proposition 5.** It is clear than in order for the reverse pecking order to hold, it is necessary that  $\mathcal{D}_D > \mathcal{D}_E$ . This condition, if  $\hat{z} > \bar{z}$  (i.e., the UNC does not hold), can be

written as

$$\int_0^{\bar{z}} \underbrace{(\min(K, z) - \lambda z)}_{>0} \underbrace{c(z)}_{<0} dz + \int_{\bar{z}}^{\hat{z}} \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{<0} dz + \int_{\hat{z}}^{\infty} \underbrace{(K - \lambda z)}_{<0} \underbrace{c(z)}_{>0} dz > 0 \quad (34)$$

We note that since  $g$  is the difference of two densities, it must be the case that

$$\int_0^{\infty} c(z) dz = 0; \quad \Rightarrow \quad - \int_0^{\hat{z}} c(z) dz = \int_{\hat{z}}^{\infty} c(z) dz$$

Further, we have

$$\begin{aligned} \int_{\hat{z}}^{\infty} (\lambda z - K) c(z) dz &> \int_{\hat{z}}^{\infty} (\lambda \hat{z} - K) c(z) dz \\ &= (\lambda \hat{z} - K) \int_{\hat{z}}^{\infty} c(z) dz \\ &= (K - \lambda \hat{z}) \int_0^{\hat{z}} c(z) dz \\ &> (K - \lambda \hat{z}) \int_{\bar{z}}^{\hat{z}} c(z) dz \\ &> \int_{\bar{z}}^{\hat{z}} (K - \lambda z) c(z) dz \end{aligned}$$

Therefore, the sum of the last two terms in (34) are negative, and since the first one is negative as well it is clear that  $\mathcal{D}_D - \mathcal{D}_E < 0$ , i.e., a reversal of the pecking order cannot obtain if UNC is not true. The statement in the proposition is immediate from (15), the definitions of  $\hat{z}$  and  $\bar{z}$ , and the discussion in the text.  $\blacksquare$

**Proof of Proposition 6.** The expression (20) is from Margrabe (1978), where  $\hat{\Delta}_{x\theta} \equiv N(a_{x\theta})$ ,  $\hat{\Delta}_{y\theta} \equiv N(a_{y\theta})$ ,  $\Sigma^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y\rho$ ,  $N(\cdot)$  denotes the cumulative distribution function of a standard normal random variable, and

$$a_{x\theta} = \frac{\log(X_\theta/Y_\theta)}{\Sigma\sqrt{T}} + \frac{1}{2}\Sigma\sqrt{T}, \quad (35)$$

$$a_{y\theta} = \frac{\log(Y_\theta/X_\theta)}{\Sigma\sqrt{T}} + \frac{1}{2}\Sigma\sqrt{T}, \quad (36)$$

The expression in (21) follows from Stulz (1982) and Rubinstein (1991), where  $\Delta_{x\theta}^* \equiv \Gamma(b_{x\theta}, a_{x\theta}, \rho_x)$ ,  $\Delta_{y\theta}^* \equiv \Gamma(b_{y\theta}, a_{y\theta}, \rho_y)$ ,  $\Gamma_\theta \equiv \Gamma(b_{x\theta} + \sigma_x\sqrt{T}, b_{y\theta} + \sigma_y\sqrt{T}, \rho)$ ,  $\rho_x = (\sigma_x - \rho\sigma_y)/\Sigma$ , and  $\rho_y = (\sigma_y - \rho\sigma_x)/\Sigma$ , the function  $\Gamma(\cdot)$  denotes the cumulative distribution function of a

bivariate standard normal random vector,<sup>37</sup> the variables  $a_{x\theta}$  and  $a_{y\theta}$  are given in (35)–(36), and the variables  $b_{x\theta}$  and  $b_{y\theta}$  are given by

$$b_{x\theta} = \frac{\log(K/X_\theta)}{\sigma_x\sqrt{T}} - \frac{1}{2}\sigma_x\sqrt{T}, \quad (37)$$

$$b_{y\theta} = \frac{\log(K/Y_\theta)}{\sigma_y\sqrt{T}} - \frac{1}{2}\sigma_y\sqrt{T}. \quad (38)$$

Condition (19) follows by noting that the dilution of equity is  $\lambda(\mathcal{A}_G - \mathcal{A}_B)$ , whereas that of debt is  $\mathcal{P}_B - \mathcal{P}_G$ . ■

**Proof of Proposition 7.** We first note that letting  $\epsilon \downarrow 0$  the statements in the Proposition boil down to the delta condition given in (23). When  $\sigma_x = \rho\sigma_y$ , it is easy to see that  $\Delta_x^* \equiv \Gamma(b_x, a_x, 0) = N(b_x) \times N(a_x) = \Delta_x \times \hat{\Delta}_x$ , so that  $\Delta_x = N(b_x)$ . Under these conditions, the delta of the compound put option (i.e., the default option) that can be decomposed into the product of the delta of a “simple” put option written only on the assets in place,  $X$ , with a strike price equal to the face value of the debt,  $K$ , times the delta with respect to the assets in place  $X$  of the underlying exchange option. Substituting  $\Delta_x^* = \Delta_x \times \hat{\Delta}_x$  into (22) and using (17), we obtain that (19) reduces to (23).

Equation (23) can be expressed more explicitly as

$$\frac{I}{\bar{X}N\left(\frac{\log(\bar{X}/\bar{Y})}{\Sigma\sqrt{T}} + \frac{1}{2}\Sigma\sqrt{T}\right) + \bar{Y}N\left(\frac{\log(\bar{Y}/\bar{X})}{\Sigma\sqrt{T}} + \frac{1}{2}\Sigma\sqrt{T}\right)} < N\left(\frac{\log(K/\bar{X})}{\sigma_x\sqrt{T}} - \frac{1}{2}\sigma_x\sqrt{T}\right) \quad (39)$$

We note that the right-hand side is independent of  $\bar{Y}$ . The left-hand side of this condition tends to zero for  $\bar{Y}$  sufficiently large, so (23) holds in this case. For  $\bar{Y}$  sufficiently small, the financing constraint for debt reduces to

$$K(1 - N(b_x + \sigma_x\sqrt{T})) + XN(b_x) = I$$

so that

$$\frac{I}{\bar{X}} = N(b_x) + \frac{K}{\bar{X}}(1 - N(b_x + \sigma_x\sqrt{T})). \quad (40)$$

As  $\bar{Y}$  goes to zero, (23) reduces to  $I/\bar{X} < N(b_x)$ , which is impossible from the financing constraint (40). This proves (i).

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<sup>37</sup>Namely  $\Gamma(a, b, c)$  is the area under a bivariate standard normal distribution function with correlation  $c$  from  $-\infty$  to  $a$ ,  $-\infty$  to  $b$ . Thus, if  $f(x_1, x_2)$  is the density of a standard normal bivariate vector  $x = (x_1, x_2)$  with correlation  $c$ , then  $\Gamma(a, b, c) = \int_{-\infty}^a \int_{-\infty}^b f(x_1, x_2) dx_1 dx_2$ .

In order to prove (ii), we note that the limit of the left-hand side of (39) as  $\sigma_x^2 \downarrow 0$  is finite, and strictly greater than zero. On the other hand, the argument of  $N(\cdot)$  in the right-hand side of the condition tends to either positive or negative infinity, depending on whether  $X < K$  (or  $X > K$ ). In the former case (39) always holds, whereas if  $X > K$  it can never hold. This completes the proof. ■

**Proof of Proposition 8.** Immediate from the discussion in the text. ■

**Proof of Proposition 9.** See Theorem 8 in Nachman and Noe (1994). ■

**Proof of Proposition 10.** Since  $H$  is increasing in (a), there is a single crossing point  $z$  such that  $H(z) = \gamma$ , for any  $\gamma \in \mathbb{R}_+$ . The claim in (a) follows immediately from Proposition 9. Assuming the NICRT, and that  $H'(z^*) = 0$  at most once, it is immediate that there are two crossing points for  $H(z^*) = \gamma$ , for any  $\gamma \in \mathbb{R}_+$ . The claim is immediate from Proposition 9. Case (c) is analogous, but noting that for  $\gamma \leq \bar{\gamma}$  there is a single point satisfying  $H(z^*) = \gamma$ , but two such points for  $\gamma$  sufficiently large. ■

**Proof of Proposition 11.** The proof is analogous to that of Proposition 9. The first-order conditions require  $s'(z)$  to be either one (or zero) at points for which  $H(z) < \gamma$  (or  $H(z) > \gamma$ ). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of  $\gamma$ , or equivalently of the investment  $I$ . The claim in (a) mirrors case (b) from Proposition 10. ■

**Proof of Proposition 12.** Using l'Hopital's rule, one has

$$\lim_{z \uparrow \infty} H(z) = \lim_{z \uparrow \infty} \frac{F_B(z) - F_G(z)}{1 - F(z)} \quad (41)$$

$$= \lim_{z \uparrow \infty} \frac{f_G(z) - f_B(z)}{pf_G(z) + (1-p)f_B(z)}. \quad (42)$$

From basic principles it is clear that:

$$\mathbb{P}(Z_{\theta T} = z) \equiv f_{\theta}(z) = f_{x\theta}(z) + f_{y\theta}(z)$$

with

$$f_{x\theta}(z) = \frac{1}{z\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x}\right)^2} N\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y\sqrt{1-\rho^2}} - \rho\frac{\log(z)-\mu_{x\theta}}{\sigma_x\sqrt{1-\rho^2}}\right)$$

$$f_{y\theta}(z) = \frac{1}{z\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_{y\theta}}{\sigma_y}\right)^2} N\left(\frac{\log(z)-\mu_{x\theta}}{\sigma_x\sqrt{1-\rho^2}} - \rho\frac{\log(z)-\mu_{y\theta}}{\sigma_y\sqrt{1-\rho^2}}\right)$$

where  $\mu_{x\theta} = \log(X_\theta)$  and  $\mu_{y\theta} = \log(Y_\theta)$ .

The limit in (42) is easy to compute by factoring out leading terms. We note that when  $\sigma_y > \sigma_x$  the right-tail behavior is determined by the piece of the densities  $f_\theta(z)$  that corresponds to the density of  $Y$ . When  $c_y = 0$ , the limit of these densities is zero. This completes the proof. ■

## References

- Admati, A., and P. Pfleiderer, 1994, “Robust financial contracting and the role of venture capitalists,” *Journal of Finance*, 49, 371–402.
- Berk, J. B., R. C. Green, and V. Naik, 1999, “Optimal investment, growth options, and security returns,” *Journal of Finance*, 54(5), 1553–1607.
- Bharath, S. T., P. Pasquariello, and G. Wu, 2010, “Does Asymmetric Information Drive Capital Structure Decisions?,” *Review of Financial Studies*, 22(8), 332–355.
- Black, F., and M. Scholes, 1973, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, 81(3), 637–654.
- Bond, P., and H. Zhong, 2014, “Buying high and selling low: Stock repurchases and persistent asymmetric information,” working paper, University of Minnesota.
- Brennan, M., and A. Kraus, 1987, “Efficient financing under asymmetric information,” *Journal of Finance*, 42, 1225–1243.
- Carlson, M., A. Fisher, and R. Giammarino, 2006, “Corporate investment and asset price dynamics: implications for SEO event studies and long-run performance,” *Journal of Finance*, 61(3), 1009–1034.
- Chakraborty, A., S. Gervais, and B. Yilmaz, 2011, “Security Design in Initial Public Offerings,” *Review of Finance*, 15(2), 327–357.
- Chakraborty, A., and B. Yilmaz, 2009, “Adverse Selection and Convertible Bonds,” *Review of Economic Studies*, 78(1), 148–175.
- Chemmanur, T., and P. Fulghieri, 1997, “Why Include Warrants in New Equity Issues? A Theory of Unit IPOs,” *Journal of Financial and Quantitative Analysis*, 32(1), 1–24.
- Constantinides, G. M., and B. D. Grundy, 1989, “Optimal investment with stock repurchase and financing as signals,” *Review of Financial Studies*, 2, 445–465.
- Cooney, J., and A. Kalay, 1993, “Positive information from equity issue announcements,” *Journal of Financial Economics*, 33, 149–172.
- DeMarzo, P., 2005, “The Pooling and Tranching of Securities: A Model of Informed Intermediation,” *Review of Financial Studies*, 18, 1–35.
- DeMarzo, P., and D. Duffie, 1999, “A Liquidity-Based Model of Security Design,” *Econometrica*, 67, 65–99.
- DeMarzo, P., I. Kremer, and A. Skrzypacz, 2005, “Bidding with Securities: Auctions and Security Design,” *American Economic Review*, 95, 936–959.

- Dybvig, P., and J. Zender, 1991, “Capital structure and dividend irrelevance with asymmetric information,” *Review of Financial Studies*, 4(1), 201–219.
- Edmans, A., and W. Mann, 2012, “Financing through asset sales,” working paper, University of Pennsylvania.
- Fama, E., and K. French, 2005, “Financing Decisions: Who Issues Stock?,” *Journal of Financial Economics*, 76, 549–582.
- Fischer, E. O., R. Heinkel, and J. Zechner, 1989, “Dynamic Capital Structure Choice: Theory and Tests,” *Journal of Finance*, 40, 19–40.
- Frank, M. Z., and V. K. Goyal, 2003, “Testing the pecking order theory of capital structure,” *Journal of Financial Economics*, 67, 217–248.
- Fulghieri, P., and D. Lukin, 2001, “Information production, dilution costs, and optimal security design,” *Journal of Financial Economics*, 61, 3–41.
- Hennessy, C. A., D. Livdan, and B. Miranda, 2010, “Repeated signaling and firm dynamics,” *Review of Financial Studies*, 23(5), 1981–2023.
- Hennessy, C. A., and T. M. Whited, 2005, “Debt dynamics,” *Journal of Finance*, 60(3), 1129–1165.
- Innes, R. D., 1990, “Limited liability and incentive contracting with ex-ante action choices,” *Journal of Economic Theory*, 52, 45–67.
- Jung, K., Y.-C. Kim, and R. M. Stulz, 1996, “Timing, investment opportunities, managerial discretion, and the security issue decision,” *Journal of Financial Economics*, 42(2), 159–186.
- Leary, M. T., and M. R. Roberts, 2005, “Do firms rebalance their capital structures?,” *Journal of Finance*, 60, 2575–2619.
- , 2010, “The pecking order, debt capacity, and information asymmetry,” *Journal of Financial Economics*, 95(3), 332–355.
- Leland, H., 1994, “Corporate Debt Value, Bond Covenants, and Optimal Capital Structure,” *Journal of Finance*, 49, 1213–1252.
- Lemmon, M. L., and J. F. Zender, 2010, “Debt Capacity and Tests of Capital Structure Theories,” *Journal of Financial and Quantitative Analysis*, 45(5), 1161–1187.
- Margrabe, W., 1978, “The value of an option to exchange one asset for another,” *Journal of Finance*, 33(1), 177–186.
- Merton, R., 1974, “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance*, 29(2), 449–470.

- Morellec, E., and N. Schürhoff, 2011, “Corporate investment and financing under asymmetric information,” *Journal of Financial Economics*, 99, 262–288.
- Myers, S., 1977, “Determinants of corporate borrowing,” *Journal of Financial Economics*, 5, 147–175.
- , 1984, “The capital structure puzzle,” *Journal of Finance*, 39, 575–592.
- Myers, S., and N. S. Majluf, 1984, “Corporate Financing and Investment Decisions when Firms have Information that Investors Do Not Have,” *Journal of Financial Economics*, 13(2), 187–221.
- Nachman, D., and T. Noe, 1994, “Optimal design of securities under asymmetric information,” *Review of Financial Studies*, 7(1), 1–44.
- Noe, T., 1988, “Capital structure and signaling game equilibria,” *Review of Financial Studies*, 1, 331–356.
- Rubinstein, M., 1991, “Somewhere over the rainbow,” *RISK*, November.
- Shaked, M., and J. G. Shanthikumar, 2007, *Stochastic orders*. Springer, New York.
- Shyam-Sunder, L., and S. C. Myers, 1999, “Testing static tradeoff against pecking order models of capital structure,” *Journal of Financial Economics*, 51(2), 219–244.
- Strebulaev, I. A., 2007, “Do Tests of Capital Structure Theory Mean What They Say?,” *Journal of Finance*, 62(4), 1747–1787.
- Stulz, R., 1982, “Options on the Minimum or the Maximum of Two Risky Assets: Analysis and Applications,” *Journal of Financial Economics*, 10(2), 161–185.

**Table 1: A simple example**

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 2. The payoff of the firm is given by a trinomial random variable  $Z \in \{z_1, z_2, z_3\}$ . The growth opportunity requires an investment of  $I = 60$ , and generates an extra cash flow of 200 in the high state. The payoff and the state probabilities are summarized below.

<b>Payoffs</b>	$z_1$	$z_2$	$z_3$
Assets in place	10	100	100
Growth opportunity	0	0	200
Total payoff	10	100	300
<b>Distributions</b>	$z_1$	$z_2$	$z_3$
Good-type, $f_G$	0.2	0.4	0.4
Bad-type, $f_B$	0.3	$0.4 - x$	$0.3 + x$

The column labelled “Pooled value” below computes the expected value of the firm,  $\mathbb{E}[Z]$ , where each type is assumed equally likely. The variable  $x$  can take values in  $[0, 0.10]$ , to guarantee that the distribution  $f_G$  first-order stochastically dominates  $f_B$ . The variable  $\lambda$  denotes the fraction of equity the firm needs to issue to finance the investment of  $I = 60$ . The column labelled  $\mathcal{D}_E$  denotes the dilution costs of equity, namely  $\lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B])$ . For all values of  $x$ , the firm can also finance the project with a debt security with a face value  $K = 76.7$ , for which the dilution costs,  $\mathcal{D}_D \equiv \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)]$ , are 6.7 (last column).

$x$	$\mathbb{E}[Z_G]$	$\mathbb{E}[Z_B]$	Pooled value	$\lambda$	$\mathcal{D}_E$	$\mathcal{D}_D$
0.00	162	133	147.5	0.407	11.8	6.7
0.01	162	135	148.5	0.404	10.9	6.7
0.02	162	137	149.5	0.401	10.0	6.7
0.03	162	139	150.5	0.399	9.2	6.7
0.04	162	141	151.5	0.396	8.3	6.7
0.05	162	143	152.5	0.393	7.5	6.7
0.06	162	145	153.5	0.391	6.6	6.7
0.07	162	147	154.5	0.388	5.8	6.7
0.08	162	149	155.5	0.386	5.0	6.7
0.09	162	151	156.5	0.383	4.2	6.7
0.10	162	153	157.5	0.381	3.4	6.7

**Table 2: Optimal debt-equity choice**

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 4. The payoff of the firm for type  $\theta$  is given by  $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$ , where both  $X_{\theta T}$  and  $Y_{\theta T}$  are lognormal, with  $\mathbb{E}[X_{\theta T}] = X_{\theta}$ ,  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ . We further denote  $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$ ,  $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$ , and  $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$ . Figure 1 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

	Symbol	Value
<b>Primitives</b>		
Value of assets in place for the good type	$X_G$	125
Value of assets in place for the bad type	$X_B$	75
Value of new assets for the good type	$Y_G$	175
Value of new assets for the bad type	$Y_B$	175
Time to maturity	$T$	10
Volatility of assets in place	$\sigma_x$	0.30
Volatility of new assets	$\sigma_y$	0.60
Probability of the good type	$p$	0.50
Correlation between assets	$\rho$	0
Investment amount	$I$	110
<b>Equilibrium outcomes</b>		
Value of firm post-investment	$p\mathbb{E}[Z_{GT}] + (1-p)\mathbb{E}[Z_{BT}]$	237.9
Value of good-type firm	$\mathbb{E}[Z_{GT}]$	257.6
Value of bad-type firm	$\mathbb{E}[Z_{BT}]$	218.3
Equity fraction issued	$\lambda$	0.462
Face value of debt	$K$	198.3
Credit spread	$r_D = (K/D)^{1/T} - 1$	6.1%
Value of debt, good-type firm	$\mathbb{E}[\min(Z_{GT}, K)]$	120.6
Value of debt, bad-type firm	$\mathbb{E}[\min(Z_{BT}, K)]$	99.4
Dilution costs of equity	$\mathcal{D}_E = \lambda(\mathbb{E}[Z_{GT}] - \mathbb{E}[Z_{BT}])$	18.2
Dilution costs of debt	$\mathcal{D}_D = \mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$	21.1

**Table 3: Optimal debt-equity choice with existing debt**

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5.1. The payoff of the firm for type  $\theta$  is given by  $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$ , where both  $X_{\theta T}$  and  $Y_{\theta T}$  are lognormal, with  $\mathbb{E}[X_{\theta T}] = X_{\theta}$ ,  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ . We further denote  $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$ ,  $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$ , and  $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$ . The firm already has debt outstanding with principal payment of  $K_0$ . The dilution costs of equity are defined as

$$\mathcal{D}_E = \lambda(\mathbb{E}[\max(Z_{GT} - K_0, 0)] - \mathbb{E}[\max(Z_{BT} - K_0, 0)]),$$

where  $\lambda$  satisfies  $I = \lambda(p\mathbb{E}[\max(Z_{GT} - K_0, 0)] + (1-p)\mathbb{E}[\max(Z_{BT} - K_0, 0)])$ . The dilution costs of equity are defined as

$$\mathcal{D}_D = \mathbb{E}[\max(\min(K, Z_{GT} - K_0), 0)] - \mathbb{E}[\max(\min(K, Z_{BT} - K_0), 0)]$$

where  $K$  satisfies  $I = p\mathbb{E}[\max(\min(K, Z_{GT} - K_0), 0)] + (1-p)\mathbb{E}[\max(\min(K, Z_{BT} - K_0), 0)]$ . Figure 4 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

	Symbol	Value
<b>Primitives</b>		
Value of assets in place for the good type	$X_G$	110
Value of assets in place for the bad type	$X_B$	90
Value of new assets for the good type	$Y_G$	120
Value of new assets for the bad type	$Y_B$	120
Time to maturity	$T$	10
Volatility of assets in place	$\sigma_x$	0.25
Volatility of new assets	$\sigma_y$	0.50
Probability of the good type	$p$	0.50
Correlation between assets	$\rho$	0
Investment amount	$I$	50
Existing debt	$K_0$	50
<b>Equilibrium outcomes</b>		
Value of firm post-investment	$p\mathbb{E}[Z_{GT}] + (1-p)\mathbb{E}[Z_{BT}]$	178.9
Value of good-type firm	$\mathbb{E}[Z_{GT}]$	186.7
Value of bad-type firm	$\mathbb{E}[Z_{BT}]$	171.1
Equity fraction issued	$\lambda$	0.379
Face value of debt	$K$	91.3
Credit spread	$r_D = (K/D)^{1/T} - 1$	6.2%
Value of good-type new equity	$\lambda\mathbb{E}[\max(Z_{GT} - K_0, 0)]$	52.8
Value of bad-type new equity	$\lambda\mathbb{E}[\max(Z_{BT} - K_0, 0)]$	47.2
Value of good-type new debt	$\mathbb{E}[\max(\min(K, Z_{GT} - K_0), 0)]$	53.0
Value of bad-type new debt	$\mathbb{E}[\max(\min(K, Z_{BT} - K_0), 0)]$	47.0
Dilution costs of equity	$\mathcal{D}_E$	5.5
Dilution costs of debt	$\mathcal{D}_D$	6.0

**Table 4: Optimal security design problem**

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 5.2. The payoff of the firm for type  $\theta$  is given by  $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$ , where both  $X_{\theta T}$  and  $Y_{\theta T}$  are lognormal, with  $\mathbb{E}[X_{\theta T}] = X_{\theta}$ ,  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ . We further denote  $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$ ,  $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$ , and  $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$ . The labels “Straight debt,” “Convertibles,” and “Warrants” refer to the functions  $s(z) = \min(K, z)$ ,  $s(z) = \min(K, z) + \max(z - \kappa, 0)$ , and  $s(z) = \max(z - \kappa, 0)$  respectively.

	Symbol	Case A	Case B	Case C
<b>Primitives</b>				
Value of assets in place type $G$	$X_G$	150	120	110
Value of assets in place type $B$	$X_B$	150	100	90
Value of new assets type $G$	$Y_G$	120	175	140
Value of new assets type $B$	$Y_B$	100	175	140
Time to maturity	$T$	1	1	1
Volatility of assets in place	$\sigma_x$	0.20	0.25	0.20
Volatility of new assets	$\sigma_y$	0.50	0.30	0.30
Probability of the good type	$p$	0.50	0.50	0.50
Correlation between assets	$\rho$	0	0	0
Investment amount	$I$	100	90	10
Existing debt face value	$K_0$	0	0	100
<b>Equilibrium outcomes</b>				
Value of firm post-investment	$p\mathbb{E}[Z_{GT}] + (1-p)\mathbb{E}[Z_{BT}]$	218.9	186.3	150.5
Optimal security	$s(z)$	Straight debt	Convertibles	Warrants
Face value	$K$	101.9	88.9	–
Conversion trigger/exercise price	$\kappa$	–	309.4	167.9

**Table 5: Financing choice with investment cost at exercise decision**

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5.3. The payoff of the firm for type  $\theta$  is given by  $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T} - I_T)$ , where both  $X_{\theta T}$  and  $Y_{\theta T}$  are lognormal, with  $\mathbb{E}[X_{\theta T}] = X_{\theta}$ ,  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ . We further denote  $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$ ,  $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$ , and  $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$ .

	Symbol	Value		
<b>Primitives</b>				
Value of assets in place for the good type	$X_G$	125		
Value of assets in place for the bad type	$X_B$	75		
Value of new assets for the good type	$Y_G$	200		
Value of new assets for the bad type	$Y_B$	200		
Time to maturity	$T$	10		
Volatility of assets in place	$\sigma_x$	0.30		
Volatility of new assets	$\sigma_y$	0.50		
Probability of the good type	$p$	0.50		
Correlation between assets	$\rho$	0		
Investment amount	$I$	100		
<b>Equilibrium outcomes</b>				
Investment at exercise ( $I_T$ )	Equity offered ( $\lambda$ )	Face value ( $K$ )	Debt dilution $\mathcal{D}_D$	Equity dilution $\mathcal{D}_E$
0	0.397	140.0	12.4	14.0
25	0.415	151.6	15.5	15.6
50	0.430	161.7	17.9	16.9
75	0.445	170.9	19.9	18.0
100	0.458	179.4	21.6	19.0

**Table 6: Robustness, additive cash-flows and the optimal debt-equity choice**

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5.3. The payoff of the firm for type  $\theta$  is given by  $Z_{\theta T} = X_{\theta T} + Y_{\theta T}$ , where both  $X_{\theta T}$  and  $Y_{\theta T}$  are lognormal, with  $\mathbb{E}[X_{\theta T}] = X_{\theta}$ ,  $\mathbb{E}[Y_{\theta T}] = Y_{\theta}$ . We further denote  $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$ ,  $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$ , and  $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$ .

	Symbol	Value
<b>Primitives</b>		
Value of assets in place for the good type	$X_G$	125
Value of assets in place for the bad type	$X_B$	75
Value of new assets for the good type	$Y_G$	175
Value of new assets for the bad type	$Y_B$	175
Time to maturity	$T$	15
Volatility of assets in place	$\sigma_x$	0.30
Volatility of new assets	$\sigma_y$	0.60
Probability of the good type	$p$	0.50
Correlation between assets	$\rho$	0
Investment amount	$I$	110
<b>Equilibrium outcomes</b>		
Value of firm post-investment	$\mathbb{E}[Z_T]$	274.5
Equity fraction issued	$\lambda$	0.40
Face value of debt	$K$	213.2
Credit spread	$r_D = (K/D)^{1/T} - 1$	4.5%
Dilution costs of equity	$\mathcal{D}_E = \lambda(\mathbb{E}[Z_{GT}] - \mathbb{E}[Z_{BT}])$	20.1
Dilution costs of debt	$\mathcal{D}_D = \mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$	21.2

**Table 7: Comparative statics via regression**

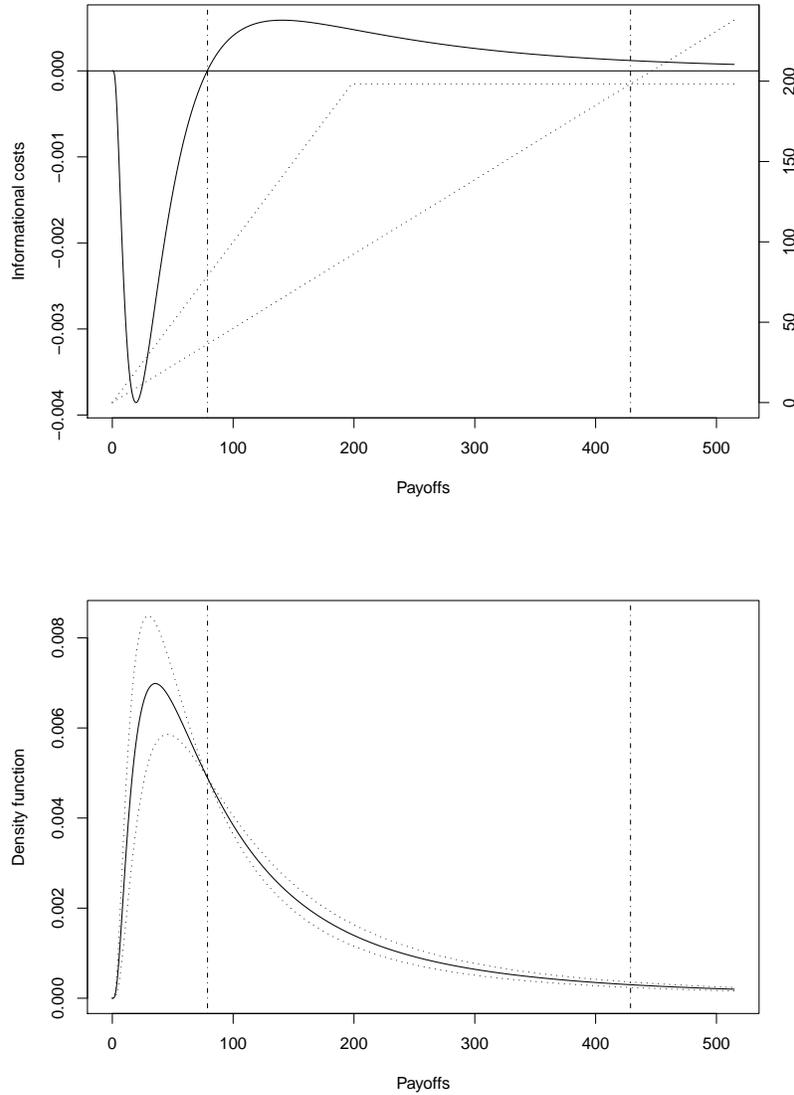
The table presents estimates of: (a) a logit regression model where the dependent variable is a dummy that equals to one if the firm optimal chooses debt, zero if the firm prefers equity, as a function of a set of explanatory variables from the model (Panel A); (b) a classical regression model of the form  $\mathcal{R}_i = \beta^\top X_i + \epsilon_i$  in Panel B, where  $\mathcal{R}_i$  denotes the relative dilution of debt versus equity,  $\mathcal{R}_i = \mathcal{D}_{Ei}/\mathcal{D}_{Di}$ , and  $X_i$  denotes a the set of explanatory variables (Panel B). The set of explanatory variables include: measures of asymmetric information on the assets in place and the new assets ( $c_x$  and  $c_y$ ), the two parameters on volatility ( $\sigma_x$  and  $\sigma_y$ ), the level of the cash flows ( $\mu_x = \log(\bar{X})$  and  $\mu_y = \log(\bar{Y})$ ), the probability of the good type ( $p$ ), the amount of existing debt ( $K_0$ ), as well as the investment amount ( $I$ ). Details on the construction of the simulated dataset are given in the Appendix.

**A. Logit regressions (success if straight debt issued)**

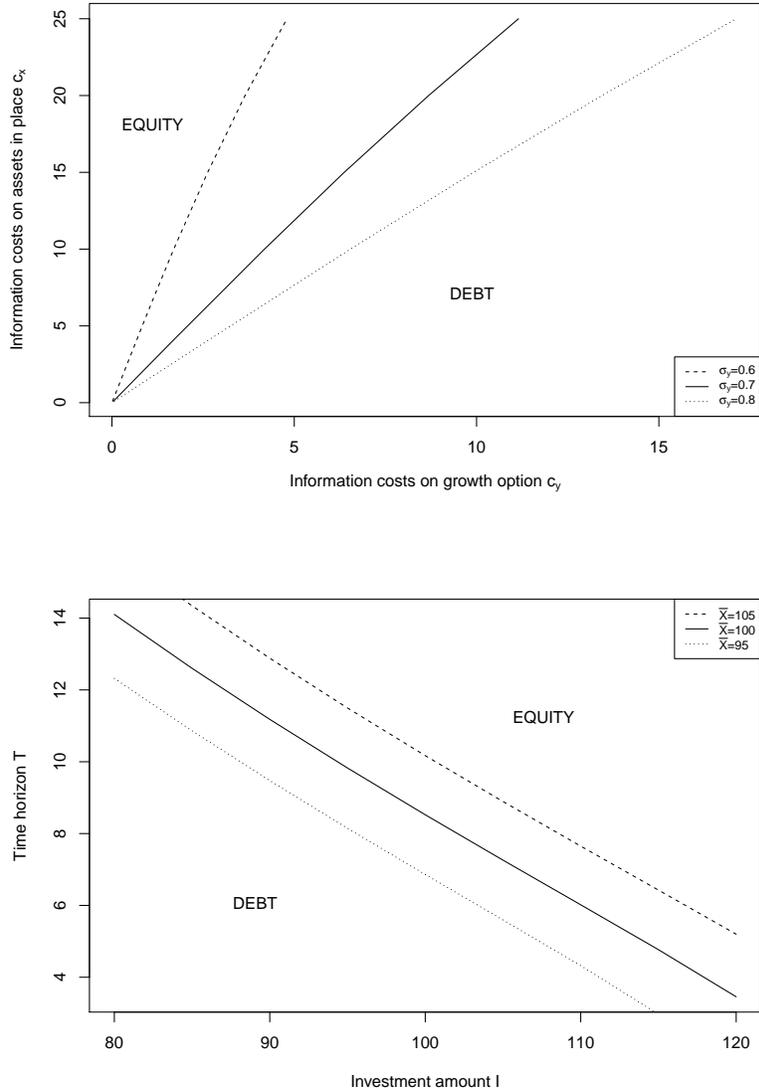
	Base case	Existing debt		Info. asy. $X$		Info. asy. $Y$	
		Low $K_0$	High $K_0$	Low $c_x$	High $c_x$	Low $c_y$	High $c_y$
Asy. info on existing assets $c_x$	-1.8	-2.0	-1.8	-6.3	-1.5	-1.1	-3.0
Asy. info on new assets $c_y$	3.2	3.7	2.9	8.7	3.4	5.0	3.5
Size existing assets $\mu_x$	1.5	2.1	1.0	1.4	2.4	1.6	2.1
Size new assets $\mu_y$	-1.1	-1.1	-1.2	-1.1	-1.6	-0.7	-1.5
Volatility existing assets $\sigma_x$	2.5	2.5	2.7	2.5	4.5	3.1	2.9
Volatility new assets $\sigma_y$	-1.9	-2.0	-2.0	-2.0	-3.2	-2.5	-2.1
Correlation $\rho$	-0.2	-0.4	-0.0	-0.1	-0.6	-0.3	-0.1
Prob. high type $p$	-0.2	-0.2	-0.1	-0.0	-0.5	-0.2	-0.2
Debt's principal $K_0$	-0.3	-0.7	-0.0	-0.2	-0.7	-0.4	-0.3
Investment $I$	-0.2	-0.5	-0.1	0.0	-0.8	-0.6	0.0
Time to maturity $T$	-0.5	-0.6	-0.5	-0.6	-0.6	-0.5	-0.6

**B. Relative dilution regressions (dilution equity/dilution debt)**

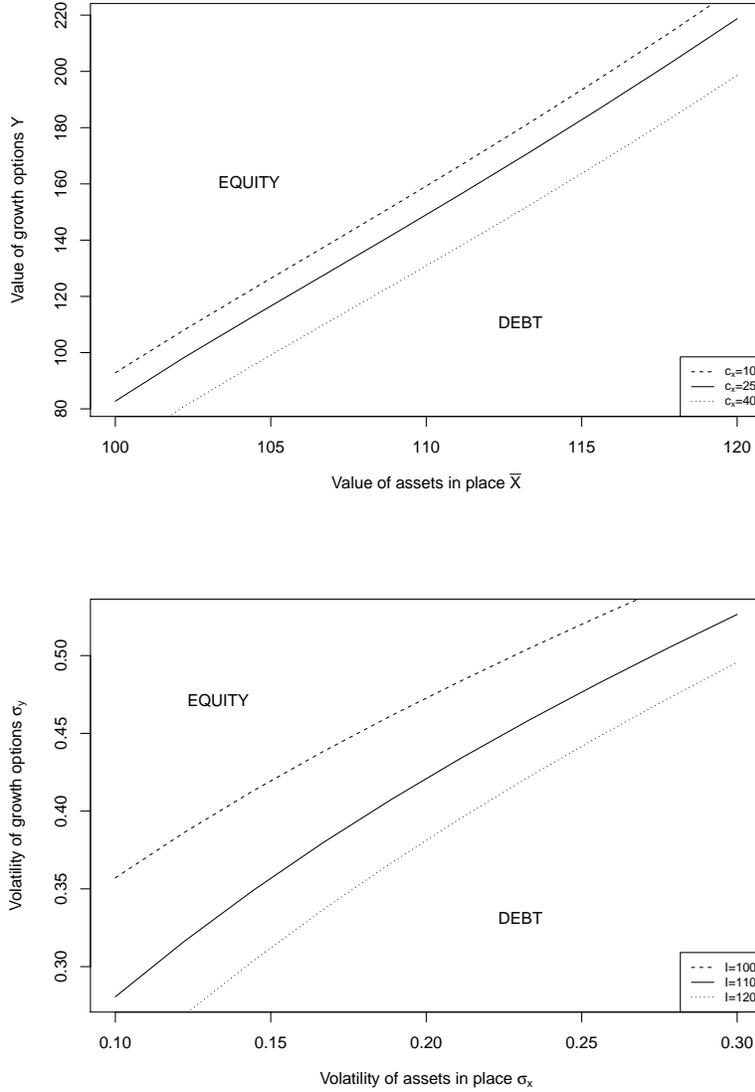
	Base case	Existing debt		Info. asy. $X$		Info. asy. $Y$	
		Low $K_0$	High $K_0$	Low $c_x$	High $c_x$	Low $c_y$	High $c_y$
Asy. info on existing assets $c_x$	-4.8	-5.3	-4.4	-7.3	-3.1	-4.0	-5.0
Asy. info on new assets $c_y$	7.4	8.9	6.4	7.5	6.8	18.9	3.9
Size existing assets $\mu_x$	4.3	5.9	3.1	1.9	5.6	5.5	3.4
Size new assets $\mu_y$	-2.4	-2.6	-2.4	-1.5	-3.2	-2.3	-1.8
Volatility existing assets $\sigma_x$	6.7	6.7	6.9	3.4	9.9	10.6	4.2
Volatility new assets $\sigma_y$	-5.0	-5.6	-4.7	-2.7	-7.1	-8.5	-3.1
Correlation $\rho$	-0.5	-1.1	-0.2	-0.1	-1.3	-1.1	-0.0
Prob. high type $p$	-0.5	-0.6	-0.3	-0.1	-1.2	-0.7	-0.4
Debt's principal $K_0$	-1.0	-2.3	-0.3	-0.4	-1.7	-1.3	-0.8
Investment $I$	-1.8	-2.6	-1.6	-0.9	-2.8	-2.2	-1.2
Time to maturity $T$	-0.9	-1.4	-0.5	-0.6	-0.9	-1.6	-0.5



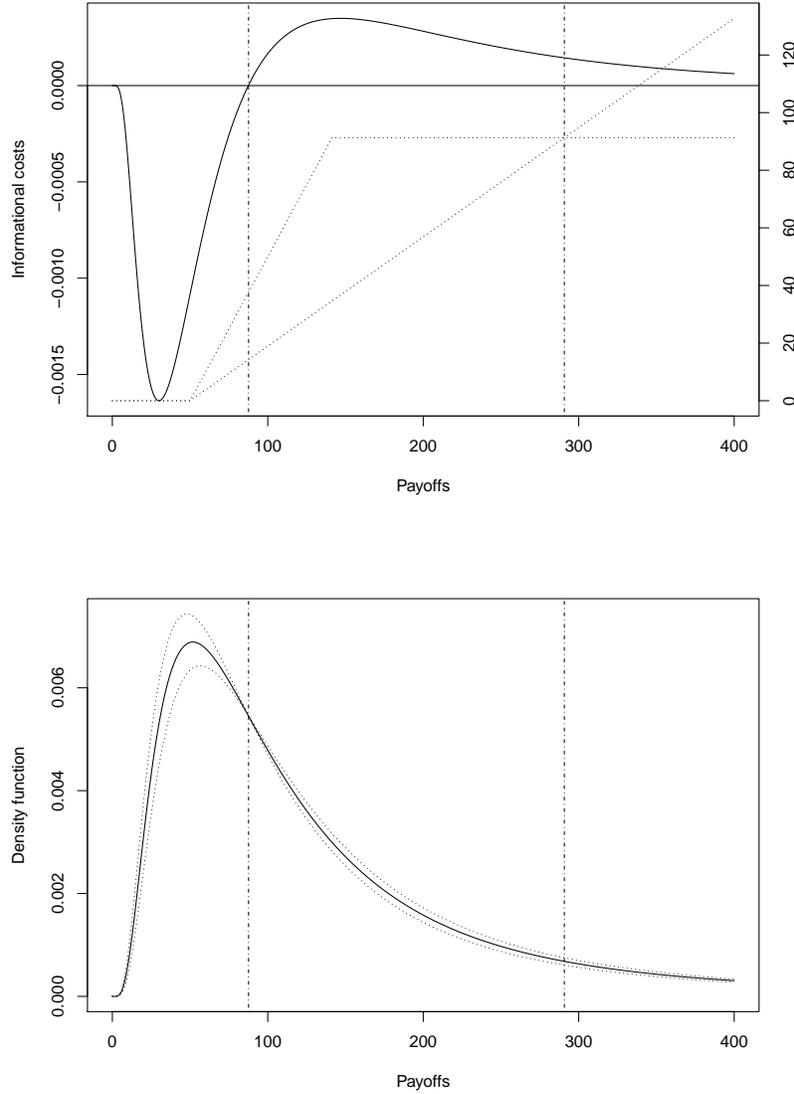
**Figure 1:** The top graph plots on the  $x$ -axis the payoffs from the firm at maturity, and in the  $y$ -axis it plots as a solid line the difference in the densities of the good and bad type firms,  $f_G(z) - f_B(z)$  ( $y$ -axis labels on the left), and as dotted lines the payoffs from debt and equity ( $y$ -axis labels on the right). The left-most vertical dashed line is the point  $\hat{z}$  for which  $f_G(\hat{z}) = f_B(\hat{z})$ , so points to the right of that line have positive information costs. The right-most vertical dashed line is the point  $\bar{z}$  for which  $K = \lambda\bar{z}$ , so for payoffs to the right of that line equityholders receive more than debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 2.



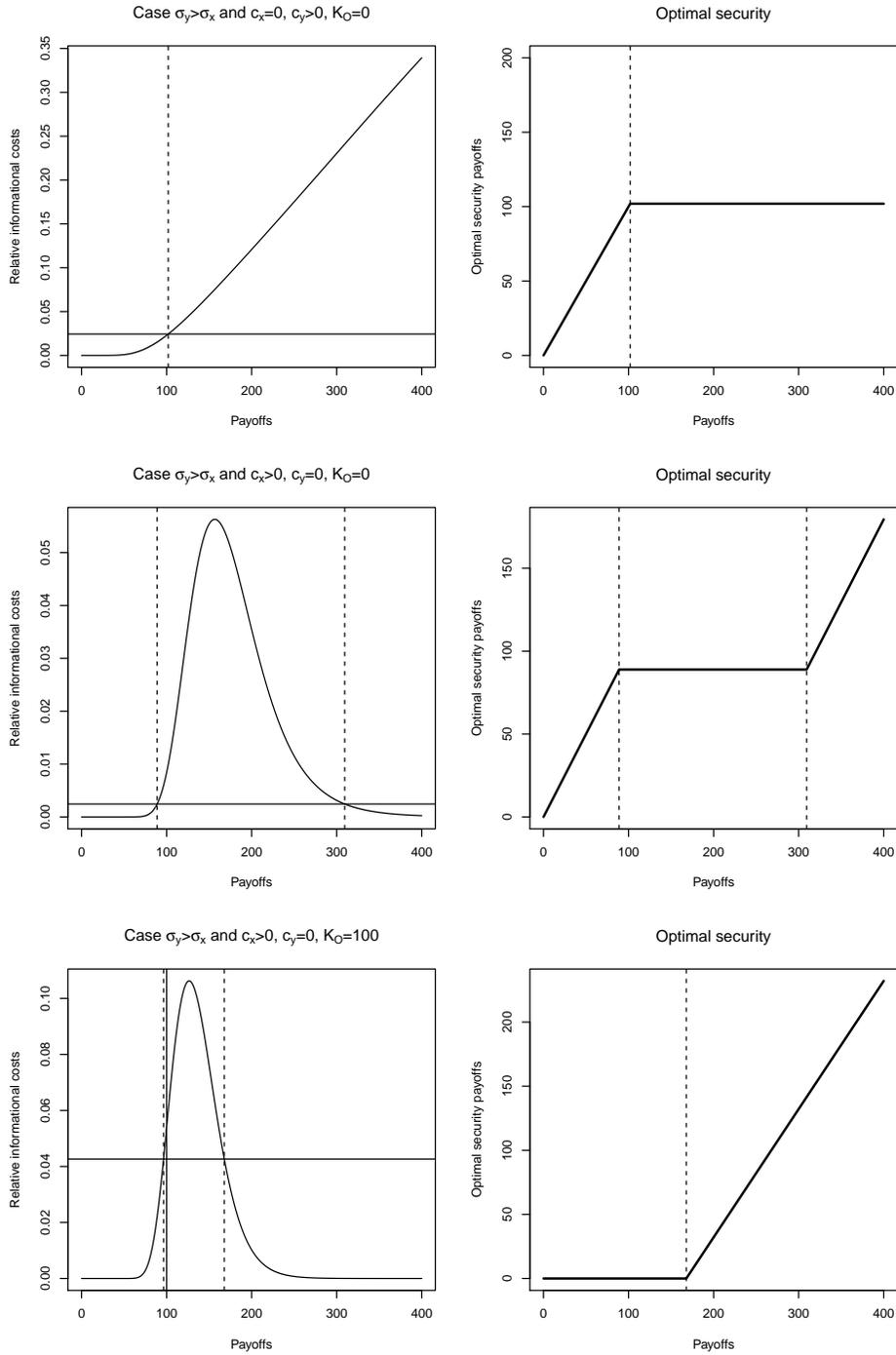
**Figure 2:** The top graph plots the set of points  $(c_x, c_y)$  for which the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ . We consider the following parameter values:  $\bar{X} = 100$ ,  $\bar{Y} = 175$ ,  $\sigma_x = 0.3$ ,  $\sigma_y = 0.6$ ,  $I = 110$ ,  $T = 10$ ,  $\rho = 0$  and  $p = 0.5$ . Recall we set  $X_G = \bar{X} + c_x$  and  $X_B = \bar{X} - c_x$ , and similarly for  $Y_G$  and  $Y_B$ . The dashed line corresponds to the base case parameters from Table 2, namely sets  $\sigma_y = 0.6$ , whereas the other two lines correspond to  $\sigma_y = 0.7$  and  $\sigma_y = 0.8$ . For pairs of  $(c_x, c_y)$  below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes  $c_x = 25$  and  $c_y = 0$ , as in Table 2, and plots the set of points  $(I, T)$  for which the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ . The solid line corresponds to the base case of Table 2, where  $\bar{X} = 100$ , whereas the dashed and dotted lines consider the cases  $\bar{X} = 105$  and  $\bar{X} = 95$ . For pairs of  $(I, T)$  below the lines debt is optimal, whereas equity is optimal above the lines.



**Figure 3:** The top graph plots the set of points  $(\bar{X}, \bar{Y})$  for which the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ . We consider the following parameter values:  $c_x = 25$ ,  $c_y = 0$ ,  $\sigma_x = 0.3$ ,  $\sigma_y = 0.6$ ,  $I = 110$ ,  $T = 10$ ,  $\rho = 0$  and  $p = 0.5$ . Recall we set  $X_G = \bar{X} + c_x$  and  $X_B = \bar{X} - c_x$ , and similarly for  $Y_G$  and  $Y_B$ . The solid line corresponds to the base case parameters from Table 2, namely sets  $c_x = 25$ , whereas the other two lines correspond to  $c_x = 10$  and  $c_x = 40$ . For pairs of  $(\bar{X}, \bar{Y})$  below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes  $\bar{X} = 100$  and  $\bar{Y} = 175$ , as in Table 2, and plots the set of points  $(\sigma_x, \sigma_y)$  for which the dilution costs of equity and debt are the same, i.e.  $\mathcal{D}_E = \mathcal{D}_D$ . The solid line corresponds to the base case of 2, where  $I = 110$ , whereas the dashed and dotted lines consider the cases  $I = 100$  and  $I = 120$ . For pairs of  $(\sigma_x, \sigma_y)$  below the lines debt is optimal, whereas equity is optimal above the lines.



**Figure 4:** The top graph plots on the  $x$ -axis the payoffs from the firm at maturity, and in the  $y$ -axis it plots as a solid line the difference in the densities of the good and bad type firms,  $f_G(z) - f_B(z)$  ( $y$ -axis labels on the left), and as dotted lines the payoffs from debt and equity ( $y$ -axis labels on the right). The left-most vertical dashed line is the point  $\hat{z}$  for which  $f_G(\hat{z}) = f_B(\hat{z})$ , so points to the right of that line have positive information costs. The right-most vertical dashed line is the point  $\bar{z}$  for which  $K = \lambda(\bar{z} - K_0)$ , so for payoffs to the right of that line equityholders receive more than the new debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 3.



**Figure 5:** The left panels plot the function  $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$ , whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 4.