Bubbles and Long-term Investors*

Nadja Guenster†

Maastricht University, The Netherlands and Haas School of Business, University of California, Berkeley

Erik Kole

Econometric Institute, Erasmus School of Economics, Erasmus University Rotterdam, The Netherlands

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†Corresponding author: Tongersestraat 53, 6200 MD Maastricht, The Netherlands, Tel. +31 43 388 4947.

Abstract

In theory, long-term investors are expected to short overvalued assets and trade against bubbles. We empirically investigate the optimal strategy of a utility-maximizing long-term investor, who learns of a bubble. We identify bubbles using a Markov-regime switching model and proxy the growth rate of fundamental value using conventional asset pricing models. Applying our method to US industry returns, we find that buy-and-hold investors with horizons longer than 6 months should short the asset bubble. However, long-term investors who can rebalance their portfolios within 4 months or less should actually ride the bubble. For these investors, a longer horizon is even associated with an increasingly large investment in the bubble. Our findings differ from the theoretical predictions because we empirically find that bubbles deflate over several months, while they burst in a single month in most theoretical models.

Keywords: asset price bubbles, long-term investors, regime switching
JEL codes: G14,G11,C13

1 Introduction

Bubbles pose a serious risk to investors' wealth if they crash, but might also offer profitable trading opportunities. From a theoretical perspective, the optimal strategy of a long-term investor who learns of a bubble is not obvious. The efficient market hypothesis predicts that investors short overpriced assets, independent of their horizon. However, in the limits-to-arbitrage literature (see, for example, De Long et al. (1990a) and Shleifer and Vishny (1997)), short horizons induce investors to refrain from trading against the bubble. If noise traders cause prices to deviate from fundamental value for prolonged periods, investors with short horizons might be forced to unwind their positions before the asset's price returns to fundamental value. Therefore, it is only optimal for long-term investors to trade against the mispricing. Indeed, an infinitely lived agent would take sufficiently extreme short positions to burst the bubble. De Long et al. (1990b) and Abreu and Brunnermeier (2003) suggest that investors should ride the bubble at short horizons and sell out as the risk of the crash increases. Empirically, Brunnermeier and Nagel (2004) show that hedge funds were profitably riding the internet bubble, as they managed to time the crash and sold out quickly.

In this paper, we investigate empirically the optimal strategy of a long-term utility-maximizing investor, who learns of a bubble. We investigate the investor's optimal asset allocation decision for horizons of up to five years and varying rebalancing frequencies. Arguably institutional investors with a long horizon likes pension funds, banks and insurances might not be able to rebalance their portfolios as quickly as the hedge funds in Brunnermeier and Nagel (2004). In addition, they might also not have sufficient timing skills to anticipate the crash.

We identify bubbles using a Markov-regime switching model as proposed by Hamilton (1989, 1990). This methodology allows us to replicate the uncertainty a real-world investor faces, it allows for sudden switches (in particular, from a bubble to a deflation regime), and we can derive forecasts of the risk and return distributions over time. For our analysis we use the 48 US industry portfolios from 1964 to 2009, as in Fama and French (1997). Famous (perceived) bubbles often started in a specific industry.[1] Examples are the recent

[1]For historical periods perceived to be a "bubble", there is usually no perfect agreement on whether

housing bubble (1998–2009), the internet bubble (1995–2000), the tronics boom (1959–1962) or the railway mania (1840s).

Before the start of a bubble, one often observes an initial rise in prices that can be attributed to fundamentally good news, for example the development of a new technology or another significant innovation (Abreu and Brunnermeier (2003) or Kindleberger (2000)). After some time however, investors become overenthusiastic or 'irrationally exuberant'. They start extrapolating the higher growth rate too far into the future. A bubble starts developing. To accommodate this pattern, we model a sequence of good news regimes, before we allow for the possible identification of a bubble regime. Although bubbles generally burst in theoretical models, in reality bubbles deflate over several months (Brunnermeier (2008)). We model the deflation of a bubble as a combination of a deflation-crash regime which is associated with sharp price declines, and a deflation-normal regime, that captures more tranquil periods. The deflation of bubble is always initiated by the deflation-crash regime, but thereafter both deflation regimes can alternate. Besides the good news regime, the bubble regime, and the two deflation regimes, our model also contains a normal regime and a crash regime. We additionally include the crash regime to allow for crashes that are not associated with bubbles, but, for example, simply due to bad news.

The investor infers the different regimes from past abnormal returns. To compute the abnormal returns, we use three asset pricing models: the CAPM, the 3-factor model of Fama and French (1993) and the 4-factor model of Carhart (1997). The good news regime and the subsequent bubble regime are characterized by large positive mean abnormal returns. The abnormal returns range from 3.05% per month for the 3-factor model to 3.41% per month for the CAPM. Both regimes are rare. The ergodic probabilities show that one can only expect the good news regime with a probability of about 2.5%. The probability to be in the bubble regime is 2.67% for the CAPM-based results and 2.31% for the other two models. The normal regime is by far the most common one with an ergodic probablity of close to 90%. Mean abnormal returns in this regime are close to zero. The crash regime is characterized by large negative abnormal returns, which are around -10% per month.

The predicted risk-and return distributions following the bubble regime provide first

the period was truly an overvaluation or whether the high prices could be justified by fundamentals. See for example Malkiel (2010) for a discussion on historical bubble periods.

insights into the investment implications of bubbles. Following the bubble regime, predicted abnormal returns are very high at first, but they decrease quickly and become very negative. For example, for the Fama-French Model, in the first month following the bubble regime, the abnormal return is 57 basis points, and in the second month, it is still 10 basis points. However, already in the third month, the abnormal return becomes negative and it is close to -30 basis points. After about ten months, it reaches a low of -1.2%. The forecast probabilities show that the sharp decline in returns can be attributed to a substantial increase in the probability of the deflation regimes, in particular the deflation-crash regime. Consequently, we then also find that volatility following the bubble regime is at first very high. Over time volatility declines as the normal regime becomes more again more likely. The findings are qualitatively similar for the three different asset pricing models.

We analyze the asset allocation decision of a power-utility investor who can choose between the market and the typical industry. The optimal weights mentioned are for an investor, who uses the Fama-French model to compute abnormal returns, but our findings are similar across models. At first, we compare the optimal weight allocated by a buy-and hold investor who is with certainty in the normal regime to a buy-and hold investor who is with certainty in the bubble regime when he makes his allocation decision. An one-month buy-and-hold investor who observes the normal regime allocates about 8% of his wealth to the typical industry. His optimal weight increases as his horizon becomes longer, and reaches a maximum of 14.5% for a two-year horizon. Thereafter, the optimal weight decreases slightly to 11.6% for a five-year buy-and-hold strategy. Thus, coming from the normal regime, we do not find any strong evidence of horizon effects for buy-and hold investors. However, if the investor's believes to be in the bubble regime with certainty when he makes his investment decision, his allocation strongly depends on his horizon. At very short horizons, the investor allocates a substantial fraction of his portfolio to the industry bubble. For example, for a one-month horizon, the optimal weight is 21.2%. The optimal weight declines as the investors horizon increases. It is 13.7% for a two-months, buy-and-hold investor and only 2.8% for a four-months, buy-and-hold investor. A buy-and-hold investor with a horizon longer than six months would take a short position. His optimal weight is -3.8% and becomes more negative as his horizon becomes longer. For a five-year horizon, his optimal weight declines to -14.6%. These findings suggest that long-term investors indeed optimally take a short position in bubbles.

However, this conclusion only holds for investors who are not able to rebalance at reasonable frequencies. If we investigate the optimal strategy of investors who can rebalance and learn, as in Guidolin and Timmermann (2007), we reach very different conclusions. An investor who believes with certainty that he is in the bubble regime, has a one-month horizon and can rebalance monthly, would allocate 21.2% to the typical industry (just the same as the one-month buy-and-hold investor). If his horizon increases to one year, his optimal weight rises to 25.5%. An investor who can only rebalance every four months still has an optimal weight of 4% if his horizon is four months as well. Again, as his horizon increases to one year, his optimal weight increases to 6.5%. For very long horizons of five years, we find that the optimal weights decline again, but this decline is small.

We conclude that for investors who can rebalance at reasonable frequencies, longer horizons might even induce them to ride bubbles more aggressively. Our findings suggest that riding bubbles is not only optimal for short-term speculators, like hedge funds. Even for investors who are not as sophisticated at hedge funds and have no timing ability, investing in the bubble is profitable and optimal. Because bubbles empirically deflate slowly, investors have time to liquidate their positions before they would lose all the gains from participating in the rise of the bubble.

2 A Regime-Switching Model for Bubbles

2.1 Model Design

Our analysis is based on a regime switching model since it allows us to separately describe the price process in case a bubble continues to inflate, in case a a bubble deflates by a series of crashes, and the base case in which no bubble is present. Evans (1991), van Norden and Schaller (1999) and Brooks and Katsaris (2005) are examples of the use of regime switching models to study asset price bubbles.

An advantage of using a regime switching model is the ease with which the actual presence of the bubble can remain latent. As in reality, the investor does not know for sure whether a bubble is present but has to make a probabilistic inference. In determining his optimal allocation he has to take into account that his inference may be wrong. This approach allows us to describe a more realistic setting than most theoretical models, where at least a fraction of investors knows with certainty that the price contains a bubble component (see among others Abreu and Brunnermeier (2003), De Long et al. (1990b), or De Long et al. (1990a)).

We let the latent process for the presence of a bubble be governed by a first order Markov chain. With a certain probability, the process can switch from one state to another and eventually to the bubble state. This switch can correspond with a displacement in a Minsky model (see Kindleberger, 2000) or "new economy thinking" as in Shiller (2000). Once the process switched to the bubble state, it can remain there for the following periods or leave it with a crash. We deviate from van Norden and Schaller (1999) and Brooks and Katsaris (2005), who do not use a Markov chain. In their studies, the latent process of a bubble evolves much more gradually and cannot accommodate the sudden switches that are considered typical characteristics of bubbles (see for example Figures 2 and 3 in Brooks and Katsaris, 2005).

Besides by a sudden change, bubbles are characterized by a price that grows faster than fundamental value. While such exuberant growth is present in all bubble models, we tie it directly to an asset pricing model like the CAPM or a multi-factor model. We do not assume that the fundamental growth rate is simply given, as is typical for the theoretical rational bubble literature (see, for example, Blanchard and Watson, 1982), nor do we tie it to dividends as many articles on testing for bubbles propose (see Flood and Hodrick, 1990, for an overview).

In our setting, structural growth beyond what can be explained from covariance with systematic risk factors (or, equivalently, the pricing kernel) is considered a bubble. We do not require that bubble growth is constant over time. Instead, we allow a stochastic growth rate which is strictly positive in expectation as in Brooks and Katsaris (2005).

Mathematically, the asset return $r_{i,t}$ obeys

$$r_{i,t} = r_{\mathrm{f},t} + \boldsymbol{\beta}_i' \boldsymbol{f}_t + \omega_i u_{i,t}(S_{i,t}), \tag{1}$$

where $r_{\mathrm{f},t}$ is the risk-free rate, $\boldsymbol{f}_t$ denotes the vector of realizations of the (traded) risk factors, $\boldsymbol{\beta}_i$ the vector of sensitivities to the risk factor, ω_i an asset-specific scale factor that influences the idiosyncratic volatility, and $u_{i,t}(S_{i,t})$ an innovation, independent from $\boldsymbol{f}_t$, and depending on a latent state variable $S_{i,t}$. The first two terms capture the systematic part of the asset return. The last term captures the idiosyncratic part of the asset return, which may contain a bubble depending on the realization of $S_{i,t}$.

The latent process $S_{i,t}$ can be in one out of fixed set of regimes. In the normal regime N, no bubble is present, and the asset price grows at the fundamental growth rate. Under the normal regime, the innovation $u_{i,t}$ follows a normal distribution. Its expectation, denoted by μ_{N}, will be close to zero. To ensure identification, we impose that the variance is equal to 1. This restriction gives $u_{i,t}$ in regime N the interpretation of a standardized abnormal return.

The next regime we consider is the good news regime. When good news arrives, the asset moves to the good news regime. It takes L periods before good news is incorporated into prices. To capture this aspect in our model, we introduce L copies of the good news regime, $G_l, l = 1, 2, \ldots, L$. If the process is in good news regime G_l at time t, it moves to G_{l+1} with certainty at time $t+1$. In each of these regimes, the innovation follows a normal distribution whose mean is higher than in the normal regime, and whose variance is equal to 1. A switch to the good news regime thus means that the distribution of $u_{i,t}$ is shifted to the right.

After L periods of good news regimes, the good news has been incorporated into prices. Because investors extrapolate price growth, a bubble occurs. The regime process then moves to the bubble regime B. Because this regime is an extrapolation of the good news regimes, it has the same mean and variance as these regimes. Because the bubble need not continue in the next period, the process can either stay in the bubble regime for another period or exit by switching to the crash state, which marks the beginning of the deflation.

The crash state marks the beginning of the deflation of the bubble. The deflation can take place in two forms. It always starts with a crash, but is also possible that the asset shows relatively normal behavior. Therefore we introduce two regimes: a deflating crash regime D_{C} and a deflating normal regime D_{N}. When the deflating crash regime takes place, the innovations should be substantially negative. To ensure that they fall below a specific value k, the innovations follow a linear transformation of a lognormal distribution, $Y = k - \exp(Z)$, where Z follows a normal distribution. This approach is commonly used in articles that model negative jumps in asset returns, such as Das and Uppal (2004). For a discussion on the relation between bubbles and crashes, see McQueen and Thorley (1994).

In the deflating normal regime, innovations show the same behavior as in the normal regime. The difference between the normal regime are the transitions. When in the deflation normal regime, the process can either stay in it, or switch back to deflation crash regime. When the deflation is completed the process switches from the deflation crash to the normal regime.

Because not all crashes have to be attributed to bubbles, we include a second crash regime C. This regime has the same distributional parameters as the deflating crash regime, but the process can enter this regime from the normal regime, and exit it to the normal and good news regime.

We summarize the distribution for $u_{i,t}(S_{i,t})$:

$$u_{i,t} \sim \begin{cases} N(\mu_{\mathrm{N}}, 1) & \text{if } S_{i,t} = \mathrm{N}, \mathrm{D_N} \\ N(\mu_{\mathrm{B}}, 1) & \text{if } S_{i,t} = \mathrm{B}, \mathrm{G}_l, l = 1, 2, \ldots, L \\ k - \mathrm{e}^Z, \quad Z \sim N(\mu_{\mathrm{C}}, \sigma_{\mathrm{C}}^2) & \text{if } S_{i,t} = \mathrm{D_C}, \mathrm{C} \end{cases} \tag{2}$$

While the distributions of $u_{i,t}$ conditional on $S_{i,t}$ have well-defined characteristics, the distribution of $u_{i,t}$ unconditional on a specific regime will show time-varying volatility, and exhibit skewness and excess kurtosis (cf. Timmermann, 2000).

We summarize the transitions in a graph in Figure 1, and the corresponding transition matrix in Table 1. Our model contains $5 + L$ states, but we impose a specific structure on the transitions to ensure identification. As we expect the occurrence of good news to be unrelated to whether the preceding regime was a normal or crash regime, we impose $p_{\mathrm{NG}} = p_{\mathrm{CG}}$. The total number of transition probabilities to estimate is seven.

[Figure 1 about here.]

[Table 1 about here.]

By including these restrictions, we intend to put enough structure on our model to ensure that we indeed detect bubbles, and to preserve enough flexibility to infer from return series how bubbles actually occur. The restrictions prevent that a single-period large return is identified as a bubble. By explicitly imposing that bubbles are ended by one or more crashes, a prolonged adjustment of fundamental value due to a market under-reaction is not likely to be identified as a bubble. The persistence of bubbles and their average growth rate are free parameters to be estimated.

2.2 Estimation and Inference

The investor does not know in which regime the process is at any point in time. Instead, he has to infer the current regime and form an expectation on future regimes and their risk-return trade-off. His information set Ψ_t at time t contains the time-series of returns and risk factors from t_0, the beginning of the sample period, to t. He applies a filtering procedure to infer with which probabilities the different states currently prevail. This procedure uses the following recursive relation to construct a times series of vectors of forecast probabilities $\boldsymbol{\xi}_{\tau|\tau-1}$ and inference probabilities $\boldsymbol{\xi}_{\tau|\tau}$ for each state s (see Hamilton, 1994, Ch. 22), where τ ranges from t_0 to t:

$$\boldsymbol{\xi}_{\tau|\tau-1} = \boldsymbol{P}\boldsymbol{\xi}_{\tau-1|\tau-1} \tag{3}$$

$$\boldsymbol{\xi}_{\tau|\tau} = \frac{1}{\boldsymbol{\xi}'_{\tau|\tau-1}\boldsymbol{g}(u_\tau)}\boldsymbol{\xi}_{\tau|\tau-1} \odot \boldsymbol{g}(u_\tau), \tag{4}$$

where $\boldsymbol{g}()$ is the vector of the probability density functions of the different states, $\boldsymbol{P}$ is the transition matrix, and $\odot$ denotes the Hadamard product. The procedure starts with inference probabilities for $t_0 - 1$. The forecast probabilities give a forecast of the state process for period τ, conditional on information up to period $\tau - 1$. When the information (i.e. the returns) of period τ becomes known, a Bayesian update is applied to arrive at the inference probabilities. We estimate the distribution parameters, transition probabilities and initial regimes probabilities at $t_0 - 1$ by recursively applying the Expectation Maximization (EM) algorithm of Dempster et al. (1977) which yields maximum likelihood estimates (see also Hamilton, 1993). Since no history before t_0 is available we restrict the initial probabilities for the good news and bubble regime to be equal. We also put this equality restriction on the initial probabilities for the crash and deflation crash regime.

2.3 Estimation results

We estimate the model for monthly returns of the 48 industries as used by Fama and French (1997), which are available on French's website[2]. Our dataset starts in July 1963 when the CRSP database is extended by stocks traded on the AMEX. It ends in December 2010. We consider the CAPM and the models by Fama and French (1993) and Carhart (1997) as asset pricing models. We put the sequence of good news equal to $L = 6$ months, and set the threshold for crashes at $k = -1$. We report the industry-specific results in Tables 2 to 4 and the regime-switching parameters in Table 5.

[Table 2 about here.]

The industry-specific estimation results for the CAPM in Table 2 shows market-βs that are centered around one, ranging from 0.55 to 1.39. Their average equals 1.05. The estimates for the scale factor ω indicate considerable cross-sectional variation in volatility, as they range from 2.54% to 9.74%. The average equals 4.06%.

The regime-switching part of Equation (1) relates to the idiosyncratic part of the returns. To have a view on the properties that the regime switching part should capture, we construct the abnormal returns as $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$. The averages of the abnormal returns which vary from -0.34% to 0.67% per month indicate that some industries have large pricing errors, but the pooled average pricing error of 0.12% per month is close to zero.[3] So averaged over time and industries, the effect of bubbles should be limited. The standard deviations of the abnormal returns exceed the corresponding scale factors, because the regime switching model for the innovations contributes to the idiosyncratic volatility. Moreover, it makes that volatilities time-varying. The coefficients of skewness and kurtosis

[2]The data can be downloaded from the Kenneth French Data Library at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. We have used the set of industry returns constructed with the new specifications.

[3]We cannot conduct a traditional GRS tests, because our model does not assume that residuals are i.i.d and normally distributed. Neith can we use the J-test advocated by Cochrane (2005). This test tests whether the moment conditions of zero pricing errors overidentify the model. Our model is estimated by maximum likelihood, and its first order conditions do not include conditions on the pricing errors per industry.

varies from -0.41 to 1.41 and from 3.32 to 16, with averages equal to 0.23 and 5.86. The positive skewness may point at positive returns due to bubbles. The excess kurtosis indicates more extreme returns than a single regime with a normal distribution would imply. As shown by Timmermann (2000), regime switching models can flexibly accommodate non-zero skewness and excess kurtosis.

Because the CAPM may fail to explain the cross-section of industry returns, we next consider the three-factor model of Fama and French (1993). The vector of risk factors contains the excess market return, the SMB-factor for size and the HML-factor for value. Table 3 shows that many industries have sizeable exposures to these factors. For the size factor, they range from -0.32 to 0.90, and for value from -0.54 to 0.73. The averages of 0.20 and 0.17 indicate that our set of industries is slightly tilted towards small value industries. Despite the inclusion of additional factors the scale factors in the residuals decrease only slightly to on average 3.87% (range from 2.38% to 9.68%).

[Table 3 about here.]

The additional factors leads to a decrease of the residuals. Their averages range from -0.83 to 0.63 with an average of 0. Also for the Fama-French model, some industries have large pricing errors, but the overall error is small. Skewness and excess kurtosis remain defining features of the abnormal returns. They do not seem to change much compared to the CAPM results, so the regime switching models are likely to have similar properties.

Because the sequence of good news and bubbles in our model bares some resemblance to momentum, we also consider the model of Carhart (1997), in which a momentum factor is added to the three risk factors of the Fama-Frech model. The exposures to this factor in Table 4 are relatively small (between -0.36 and 0.21) and on average negative (-0.05). Compared to the Fama-French model, the residuals show only slight changes.

[Table 4 about here.]

Table 5 shows the estimation results of the regime switching model. Panel A shows the distribution parameters of the different regimes. In Panel B, these parameters are transformed into means and volatilities. The transition probabilities are reported in Panel

C. The expectation of the standardized abnormal returns in the normal regime are very close to zero. They range from 0.013 for the Fama-French model to 0.036 for the CAPM. Using the average value of the scale factors to transfer these estimates to expected abnormal returns in Panel B, we find expected values between 0.05% (Fama-French model) and 0.15% per month (CAPM). We conclude that during the normal regime, innovations do not systematically deviate from the fundamental part of the asset pricing model.

[Table 5 about here.]

When the good news or bubble regimes prevail, the expectations of the standardized returns exceed those for the normal regime by far with values between 0.789 (Fama-French model) and 0.841 (CAPM). These estimates imply that the abnormal returns are on average between 3.05% and 3.41% per month. The standard errors indicate that the expected values are estimated quite precise. So, during sequences of good news and bubbles, we can expect to see sizeable increases in price.

The flip side of bubbles in our model are crashes. The location parameter μ_{C} and scale parameter σ_{C} of the lognormal distribution that prevails during crashes vary between 0.278 (CAPM) and 0.281 (Carhart model), and 0.638 (Fama-French model) and 0.642 (Carhart model). Based on these numbers, the expected abnormal return during crashes ranges from -10.04% (CAPM to -10.64% (Carhart) model. The volatility is also higher during crash than during the other regimes with values between 4.44% (Carhart model) and 4.68%.[4] Crashes pose a serious danger for each investors who wants to speculate on bubbles.

Panel C of Table 5 contains the transition probabilities that result from the estimation (we exclude the probabilities that are fixed at zero or one). The normal regime is highly persistent. The process stays in the normal regime for another period with a probability of 0.0994 (CAPM and Carhart model) or 0.995 (Fama and French model). A switch to the good news regime G_1 is rare with a probability of approximately 0.005 (once per 200 months). So a switch to a sequence of good news, bubbles and eventually crashes does not happen very frequently in our sample.

[4]For a transformed lognormal random variable $Y = \omega(k - \mathrm{e}^Z)$ with $Z \sim N(\mu, \sigma^2)$, the expectation follows as $\omega(k - \exp(\mu + \frac{1}{2}\sigma^2))$ and the standard deviation as $\omega \exp(\mu + \frac{1}{2}\sigma^2)\sqrt{\exp(\sigma^2) - 1}$.

The bubble regime exhibits persistence, too. With estimates ranging from 0.797 (Carhart model) to 0.833 (CAPM), this persistence is much smaller than for the normal regime. The average length of a bubble ranges from $1/(1-0.797) = 4.93$ to $1/(1-0.833) = 5.98$ months. These months come after the fixed sequence of $L = 6$ months of good news. The expected gain due to a bubble is consequently some $4.93 \times 3.16 = 15.6\%$ to $5.98 \times 3.41 = 20.4\%$. Of course, the bubble length can deviate from these expected values.

When the bubble ends, a crash marks the start of a period of deflation. With a probability of 0.224 (Fama-French model) to 0.251 (CAPM), the process switches to the normal regime, which indicates that the deflation is finished. With a probability of 0.175 (CAPM) to 0.191 (Fama-French model) another crash occurs. So the deflation crash regime is not as persistent as the normal regime or the bubble regime. Still, the risk of another crash is non-negligible. With the remaining probability of 0.574 (CAPM) or 0.585 (the other two models), the process switches to the deflation normal regime. This regime has the same distributional properties as the normal regime, but a much higher probability of a switch to the deflation crash regime of 0.268 (Carhart model) to 0.289 (CAPM). In both deflation regimes, the risk of future negative returns stays high.

The standard errors on the transition probabilities indicate that they are well identified. The only exception is the crash regime that is unrelated to bubbles. This regime has a very small probability to occur after the normal regimes, with values of 0.0014 (CAPM) or lower. When such a switch occurs, more crashes can follow, but the estimates for that switch have relatively large standard errors. We conclude that our model relates most of the crashes in our sample to bubbles.

In Table 6 the ergodic probabilities are shown. They present the probability that the investor assigns to the different regimes if he has no historical information. They can be interpreted as steady-state probabilities. In line with our findings for the transition probabilities, the normal regime is most likely with probabilities between 0.885 (Carhart) and 0.891 (CAPM). Each good news regime has an ergodic probability of around 0.0045, so together this adds up to a probability of around 2.7%. The ergodic probabilities for the bubble regimes are slightly lower and range from 0.0231 (Fama-French and Carhart models) to 0.0267 (CAPM). The steady state probability of the deflation crash regime has the same order of magnitude (between 0.0178 and 0.0197), whereas the deflation normal regime is approximately double (between 0.0354 and 0.0430). So, in the long run, deviations from the normal regime do not have a high probability. The multiplication of the ergodic probabilities by the means of the different regimes gives the model implied expected returns. They equal 0.10% per month for the CAPM, -0.01% for the Fama-French model, and 0.04% for the Carhart model. So, in the long run, the industries do not offer abnormal returns that deviate much from zero in expectation. Of course, in the short run information from prices may indicate differently.

[Table 6 about here.]

Figure 2 shows the distribution of the different regimes over time. We see a pronounced increase in the number of industries experiencing a bubble at the end of the century when the internet bubble occurred. It seems that it was also a period of higher than average volatility as several industries experienced crashes during these years as well. Especially for the CAPM and the 3-Factor Model, we find, in line with the bursting of the bubble, a sharp increase in the number of industries experiencing deflation around 2000. Our model also relates the recent credit crisis to some extent to bubbles. Our model also identifies earlier bubbles, like the "Bubble in Concept Stocks" in 1967–68 as coined by Malkiel (1996) (see also Baker and Wurgler, 2006). The distribution of bubbles across industries in Table 7 shows that the different regimes are well distributed across the different industries. It seems that no single industry in particular is driving our findings.

[Figure 2 about here.]

[Table 7 about here.]

2.4 Forecasts

The investor uses the model to make forecasts for future periods m. For the current probability distribution over the regimes $\boldsymbol{\xi}_t$, the m-period ahead forecast probabilities can be calculated as $\boldsymbol{\xi}_{t+m|t} = \boldsymbol{P}^m \boldsymbol{\xi}_t$. When the investor only has price information, his current probability distribution equals the inference probabilities in Equation (4). Based on the forecast probabilities and the probability density functions, he constructs the forecasted distribution of the innovations $g_{t+m}(u)$:

$$g_{t+m}(u) = \boldsymbol{\xi}'_{t+m|t}\boldsymbol{g}(u). \tag{5}$$

The m-period-ahead forecast of the innovation distribution consists of the probabilities of the different states and their respective distributions. Along the same lines, any raw moment of order n can be calculated as a sum of state-specific moment weighted by the states' forecast probabilities:

$$\mathrm{E}_t\left[u^n_{t+m}\right] = \sum_{s\in\mathcal{S}} \xi_{t+m|t}(s)\,\mathrm{E}[u^n_{t+m}|S_{t+m} = s]. \tag{6}$$

Figure 3 shows the forecast probabilities when the investor is certain that the asset currently experiences a bubble, $\Pr[S_t = \mathrm{B}] = 1$. In the short run, the investor expects the bubble to continue. The probability that the return process stays in the bubble regime for the following month, ranges from 0.797 (Carhart model) to 0.833 (CAPM). Two months later, the probability is still between 0.64 and 0.69. The probability that the bubble continues declines rapidly over time. After 4 months, the probability that the bubble has ended is larger than the probability that it continues.

[Figure 3 about here.]

The probability of a crash is close to 0.20 for the first few months. The forecast for the month $t + 1$ follows from Table 5 and varies from 0.167 (CAPM) to 0.203 (Fama-French model). In $t + 2$ this probability is the sum of the probability that a second crash follows the first, and that the bubble ends in this period. For the CAPM this yields $0.833 \times 0.167 + 0.167 \times 0.175 = 0.169$, which is a bit higher than the probability for first month. For the Fama-French and Carhart models these probabilities are 0.188 and 0.198. In the first five to ten months, the probability of the deflation crash regime stays high. After month 10 it slowly declines.

The deflation normal regime can show up for the first time in month $t + 2$, when the bubble has burst in month $t + 1$. For the CAPM, this probability equals $0.167 \times 0.574 = 0.096$. For the other two models we find 0.110 and 0.119. The probability that the process is in the deflation normal regime steadily increases to a maximum between 0.312 (CAPM) and 0.357 (Carhart model) in month 10. After month 10 we see a slow decline setting in.

The normal regime can also show up for the first time in month $t+2$, which means that the deflation is over after month $t+1$. For the CAPM, this probability equals $0.167 \times 0.251 = 0.042$ (Fama-French model: 0.042; Carhart model: 0.048). The probability that the deflation is completed after month $t+2$ is 0.095 for the Carhart model and 0.084 for the other two. This probability increases slowly at first. It takes 15 to 16 months before the probability of the normal regime exceeds 0.5

After month $t+45$ the predictions are close to the steady-state distribution that follows from the ergodic probabilities in Table 6. From month $t+45$ to month $t+60$, the forecasts do not change much anymore, which is why we do not look beyond a horizon of 5 years.

Combining the forecast probabilities and the expected abnormal returns of the different regimes produces the forecasts for the expected abnormal returns and their volatilities in Figure 4. We consider three different initial probability distributions, corresponding with a certain start from the bubble regime ($\Pr[S_t = \mathrm{B}] = 1$), a certain start from the normal regime ($\Pr[S_t = \mathrm{N}] = 1$), and a start where the initial probability distribution is equal to the ergodic probabilities.

[Figure 4 about here.]

For all three asset pricing models, the forecasts for the expected returns starting from the bubble regime show the same pattern. Expected abnormal returns for month $t+1$ are positive. They steeply decline and become negative after two or three months. They reach their minimum around month 12, after which they slowly increase again. The speed of their increase goes down, and it takes quite some months before they become close to zero. After 60 months, the expected returns are close to the ergodic predictions, which are constant by construction. While the structure of the forecasts is the same for each model, the actual forecasts show some variation. The forecast for the first month for the CAPM equals 1.06%. The most negative forecast is -0.94 in month $t+11$. The forecasts based on the Fama-French or Carhart models start much lower (0.57% and 0.48%), and become a bit more negative at -1.17% and -1.09%. For a bubble riding strategy, the forecasts of the latter two models are hence less favorable.

For a comparison, we also show the forecasts from the normal regime. These forecasts also show similar forecasting patterns for the different asset pricing models. First they increase, which is related to switches to the good news regimes. After some 10 months, these favorable switches are offset by switches to the deflation regimes. The forecasts then decline towards the ergodic forecasts. The forecast for month $t + 1$ equals 0.15% for the CAPM, 0.06% for the Fama-French model and 0.10% for the Carhart model. These forecasts increase maximally by aproximately 0.08%. The dynamics in our model are clearly concentrated in the bubble and crash regime.

The forecasted volatilities after the bubble regime also show similar properties (the right panels of Figure 4). They start high (CAPM: 6.70%; Fama-French model: 6.51%; Carhart model: 6.67%), because only the two extremes of bubble continuation and deflation crash are possible. Over time, the volatility forecasts decline, because the probability of the more neutral normal and deflation normal regimes increase. After 60 months the forecasted volatilities are close to the ergodic volatility forecasts.

The forecasted volatilities after the normal regime show the opposite properties. They start low, then increase with a pace that goes up after month $t+6$, and eventually converge to the ergodic volatility forecasts. The slow increase in the first few months is related to the small increase of the probability of the good news and bubble regimes. After month $t + 6$, the deflation crash regime can occur, which causes a slightly sharper increase in volatility.

Our finding that bubbles initially lead to positive expected abnormal returns, but also leads to higher volatility is in line with the evidence in Guenster et al. (2012) for a one month horizon. We show that this result extends to month $t + 2$ as well (and month $t + 3$ in case of the CAPM), though the risk-return trade-off has declined. In line with the limits-to-arbitrage literature, we show that the correction of a bubble is accompanied by higher volatility, in particular for short horizons. We investigate the implications of these return patterns for investors in the next section.

3 Allocations in the presence of a bubble

3.1 The investment problem

We derive the optimal response of an investor to the presence of a bubble in an expected utility framework. We investigate how his decision depends on the investment horizon, and on the possibilities to rebalance. The investor is currently at time t, and has a power utility function defined over terminal wealth M periods ahead,

$$U(W_{t+M}) = \frac{W_{t+M}^{1-\gamma}}{1-\gamma}, \quad W_{t+M} > 0, \gamma \neq 1, \tag{7}$$

where γ is the investor's coefficient of relative risk aversion. For $W_{t+M} \leq 0$, the investor is bankrupt and we set his utility to $-\infty$. When $\gamma = 1$ the investor has log utility, $U(W_{t+M}) = \log W_{t+M}$.

The investor maximizes this utility function by choosing a sequence of K portfolios, each for a horizon of $h = M/K$ months at K equally spaced points $t, t{+}h, t{+}2h, \ldots, t{+}(K{-}1)h$. Our interest is mainly in the setting where the investor trades every month, $K = M$ and $h = 1$, or follows a buy-and-hold strategy, $K = 1$ and $h = M$. This framework can also handle the situation where unwinding positions takes more months $(1 < h < M)$.

At each point $t + hk$, $k = 0, 1, \ldots, K - 1$ the investor constructs a portfolio with a fraction w_k of her wealth W_{t+hk} in a possibly bubbly industry for the coming h months, and the remainder in the market. As in Guidolin and Timmermann (2007) we assume that the riskfree rate is constant and equal to r_{f}. We assume that the industry is an "average" industry, with a market beta of one, and no exposure to size, value or momentum, or any other factor,

$$r_{i,t+\tau} = r_{\mathrm{f}} + f_{\mathrm{m},t+\tau} + \omega_i u_{i,t+\tau}(S_{i,t+\tau}), \tag{8}$$

where $f_{\mathrm{m},t+\tau}$ is the excess return on the market over month $t + \tau$.

Consequently, the investor solves the following problem

$$\max_{\{w_k\}_{k=0}^{K-1}} \mathrm{E}\left[U(W_{t+M})|\,\boldsymbol{\xi}_t\right] \tag{9}$$

$$\text{s.t. } W_{t+h(k+1)} = W_{t+hk}\big(w_k R_{i,t+hk,h} + (1-w_k)R_{\mathrm{m},t+hk,h}\big) \tag{10}$$

$$R_{i,t+hk,h} \equiv \prod_{\tau=1}^{h}\big(1 + r_{i,t+hk+\tau}\big) \tag{11}$$

$$R_{\mathrm{m},t+hk,h} \equiv \prod_{\tau=1}^{h}\big(1 + r_{\mathrm{f}} + f_{\mathrm{m},t+hk+\tau}\big). \tag{12}$$

The investor maximizes her expected terminal wealth, conditional on the current probability distribution over the different regimes, denoted by $\boldsymbol{\xi}_t$. Equation (10) gives the evolution of wealth and incorporates the budget restriction. This evolution is a function of the compounded return over h periods on the industry, $R_{i,t+hk,h}$, and on the market $R_{\mathrm{m},t+hk,h}$, which are defined in Equations (11) and (12). To answer our research question we investigate how the initial portfolio weight w_0 depends on the probability of the bubble regime at time t, $\xi_{\mathrm{B},t}$.

To solve this maximization we derive the Bellmann equation and use dynamic programming as in Brandt et al. (2005); Guidolin and Timmermann (2007). First, we define the value function

$$V(W_t, \boldsymbol{\xi}_t) \equiv \max_{\{w_k\}_{k=0}^{K-1}} \mathrm{E}\left[U(W_{t+M})|\,\boldsymbol{\xi}_t\right] \text{ s.t. Equation (10).} \tag{13}$$

Because power utility has CRRA, we can without loss of generality maximize the compounded return W_{t+M}/W_t. Substituting Equation (10) yields

$$\begin{aligned} V(1, \boldsymbol{\xi}_t) &= \max_{\{w_k\}_{k=0}^{K-1}} \mathrm{E}\left[U(W_{t+M}/W_t)|\,\boldsymbol{\xi}_t\right] \\ &= \max_{\{w_k\}_{k=0}^{K-1}} \mathrm{E}\left[\frac{1}{1-\gamma}\prod_{k=0}^{K-1}\big(w_k R_{i,t+hk,h} + (1-w_k)R_{\mathrm{m},t+hk,h}\big)^{1-\gamma}\,\middle|\,\boldsymbol{\xi}_t\right] \end{aligned} \tag{14}$$

Next, we apply the law of iterated expectations

$$\begin{aligned} V(1, \boldsymbol{\xi}_t) = \max_{w_0} \mathrm{E}\Bigg[&\big(w_0 R_{i,t,h} + (1-w_k)R_{\mathrm{m},t,h}\big)^{1-\gamma} \times \\ &\max_{\{w_k\}_{k=1}^{K-1}} \mathrm{E}\left[\frac{\prod_{k=1}^{K-1}\big(w_k R_{i,t+hk,h} + (1-w_k)R_{\mathrm{m},t+hk,h}\big)^{1-\gamma}}{1-\gamma}\,\middle|\,\boldsymbol{\xi}_{t+h}\right]\Bigg|\,\boldsymbol{\xi}_t\Bigg]. \end{aligned} \tag{15}$$

We define a new function Q based on the second maximization in this equation,

$$Q(k, \boldsymbol{\xi}_{t+hk}) \equiv (1-\gamma) \max_{\{w_j\}_{j=k}^{K-1}} \mathrm{E}\left[\frac{\prod_{j=k}^{K-1} \big(w_j R_{i,t+hj,h} + (1-w_j) R_{\mathrm{m},t+hj,h}\big)^{1-\gamma}}{1-\gamma} \middle| \boldsymbol{\xi}_{t+hk} \right]. \quad (16)$$

Combining Equations (15) and (16) leads to the Bellmann equation

$$\begin{aligned} &\frac{1}{1-\gamma} Q(k, \boldsymbol{\xi}_{t+hk}) \\ &= \max_{w_k} \mathrm{E}\left[U\big(w_k R_{i,t+hk,h} + (1-w_k) R_{\mathrm{m},t+hk,h}\big) Q(k+1, \boldsymbol{\xi}_{t+h(k+1)}) \big| \boldsymbol{\xi}_{t+hk} \right], \end{aligned} \quad (17)$$

for $k = 0, 1, \ldots, K-1$ with final condition $Q(K, \boldsymbol{\xi}_{t+hK}) = 1$. The first order condition to Equation (9) can then be formulated as

$$\mathrm{E}\left[\frac{R_{i,t+hk,h} - R_{\mathrm{m},t+hk,h}}{\big(w_k^* R_{i,t+hk,h} + (1-w_k^*) R_{\mathrm{m},t+hk,h}\big)^{\gamma}} Q(k+1, \boldsymbol{\xi}_{t+h(k+1)}) \middle| \boldsymbol{\xi}_{t+hk} \right] = 0, \quad (18)$$

for $k = 0, 1, \ldots, K-1$.

Because $\boldsymbol{\xi}_{t+h(k+1)}$ and $R_{i,t+h(k+1)}$ are not independent, we cannot split the expectation of the product in the product of the expectations. As in Guidolin and Timmermann (2007) we assume optimal learning by the investor: he uses the information in $r_{i,t+h(k+1)}$ to update his belief about the probabilities of the different states. First he constructs a vector of forecast probabilities

$$\phi_{t+h(k+1)|t+hk} = \boldsymbol{P}^h \boldsymbol{\xi}_{t+hk}, \quad (19)$$

which she then updates with the last innovation $u_{i,t+h(k+1)}$ by Bayes' rule

$$\boldsymbol{\xi}_{t+h(k+1)} = \frac{1}{\phi'_{t+h(k+1)|t+hk} \boldsymbol{g}\big(u_{i,t+h(k+1)}\big)} \phi_{t+h(k+1)|t+hk} \odot \boldsymbol{g}\big(u_{i,t+h(k+1)}\big), \quad (20)$$

where $\boldsymbol{g}(u)$ is the vector with the values of the regime-specific density functions for u. The same rules are applied in the Hamilton filter for the actual observations (see Equations (3) and (4)).

The application of the Bellmann principle of optimality (see Bellmann, 1957) splits the problem of findings a series of K optimal portfolios into a series of K problems of finding a single optimal portfolio. This series of problems is solved backwards, starting with the problem in Equation (17) for portfolio for $K-1$ and using $Q(K, \boldsymbol{\xi}_{t+hK}) = 1$. Solving this problem produces a set of optimal portfolios w^*_{K-1} for different values of $\boldsymbol{\xi}_{t+h(K-1)}$, and the corresponding values for the function $Q(K-1, \boldsymbol{\xi}_{t+h(K-1)})$. These values are then used when solving the problem in Equation (17) for the optimal portfolio w^*_{K-2} and so on, until the optimal portfolio w^*_0 for time t is determined.

There are three special cases that deserve some extra attention. When the investor follows a buy-and-hold strategy, learning becomes irrelevant. The investor chooses a single portfolio w_0, which is held for M periods. The set of first order conditions given by Equation (18) reduce to

$$\mathrm{E}\left[\frac{R_{i,t,M} - R_{\mathrm{m},t,M}}{\big(w^*_0 R_{i,t,M} + (1-w^*_0) R_{\mathrm{m},t,M}\big)^\gamma} \middle| \boldsymbol{\xi}_t \right] = 0. \tag{21}$$

When the investor can rebalance every period, the evolution of wealth in Equation (10) satisfies $W_{t+k+1} = W_{t+k}\big(1 + r_\mathrm{f} + f_{\mathrm{m},t+k+1} + w_k \omega_{i,t+k+1} u_{i,t+k+1}(S_{i,t+k+1})\big)$. The first order conditions in Equation (18) reduce to

$$\mathrm{E}\left[\frac{\omega_{i,t+k+1} u_{i,t+k+1}(S_{i,t+k+1})}{\big(1 + r_\mathrm{f} + f_{\mathrm{m},t+k+1} + w^*_k \omega_{i,t+k+1} u_{i,t+k+1}(S_{i,t+k+1})\big)^\gamma} Q(k+1, \boldsymbol{\xi}_{t+k+1}) \middle| \boldsymbol{\xi}_{t+k} \right] = 0, \tag{22}$$

for $k = 0, 1, \ldots, M$.

When the investor has log utility, the investor maximizes

$$\log W_{t+M} = \log W_t + \sum_{k=0}^{K-1} \log\big(w_k R_{i,t+hk,h} + (1-w_k) R_{\mathrm{m},t+hk,h}\big). \tag{23}$$

We find the usual result that the investor takes a myopic decision. In this case, learning is not relevant for the investor. The first order conditions in Equation (18) reduce to

$$\mathrm{E}\left[\frac{R_{i,t+hk,h} - R_{\mathrm{m},t+hk,h}}{w^*_k R_{i,t+hk,h} + (1-w^*_k) R_{\mathrm{m},t+hk,h}} \middle| \boldsymbol{\xi}_{t+hk} \right] = 0, \tag{24}$$

for $k = 0, 1, \ldots, K-1$.

3.2 The buy-and-hold decision

We first analyze the decision of a buy-and-hold investor. While this strategy may not be the most realistic, because it means that investors do not update their portfolio at all during their investment horizon, this strategy is an interesting benchmark case. Decisions in which the investor takes rebalancing opportunities into account can be seen as a sequence of buy-and-hold decisions for a shorter horizon. It can clarify how learning affects the decision of an investor.

The optimal portfolio of a buy-and-hold investor follows from the first order condition in Equation (21). We cannot analytically solve for w_0^*, but use numerical techniques based on simulations instead. We simulate return paths with a length of M months. We use bootstraps to simulate from the distribution of the excess market returns, and Monte Carlo draws from the regime process and the distribution of the innovations. We fix the risk-free rate at 0.30%, which is close to its long-term average. For each simulation b we calculate the values for $R^b_{i,t,M}$ and $R^b_{\mathrm{m},t,M}$ by compounding the monthly returns. We approximate the expectations in Equation (21) by $B = 100,000$ simulations, and then solve for w_0^*.

First we consider the term structure of the risk-return trade-off following Campbell and Viceira (2005) in Figure 5. It shows the averages and the volatilities of the cumulative returns for different initial probability distributions, scaled to a monthly horizon. For all three asset pricing models, the expected returns are positive and large, when the asset is with certainty in a bubble. The expected returns for month $t+1$, varying from 0.89% to 1.14% are higher than the abnormal return for the same month in Figure 4, because the fundamental part of the return, the risk-free rate and excess market return, are taken into account. Because the expectations of the abnormal returns go down and become negative, we see a downward sloping pattern for the expectation of the cumulative returns. Because of the cumulation, it takes much longer before these expectations become negative: eight months for the Fama-French and Carhart models, and twelve for the CAPM. It takes some 45 months, before the expected cumulative returns become positive again, though they are still below the risk-free rate of 0.30%. We conclude that bubbles offer an attractive return at short horizons, thought they decline fast. For horizons of one to four years, expected returns are negative, which entices short-selling.

[Figure 5 about here.]

Deciding whether an asset forms an attractive investment opportunity also depend on the risk that it entails. Therefore we also show the volatilities in Figure 5. The volatilities start between 8.05% (Carhart model) and 8.8% (CAPM). In the first months they show a steep increase to a maximum of 8.8% after six months for the Carhart model, 9.0% for the Fama-French model after eight months or 10.3% after ten months for the CAPM. After the maximum a slow decline sets in. After sixty months, the volatilities are again close to their values for month $t+1$. As argued by Guidolin and Timmermann (2007), the increase in volatility relates to the persistence of regimes, which has a mean-averting effect and is opposite to the decreasing volatilities that result from VAR-style mean-reversion in Campbell and Viceira (2005). The decline for longer horizons relates to the dominance of the normal regime in the long-run predictions. Combining the expected returns and volatilities graphs shows that bubbles are attractive at first. However, this attraction disappears rapidly, because expected returns go down, while volatilities go up. When expected become negative, and volatilities go down again, a bubbly asset may be interesting for short selling.

When the process starts from the normal regime, or from the ergodic probability distributions, the expected returns show hardly any dynamics. The volatilities on the other hand slope upwards for both. This feature relates again to the mean-aversion described by Guidolin and Timmermann (2007). Because complete certainty on the process being in either the bubble or normal state is not realistic, we include the expected returns and volatilities when the process starts with equal probabilities in the bubble or normal regime. The expected return are then exactly an average of the expectations for the normal and bubble case. For the volatilities such a relation does not apply, and we see that the pattern is closer to the bubble case.

We show the optimal buy-and-hold allocation for a power-utility investor with a risk aversion coefficient of 5 in Figure 6. As we reasoned based on the expected returns and volatilities, riding bubbles is attractive for short horizons. A one-month investor would allocate between 22% (Carhart model) and 37% (CAPM) of his wealth to a bubbly industry, and the remainder in the market portfolio. For a two-month buy-and-hold strategy, weights range from 11% (Carhart model) to 28% (CAPM). This decline continues and the weights become negative for horizons of four (Fama-French and Carhart models) to eight months (CAPM). For horizons that exceed these lengths, expected returns are lower than the market, which makes a short position in the bubbly industry and a leveraged position in the market attractive. The largest short positions show up for horizons of 17 (Carhart model, -25%), 19 (Fama-French model, -19%) or 25 months (CAPM, -11%). For longer horizons, weights go slightly up again, reflecting the increase in expected returns. As predicted by theory, investors with a long horizon that follow a buy-and-hold strategy will short bubbles. In line with empirical and theoretical literature for short horizons, riding bubbles is the optimal strategy.

[Figure 6 about here.]

This conclusion also applies to an investor who is divided between the normal regime and the bubble regime. The allocations for this investor are less aggressive than for the investor who is sure about the asset begin in the bubble regime, but still show the same pattern. If this investor has a short horizon, he will ride bubbles. For longer horizons, he will short the asset.

3.3 Decisions with rebalancing

While the optimal buy-and-hold strategies form an interesting benchmark case, they are not entirely realistic. Most investors monitor the performance of their investments, and adjust their allocation when the investment climate changes. We expect these adjustments to be particularly relevant when bubbles are concerned. Investor who speculate on the continuation of a bubble, may want to exit as soon as signals of the bursting of a bubble reach them.

We consider the optimal decision of an investor who can adjust his position every month in Figure 7. After observing the return for a particular month, this investor updates his inferences of the regime probabilities as in Equation (20), and adjusts his allocation accordingly. His investment problem become dynamic, which we solve by the techniques described in Appendix A.

[Figure 7 about here.]

If the investor has an investment horizon of one month, the buy-and-hold decision and decision with rebalancing are of course identical, so the curves in Figure 7 start at the same point as in Figure 6. While certainty about the bubble at time t leads to declining weights for a buy-and-hold strategy, an investors who can rebalance should increases his holdings if his horizon becomes longer. For the CAPM this effect is strongest for an horizon of three months with an optimal allocation for month $t + 1$ of 43% to the bubbly industry. For the Fama-French and Carhart models the maximum allocations equal 34% in month 7 and 26% in month 5. When the horizon becomes longer, optimal weights decline first and then stabilize after month 30.

To explain why the possibility of rebalancing makes riding bubbles more attractive, we look at our results in Table 5 and Figure 4 for the Fama-French model. Riding bubbles is attractive because the expected return in month $t+1$ is positive (0.57% for the Fama-French model). However, it is also risky because a crash may occur. An investor with a two-month horizon reconsiders his portfolio after month $t + 1$. He uses the return in month $t + 1$ to update his time t forecast for this month, which was a continuation of the bubble with probability 0.812 and a switch to the deflation crash regime with probability 0.188. If the abnormal return during month $t+1$ is positive, he will adjust the forecast probability for the bubble regime upward and the probability for the crash regime downward. His predictions for month $t+2$ are based on these updated probabilities, so he will probably ride the bubble again in month $t+2$. Now suppose that the abnormal return during month $t+1$ is below k. He will then decrease the forecasted probability for the bubble regime, and increase it for the deflation crash regime. In the limiting case that the deflation crash regime prevails with probability 1, the abnormal return forecast becomes $(0.224+0.585)\times 0.05+0.191\times -10.15 = -1.89\%$ for month $t + 2$. Because the abnormal return forecast is negative, the investor will probably take a short position, which would allow him to recoup some of his loss in month $t+1$. This possibility to adjust the allocation entices the investor to take more risk in the beginning. Persistence in return patterns is a necessary condition for this behavior, which is what we find in the data based on the regime switching model.

The decline for longer horizons is related to the overall riskiness of the investment set. Because no riskfree asset is available, the investor remains exposed to risk for his complete investment horizon. This market risk may exacerbate losses that could come from a riding bubbles strategy. To mitigate the effects of this long exposure to market risk, the investor limits his initial risk taking.

The effect of learning is also present when the process starts from the steady-state distribution. In this case the investor also increases his exposure to the industry to profit from the specific form of the abnormal return distribution. Because all regimes are possible with some probability (see Table 6), the investor can take additional risk. When the regime process starts in the normal regime, opportunities for learning are small, because the process is likely to remain in the normal regime.

Learning is equally valuable to an investor who is divided 50/50 between the normal and bubble regime as it is to an investor who knows about the bubble with certainty. The optimal portfolios for the 50/50 case follow the same pattern as the case with certainty on the bubble. The main difference is that the weighs for all horizons have been shifted downward.

We analyze the effect of learning when investor can rebalance less often than every month in Table 8. When an investor assigns a probability of 50% or more to the bubble regime, the effect of learning remains important. An investor who know with certainty about the bubble and an investment horizon of two months invests 15.2% of his wealth in the industry. When his horizon is doubled to four months, he either reduces his weight to 0.7% if he cannot rebalance, or increases it to 19.7% when he can rebalance. When the investor can rebalance every three, four or six months, the effect of learning remains present, though it becomes smaller. We conclude that learning is an important aspect when determining the optimal response to bubbles.

[Table 8 about here.]

4 Conclusion

Riding bubbles is not only the optimal strategy for short-term speculator. Instead, our findings show that even long-term investor would ride bubbles, as long as they can rebalance their portfolios at reasonable frequencies. Only investors who need longer six months to rebalance their portfolios would take a short position, if they learn of a bubble. These findings are in contrast to theoretical papers such as De Long et al. (1990a) and Shleifer and Vishny (1997) which propose that investors refrain from trading against bubbles due to their short horizons. The difference between the theoretical predictions and our empirical findings might be due to the fact that bubbles burst in these theoretical models, but empirically, we find that they deflate slowly.

A Solution techniques

As Guidolin and Timmermann (2007) we use simulations to approximate the expectation in Equation (17). A simulation consist of a sequence of h market returns and h industry returns. For both we need h draws from the distribution of the excess market return, which we construct by bootstrapping. The idiosyncratic part of the industry return requires a simulated path from the regime process, $S_{i,\tau}$, $\tau = 1, 2, \ldots, h$ and random draws from the regimes that constitute the simulated path, $u_{i,\tau}(S_{i,\tau})$.

To reduce the computational burden, we construct the paths by stratified sampling. We construct a fixed number of paths B that start with a particular regime s. So, we have B paths that start from the normal regime, B paths that start from the crash regime, and so on. The next regime is simulated based on the transition probabilities that apply to the first regime, and so on. A single simulated path is thus indicated by the initial regime s and the path number b. The resulting compounded industry return is denoted by $R_{i,h}^{s,b}$. Since the compounded market return is regime independent, we denote it by $R_{\mathrm{m},h}^{b}$. Because only the length of the path h matters for the simulation, we omit the time period $t + hk$ from the notation.

To increase the precision, we use antithetic sampling to draw from the regimes. Suppose we have a drawn a particular $u_{i,\tau}^{s,b}$ from regime $S_{i,\tau}^{s,b}$ in path (s, b), with cumulative probability $\Pr\big[u \leq u_{i,\tau}^{s,b} | S_{i,\tau}^{s,b}\big]$. Then we also add the draw $u_{i,\tau}^{s,B+b}$ from the same regime $S_{i,\tau}^{s,b}$ such that $\Pr\big[u \leq u_{i,\tau}^{s,B+b} | S_{i,\tau}^{s,b}\big] = 1 - \Pr\big[u \leq u_{i,\tau}^{s,b} | S_{i,\tau}^{s,b}\big]$.

The last simulated draw $u_{i,h}^{s,b}$ is used to construct the inference probability for the next decision as in Equations (19) and (20). Each combination of $\boldsymbol{\xi}_{t+hk}$ and $u_{i,h}^{s,b}$ will generally produce a new $\boldsymbol{\xi}_{t+h(k+1)}^{s,b}$, leading to the curse of dimensionality. As in Guidolin and Timmermann (2007), we approximate $\boldsymbol{\xi}_{t+h(k+1)}^{s,b}$ over a grid of J points $\boldsymbol{\xi}^j$, $j = 1, 2, \ldots, J$, by selecting the grid point that has the lowest L_1 norm to $\boldsymbol{\xi}_{t+h(k+1)}^{s,b}$. Mathematically, we replace $\boldsymbol{\xi}_{t+h(k+1)}^{s,b}$ by

$$\tilde{\boldsymbol{\xi}}_{t+h(k+1)}^{(s,b)} = \left\{ \boldsymbol{\xi}^j : \min_j \sum_{q \in \mathcal{S}} \left| \xi_q^j - \xi_{q,t+h(k+1)}^{s,b} \right| \right\}. \tag{25}$$

These techniques lead to the following approximation of Equation (17)

$$\begin{aligned}\frac{Q\left(k,\boldsymbol{\xi}_{t+hk}\right)}{1-\gamma} \approx \max_{w_k} \frac{1}{2B}\sum_{s\in\mathcal{S}}\sum_{b=1}^{2B} U\Big(w_k R_{i,h}^{s,b} + (1-w_k)R_{\mathrm{m},h}^{b}\Big)\cdot \\ Q\Big(k+1,\tilde{\boldsymbol{\xi}}_{t+h(k+1)}^{s,b}\Big)\Pr[S_{t+hk+1}=s|\boldsymbol{\xi}_{t+hk}].\end{aligned} \tag{26}$$

We divide by $2B$ because antithetic sampling doubles the number of paths B. We multiply by $\Pr[S_{t+hk+1}=s|\boldsymbol{\xi}_{t+hk}]$ to give each stratum its proper weight in the simulation.

The last element to discuss for our solution of the dynamic program is the grid for the inference probability $\boldsymbol{\xi}$. We construct this grid such that for a given set of simulated paths, the distance between any point $\boldsymbol{\xi}_{t+h(k+1)}^{(s,b)}$ and the nearest point in the grid $\tilde{\boldsymbol{\xi}}_{t+h(k+1)}^{(s,b)}$ is smaller than a specified threshold d multiplied by the number of regimes. We use a simple algorithm to construct such a grid. We first specify a small set of grid points, which comprises the ergodic regime probabilities, and the set $\{\boldsymbol{\xi}:\xi_{\mathrm{N}}=p,\xi_{\mathrm{B}}=1-p,\xi_s=0\forall s\neq \mathrm{N,B},p=0,0.1,0.2,\ldots,1\}$. For each point in this list $\boldsymbol{\xi}^j$, and all simulated paths, we check whether the distance between the resulting point $\boldsymbol{\xi}^{s,b}(\boldsymbol{\xi}^j)$ and its nearest neighbor $\tilde{\boldsymbol{\xi}}^{s,b}(\boldsymbol{\xi}^j)$ is smaller than the threshold. If so, the algorithm stops. If not, the point $\boldsymbol{\xi}^{s,b}(\boldsymbol{\xi}^j)$ is added to the grid, and the new grid is evaluated.

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Table 1: Structure of the Transition Matrix

	N	C	G_1	G_2	$\cdots$	G_l	G_{l+1}	$\cdots$	G_L	B	D_C	D_N
N	p_{NN}	p_{CN}	0	0	$\cdots$	0	0	$\cdots$	0	0	$p_{D_C N}$	0
C	p_{NC}	p_{CC}	0	0	$\cdots$	0	0	$\cdots$	0	0	0	0
G_1	p_{NG}	p_{CG}	0	0	$\cdots$	0	0	$\cdots$	0	0	0	0
G_2	0	0	1	0	$\cdots$	0	0	$\cdots$	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_l	0	0	0	0	$\ddots$	0	0	$\cdots$	0	0	0	0
G_{l+1}	0	0	0	0	$\cdots$	1	0	$\cdots$	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
G_L	0	0	0	0	$\cdots$	0	0	$\ddots$	0	0	0	0
B	0	0	0	0	$\cdots$	0	0	$\cdots$	1	p_{BB}	0	0
D_C	0	0	0	0	$\cdots$	0	0	$\cdots$	0	p_{BD_C}	$p_{D_C D_C}$	$p_{D_N D_C}$
D_N	0	0	0	0	$\cdots$	0	0	$\cdots$	0	0	$p_{D_C D_N}$	$p_{D_N D_N}$

This table shows the structure of the matrix of transition probabilities for the different regimes: normal (N), crash (C), good news ($G_l, l = 1, 2, \ldots, L$), bubble (B), deflation crash (D_C) and deflation normal (D_N). Each column should sum to one, and we also impose $p_{NG} = p_{CG}$.

Table 2: Industry-specific Estimates for the CAPM and Abnormal Returns

Industry	Estimates of Equation (1) β_i		ω_i (×100)		Abnormal Returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$ mean	stdev.	skew.	kurt.	min.	max.
Agric	0.93	(0.045)	4.82	(0.15)	0.18	5.17	0.26	4.02	−14.4	25.0
Food	0.69	(0.028)	3.03	(0.10)	0.32	3.22	0.63	7.48	−11.7	19.9
Soda	0.82	(0.047)	4.94	(0.16)	0.39	5.38	0.54	7.06	−18.6	32.7
Beer	0.79	(0.039)	3.78	(0.12)	0.35	4.05	−0.09	4.73	−17.1	14.9
Smoke	0.71	(0.047)	4.97	(0.15)	0.67	5.44	−0.05	6.83	−29.5	27.9
Toys	1.21	(0.044)	4.58	(0.14)	−0.15	5.07	−0.22	4.27	−24.9	17.1
Fun	1.39	(0.044)	4.64	(0.15)	0.28	4.86	0.24	4.75	−19.0	25.3
Books	1.09	(0.032)	3.22	(0.12)	0.01	3.44	0.11	5.65	−16.8	18.5
Hshld	0.85	(0.032)	2.72	(0.10)	0.10	3.05	−0.41	7.22	−19.4	12.3
Clths	1.16	(0.038)	3.94	(0.13)	0.12	4.22	0.29	5.27	−15.2	19.1
Health	1.12	(0.061)	5.83	(0.20)	0.06	6.73	−0.14	6.37	−29.6	31.2
MedEq	0.90	(0.030)	3.34	(0.11)	0.26	3.52	−0.06	3.32	−12.7	12.3
Drugs	0.83	(0.032)	3.15	(0.11)	0.26	3.57	−0.09	5.19	−14.1	18.0
Chems	1.06	(0.026)	2.78	(0.09)	0.03	3.06	0.23	5.66	−11.8	16.8
Rubbr	1.10	(0.036)	3.56	(0.12)	0.14	3.81	0.41	5.10	−12.7	19.9
Txtls	1.10	(0.046)	4.92	(0.16)	0.03	5.22	1.41	16.00	−18.8	47.1
BldMt	1.20	(0.030)	3.05	(0.09)	0.02	3.35	0.48	8.53	−14.5	23.2
Cnstr	1.33	(0.039)	4.22	(0.13)	−0.03	4.46	0.44	4.00	−16.3	18.2
Steel	1.30	(0.043)	4.29	(0.14)	−0.24	4.55	0.50	5.12	−18.1	24.4
FabPr	0.92	(0.048)	5.05	(0.15)	0.25	5.62	−0.10	7.29	−35.8	24.3
Mach	1.24	(0.027)	2.76	(0.09)	0.01	3.00	−0.05	4.01	−13.0	10.6
ElcEq	1.22	(0.029)	3.16	(0.10)	0.21	3.23	0.03	3.32	−12.1	10.4
Autos	1.17	(0.044)	4.40	(0.15)	−0.16	4.62	0.63	11.31	−21.6	36.6
Aero	1.15	(0.042)	4.28	(0.15)	0.23	4.55	0.24	5.11	−18.7	17.9
Guns	0.63	(0.091)	9.74	(0.29)	0.40	9.91	0.94	8.06	−27.7	74.2
Gold	0.82	(0.047)	4.94	(0.16)	0.39	5.38	0.54	7.06	−18.6	32.7
Ships	1.10	(0.046)	5.02	(0.19)	0.08	5.40	0.33	4.45	−21.0	20.0
Mines	1.09	(0.049)	4.95	(0.17)	0.22	5.26	−0.11	3.74	−21.6	16.7
Coal	1.21	(0.075)	7.79	(0.25)	0.46	8.44	0.80	6.30	−29.1	45.6
Oil	0.79	(0.036)	3.80	(0.12)	0.30	4.05	0.31	4.09	−14.4	17.4
Util	0.55	(0.028)	3.06	(0.10)	0.16	3.26	0.11	4.10	−13.2	11.8
Telcm	0.75	(0.031)	2.97	(0.09)	0.06	3.19	0.21	4.65	−11.8	16.5
PerSv	1.16	(0.039)	4.15	(0.14)	−0.23	4.95	−0.10	3.93	−17.7	16.5
BusSv	1.29	(0.031)	2.96	(0.09)	0.11	3.13	0.45	4.41	−9.1	13.4
Comps	1.16	(0.040)	4.22	(0.13)	0.04	4.58	0.10	5.59	−21.2	21.3
Chips	1.36	(0.037)	3.64	(0.12)	−0.05	4.09	0.04	6.33	−18.1	22.6
LabEq	1.32	(0.037)	3.89	(0.13)	0.02	4.07	0.17	4.12	−13.6	18.4
Paper	0.98	(0.032)	3.41	(0.11)	0.08	3.51	0.64	5.92	−14.7	18.2
Boxes	0.99	(0.031)	3.21	(0.10)	0.10	3.74	−0.21	4.98	−16.4	17.6
Trans	1.09	(0.031)	3.34	(0.10)	0.01	3.47	0.32	4.04	−11.1	12.7
Whshl	1.08	(0.027)	2.79	(0.09)	0.12	3.09	0.13	5.37	−13.2	13.3
Rtail	1.00	(0.030)	3.09	(0.10)	0.15	3.26	−0.05	3.86	−12.9	12.9
Meals	1.03	(0.035)	3.62	(0.11)	0.32	4.13	−0.13	4.88	−18.8	15.7
Banks	1.04	(0.033)	3.51	(0.11)	−0.03	3.81	−0.15	5.71	−19.9	13.9
Insur	0.97	(0.034)	3.63	(0.11)	0.10	3.86	0.56	6.69	−16.2	21.4
RlEst	1.19	(0.054)	5.39	(0.18)	−0.33	5.50	1.14	13.10	−18.6	46.2
Fin	1.22	(0.025)	2.54	(0.09)	0.10	2.80	0.12	6.73	−16.4	14.6
Other	1.14	(0.039)	4.02	(0.12)	−0.34	4.65	−0.26	5.49	−20.2	16.4

The first four columns of this table report the estimates of the industry-specific part of the model in Equation (1), with the CAPM as asset pricing model. Based on these estimates, we construct abnormal returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$. We report their means, standard deviation (both in % per month), skewness, kurtosis, minimum and maximum (last two in % per month) in the next six columns. The estimates are based on the returns on the 48 industries of Fama and French (1997). We assume that the innovations $u_{i,t}$, and the regime processes $S_{i,t}$ for the different industries are independent. We estimate the parameters by the EM-algorithm of Dempster et al. (1977). We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$. Standard errors of the estimates are in parentheses.

Table 3: Industry-specific Estimates for the Fama-French Model and Abnormal Returns

Industry	Estimates of Equation (1)								Abnormal Returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$					
	market β_i		SMB β_i		HML β_i		ω_i ($\times 100$)		mean	stdev.	skew.	kurt.	min.	max.
Agric	0.84	(0.048)	0.44	(0.066)	0.11	(0.072)	4.62	(0.14)	0.07	5.05	0.37	4.78	−15.1	27.3
Food	0.75	(0.032)	−0.11	(0.041)	0.17	(0.049)	2.95	(0.10)	0.26	3.14	0.62	6.85	−11.1	19.0
Soda	0.89	(0.052)	−0.10	(0.074)	0.20	(0.080)	4.90	(0.16)	0.31	5.31	0.52	6.84	−18.3	32.4
Beer	0.84	(0.042)	−0.10	(0.055)	0.09	(0.065)	3.76	(0.12)	0.32	4.02	−0.09	4.42	−17.4	14.3
Smoke	0.80	(0.049)	−0.24	(0.070)	0.17	(0.075)	4.92	(0.15)	0.63	5.36	−0.13	7.20	−31.0	28.5
Toys	1.11	(0.045)	0.51	(0.069)	0.08	(0.070)	4.37	(0.13)	−0.27	4.82	−0.23	4.09	−23.7	15.5
Fun	1.34	(0.046)	0.42	(0.065)	0.24	(0.072)	4.47	(0.15)	0.10	4.66	0.17	4.26	−18.9	22.3
Books	1.07	(0.035)	0.21	(0.046)	0.20	(0.052)	3.14	(0.11)	−0.11	3.34	0.05	4.74	−13.7	16.5
Hshld	0.89	(0.028)	−0.20	(0.042)	−0.04	(0.043)	2.69	(0.09)	0.15	3.02	−0.53	9.29	−22.6	11.2
Clths	1.08	(0.037)	0.54	(0.056)	0.36	(0.063)	3.61	(0.13)	−0.12	3.92	0.20	8.36	−21.4	25.0
Health	1.05	(0.061)	0.67	(0.090)	0.08	(0.093)	5.63	(0.20)	−0.06	6.51	−0.35	7.36	−35.3	26.2
MedEq	0.84	(0.034)	0.05	(0.048)	−0.28	(0.058)	3.30	(0.12)	0.38	3.43	−0.08	3.74	−13.0	12.0
Drugs	0.83	(0.035)	−0.32	(0.045)	−0.37	(0.057)	3.17	(0.10)	0.49	3.32	0.08	5.01	−14.7	13.3
Chems	1.13	(0.027)	−0.07	(0.037)	0.31	(0.041)	2.64	(0.08)	−0.10	2.88	0.25	5.12	−10.2	16.0
Rubbr	1.03	(0.031)	0.63	(0.045)	0.33	(0.048)	3.06	(0.09)	−0.12	3.27	0.35	4.65	−10.2	15.6
Txtls	1.11	(0.042)	0.69	(0.065)	0.67	(0.070)	4.18	(0.13)	−0.41	4.50	1.34	15.23	−17.6	39.6
BldMt	1.22	(0.029)	0.28	(0.042)	0.44	(0.045)	2.77	(0.09)	−0.23	3.03	0.26	7.40	−13.7	18.9
Cnstr	1.27	(0.044)	0.51	(0.067)	0.30	(0.073)	3.96	(0.14)	−0.25	4.21	0.28	3.83	−16.5	15.7
Steel	1.27	(0.039)	0.38	(0.057)	0.40	(0.060)	3.99	(0.12)	−0.49	4.34	0.35	5.45	−20.1	22.3
FabPr	0.94	(0.050)	0.20	(0.071)	0.43	(0.076)	4.90	(0.15)	0.02	5.40	−0.08	6.54	−33.2	22.0
Mach	1.20	(0.029)	0.26	(0.038)	0.09	(0.045)	2.67	(0.08)	−0.08	2.91	−0.03	3.92	−10.7	10.0
ElcEq	1.21	(0.032)	0.06	(0.045)	0.03	(0.048)	3.15	(0.10)	0.19	3.23	0.05	3.38	−11.9	11.3
Autos	1.24	(0.046)	0.10	(0.062)	0.64	(0.067)	4.17	(0.14)	−0.45	4.25	0.73	9.04	−16.1	31.7
Aero	1.15	(0.044)	0.18	(0.073)	0.31	(0.069)	4.20	(0.14)	0.06	4.44	0.18	4.70	−17.9	17.6
Guns	0.57	(0.097)	0.40	(0.137)	0.12	(0.148)	9.68	(0.29)	0.28	9.85	1.04	8.67	−29.0	75.2
Gold	0.89	(0.052)	−0.10	(0.074)	0.20	(0.080)	4.90	(0.16)	0.31	5.31	0.52	6.84	−18.3	32.4
Ships	1.16	(0.050)	0.10	(0.075)	0.46	(0.086)	4.99	(0.19)	−0.14	5.22	0.33	4.20	−20.4	18.6
Mines	1.10	(0.049)	0.31	(0.066)	0.44	(0.075)	4.59	(0.17)	−0.04	5.11	−0.11	3.84	−19.9	16.9
Coal	1.19	(0.079)	0.31	(0.116)	0.30	(0.120)	7.74	(0.24)	0.27	8.38	0.78	6.08	−28.8	44.2
Oil	0.89	(0.039)	−0.24	(0.053)	0.30	(0.058)	3.67	(0.11)	0.21	3.89	0.25	4.11	−14.5	16.7
Util	0.66	(0.030)	−0.20	(0.041)	0.42	(0.047)	2.81	(0.09)	0.00	2.94	0.11	3.68	−10.0	9.1
Telcm	0.80	(0.030)	−0.19	(0.041)	0.14	(0.045)	2.92	(0.09)	0.03	3.12	0.29	5.16	−10.4	16.4
PerSv	1.06	(0.039)	0.50	(0.060)	0.04	(0.061)	3.92	(0.12)	−0.33	4.74	−0.31	4.72	−22.1	14.3
BusSv	1.12	(0.025)	0.45	(0.036)	−0.43	(0.038)	2.41	(0.07)	0.23	2.57	0.23	5.46	−11.4	11.4
Comps	1.04	(0.039)	0.23	(0.056)	−0.54	(0.057)	3.85	(0.11)	0.24	4.14	0.01	4.37	−15.2	16.2
Chips	1.22	(0.033)	0.44	(0.046)	−0.39	(0.050)	3.14	(0.10)	0.05	3.64	−0.21	5.23	−16.6	13.5
LabEq	1.14	(0.037)	0.49	(0.050)	−0.40	(0.059)	3.46	(0.11)	0.13	3.56	0.13	3.82	−11.2	14.0
Paper	1.05	(0.033)	−0.03	(0.047)	0.40	(0.049)	3.31	(0.10)	−0.10	3.32	0.57	5.21	−10.0	17.0
Boxes	1.02	(0.035)	−0.09	(0.050)	0.01	(0.051)	3.21	(0.10)	0.11	3.72	−0.17	4.75	−15.4	17.6
Trans	1.10	(0.033)	0.22	(0.050)	0.33	(0.051)	3.21	(0.10)	−0.17	3.30	0.29	3.85	−11.2	10.8
Whshl	1.00	(0.025)	0.55	(0.039)	0.03	(0.038)	2.38	(0.08)	0.01	2.79	−0.47	7.82	−14.6	10.8
Rtail	0.98	(0.032)	0.14	(0.048)	0.04	(0.048)	3.09	(0.10)	0.12	3.25	−0.06	3.79	−12.5	11.5
Meals	1.00	(0.039)	0.32	(0.057)	0.15	(0.061)	3.55	(0.11)	0.19	4.05	−0.26	5.38	−20.4	14.9
Banks	1.17	(0.033)	−0.18	(0.047)	0.54	(0.050)	3.26	(0.10)	−0.25	3.36	0.24	4.71	−13.6	13.0
Insur	1.07	(0.035)	−0.16	(0.050)	0.38	(0.053)	3.47	(0.10)	−0.04	3.62	0.74	6.40	−10.6	21.3
RlEst	1.14	(0.046)	0.90	(0.070)	0.73	(0.077)	4.44	(0.18)	−0.83	4.42	1.16	13.22	−15.3	37.9
Fin	1.23	(0.026)	0.13	(0.036)	0.16	(0.040)	2.50	(0.08)	0.00	2.75	0.18	6.65	−15.8	13.5
Other	1.10	(0.042)	0.21	(0.068)	0.00	(0.066)	3.94	(0.13)	−0.37	4.58	−0.21	5.52	−20.8	17.1

The first eight columns of this table report the estimates of the industry-specific part of the model in Equation (1), with the model of Fama and French (1993) as asset pricing model. Based on these estimates, we construct abnormal returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$. We report their means, standard deviation (both in % per month), skewness, kurtosis, minimum and maximum (last two in % per month) in the next six columns. The estimates are based on the returns on the 48 industries of Fama and French (1997). We assume that the innovations $u_{i,t}$, and the regime processes $S_{i,t}$ for the different industries are independent. We estimate the parameters by the EM-algorithm of Dempster et al. (1977). We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$. Standard errors of the estimates are in parentheses.

Table 4: Industry-specific Estimates for the Carhart Model and Abnormal Returns

Industry	Estimates of Equation (1)										Abnormal Returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$					
	market β_i		SMB β_i		HML β_i		MOM β_i		ω_i ($\times 100$)		mean	stdev.	skew.	kurt.	min.	max.
Agric	0.86	(0.05)	0.46	(0.08)	0.08	(0.08)	0.14	(0.05)	4.59	(0.14)	−0.03	5.03	0.34	4.99	−17.4	27.4
Food	0.75	(0.03)	−0.11	(0.04)	0.16	(0.05)	0.01	(0.03)	2.95	(0.10)	0.25	3.14	0.63	6.89	−11.1	19.1
Soda	0.89	(0.06)	−0.15	(0.07)	0.19	(0.08)	−0.10	(0.05)	4.88	(0.17)	0.40	5.29	0.49	6.59	−18.7	31.7
Beer	0.84	(0.04)	−0.09	(0.05)	0.12	(0.06)	0.13	(0.04)	3.74	(0.12)	0.21	4.02	−0.10	4.89	−18.1	15.6
Smoke	0.80	(0.05)	−0.24	(0.07)	0.18	(0.08)	0.03	(0.05)	4.91	(0.15)	0.61	5.36	−0.13	7.30	−31.2	28.8
Toys	1.10	(0.05)	0.51	(0.07)	0.06	(0.07)	−0.12	(0.05)	4.36	(0.13)	−0.17	4.79	−0.18	4.01	−22.7	17.1
Fun	1.32	(0.04)	0.42	(0.06)	0.21	(0.07)	−0.21	(0.04)	4.32	(0.15)	0.28	4.58	0.05	3.67	−18.8	15.4
Books	1.07	(0.03)	0.21	(0.05)	0.18	(0.05)	−0.06	(0.03)	3.11	(0.12)	−0.05	3.33	0.01	4.46	−13.9	14.4
Hshld	0.89	(0.03)	−0.20	(0.05)	−0.05	(0.05)	0.02	(0.04)	2.68	(0.09)	0.14	3.02	−0.51	9.21	−22.4	11.3
Clths	1.05	(0.04)	0.53	(0.06)	0.32	(0.06)	−0.19	(0.04)	3.57	(0.12)	0.04	3.81	0.28	7.25	−18.2	24.1
Health	1.04	(0.06)	0.67	(0.09)	0.07	(0.09)	−0.03	(0.06)	5.61	(0.20)	−0.03	6.52	−0.36	7.28	−34.8	25.9
MedEq	0.84	(0.03)	0.04	(0.05)	−0.27	(0.06)	0.06	(0.04)	3.27	(0.12)	0.33	3.43	−0.10	3.81	−13.3	11.8
Drugs	0.84	(0.03)	−0.31	(0.04)	−0.37	(0.05)	0.07	(0.03)	3.14	(0.10)	0.44	3.31	0.10	5.09	−15.7	13.3
Chems	1.12	(0.03)	−0.07	(0.04)	0.29	(0.04)	−0.09	(0.03)	2.60	(0.08)	−0.03	2.87	0.19	5.09	−9.5	15.3
Rubbr	1.02	(0.03)	0.63	(0.04)	0.31	(0.05)	−0.08	(0.03)	3.04	(0.09)	−0.05	3.26	0.27	4.34	−11.0	13.1
Txtls	1.09	(0.04)	0.62	(0.06)	0.67	(0.06)	−0.33	(0.04)	4.10	(0.12)	−0.14	4.31	0.60	7.94	−17.3	28.7
BldMt	1.21	(0.03)	0.28	(0.04)	0.42	(0.04)	−0.09	(0.03)	2.74	(0.09)	−0.16	3.01	0.08	6.46	−13.3	16.0
Cnstr	1.27	(0.04)	0.51	(0.07)	0.31	(0.07)	0.04	(0.04)	3.95	(0.14)	−0.28	4.21	0.26	3.85	−16.6	15.6
Steel	1.26	(0.04)	0.38	(0.06)	0.36	(0.06)	−0.12	(0.04)	3.96	(0.12)	−0.38	4.33	0.36	5.36	−20.0	21.3
FabPr	0.95	(0.05)	0.19	(0.07)	0.43	(0.08)	0.03	(0.05)	4.89	(0.15)	0.00	5.40	−0.09	6.58	−33.4	22.2
Mach	1.19	(0.03)	0.26	(0.04)	0.10	(0.04)	−0.12	(0.03)	2.63	(0.08)	0.01	2.87	−0.06	3.73	−10.2	8.9
ElcEq	1.22	(0.03)	0.06	(0.05)	0.03	(0.05)	0.02	(0.03)	3.15	(0.10)	0.17	3.23	0.05	3.41	−12.1	11.3
Autos	1.21	(0.05)	0.09	(0.06)	0.53	(0.07)	−0.36	(0.04)	3.77	(0.13)	−0.14	4.06	0.02	5.16	−16.9	20.3
Aero	1.14	(0.04)	0.16	(0.06)	0.27	(0.06)	−0.11	(0.04)	4.15	(0.13)	0.16	4.43	0.26	4.73	−16.8	18.1
Guns	0.60	(0.10)	0.40	(0.14)	0.17	(0.15)	0.21	(0.10)	9.63	(0.29)	0.10	9.81	1.07	8.82	−29.9	75.3
Gold	0.89	(0.06)	−0.15	(0.07)	0.19	(0.08)	−0.10	(0.05)	4.88	(0.17)	0.40	5.29	0.49	6.59	−18.7	31.7
Ships	1.16	(0.05)	0.10	(0.08)	0.45	(0.09)	−0.02	(0.05)	4.97	(0.19)	−0.13	5.22	0.33	4.20	−20.3	18.8
Mines	1.09	(0.05)	0.31	(0.07)	0.44	(0.08)	0.02	(0.05)	4.56	(0.16)	−0.05	5.11	−0.11	3.85	−19.9	16.8
Coal	1.21	(0.08)	0.31	(0.12)	0.33	(0.12)	0.17	(0.08)	7.69	(0.24)	0.13	8.34	0.77	6.04	−29.6	43.9
Oil	0.91	(0.04)	−0.25	(0.05)	0.33	(0.06)	0.12	(0.04)	3.61	(0.12)	0.10	3.85	0.26	3.97	−13.9	15.6
Util	0.66	(0.03)	−0.20	(0.04)	0.41	(0.05)	0.02	(0.03)	2.82	(0.09)	−0.01	2.94	0.13	3.66	−10.0	9.1
Telcm	0.79	(0.03)	−0.19	(0.04)	0.09	(0.04)	−0.09	(0.03)	2.89	(0.09)	0.13	3.10	0.20	4.74	−10.8	15.7
PerSv	1.06	(0.04)	0.50	(0.06)	0.04	(0.06)	0.03	(0.04)	3.93	(0.12)	−0.35	4.73	−0.32	4.74	−22.6	14.1
BusSv	1.12	(0.02)	0.45	(0.04)	−0.44	(0.04)	−0.02	(0.02)	2.40	(0.07)	0.25	2.56	0.24	5.37	−11.0	11.6
Comps	1.02	(0.04)	0.24	(0.05)	−0.58	(0.06)	−0.13	(0.04)	3.82	(0.11)	0.35	4.09	0.03	4.12	−15.3	14.8
Chips	1.20	(0.03)	0.44	(0.04)	−0.44	(0.05)	−0.12	(0.03)	3.11	(0.10)	0.17	3.58	−0.15	4.92	−15.9	11.5
LabEq	1.14	(0.04)	0.49	(0.05)	−0.43	(0.06)	−0.03	(0.04)	3.46	(0.11)	0.16	3.56	0.13	3.83	−11.2	14.4
Paper	1.04	(0.03)	−0.02	(0.05)	0.38	(0.05)	−0.08	(0.03)	3.29	(0.10)	−0.03	3.30	0.56	5.07	−10.4	16.4
Boxes	1.02	(0.04)	−0.10	(0.05)	0.01	(0.05)	−0.02	(0.04)	3.21	(0.10)	0.12	3.72	−0.16	4.76	−15.4	17.8
Trans	1.09	(0.03)	0.22	(0.05)	0.32	(0.05)	−0.04	(0.03)	3.21	(0.10)	−0.13	3.29	0.31	3.77	−11.3	11.0
Whshl	1.00	(0.03)	0.54	(0.04)	0.04	(0.04)	0.01	(0.03)	2.38	(0.08)	0.00	2.79	−0.45	7.82	−14.8	10.9
Rtail	0.98	(0.03)	0.12	(0.04)	0.02	(0.05)	−0.12	(0.03)	3.01	(0.09)	0.21	3.21	−0.09	3.63	−11.8	10.2
Meals	1.00	(0.04)	0.32	(0.06)	0.15	(0.06)	−0.03	(0.04)	3.54	(0.11)	0.22	4.04	−0.24	5.13	−19.7	14.5
Banks	1.15	(0.03)	−0.18	(0.05)	0.49	(0.05)	−0.18	(0.03)	3.20	(0.10)	−0.09	3.29	0.18	4.47	−14.8	12.7
Insur	1.06	(0.03)	−0.15	(0.05)	0.35	(0.05)	−0.10	(0.03)	3.43	(0.10)	0.04	3.62	0.71	6.35	−13.2	21.3
RlEst	1.10	(0.05)	0.90	(0.06)	0.68	(0.07)	−0.21	(0.04)	4.36	(0.16)	−0.64	4.37	0.71	8.09	−16.0	31.3
Fin	1.23	(0.03)	0.13	(0.04)	0.15	(0.04)	−0.05	(0.03)	2.49	(0.09)	0.04	2.74	0.20	6.84	−15.8	13.9
Other	1.09	(0.04)	0.20	(0.06)	−0.02	(0.06)	−0.06	(0.04)	3.93	(0.13)	−0.32	4.57	−0.22	5.44	−20.7	17.2

The first ten columns of this table report the estimates of the industry-specific part of the model in Equation (1), with the model of Carhart (1997) as asset pricing model. Based on these estimates, we construct abnormal returns $r_{i,t} - r_{\mathrm{f},t} - \hat{\boldsymbol{\beta}}_i' \boldsymbol{f}_t$. We report their means, standard deviation (both in % per month), skewness, kurtosis, minimum and maximum (last two in % per month) in the next six columns. The estimates are based on the returns on the 48 industries of Fama and French (1997). We assume that the innovations $u_{i,t}$, and the regime processes $S_{i,t}$ for the different industries are independent. We estimate the parameters by the EM-algorithm of Dempster et al. (1977). We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$. Standard errors of the estimates are in parentheses.

Table 5: Estimation results for the regime-switching models

a: *Estimates for the distribution parameters*

	CAPM		Fama-French Model		Carhart Model	
μ_{N}	0.036	(0.007)	0.013	(0.007)	0.024	(0.007)
μ_{B}	0.841	(0.042)	0.789	(0.048)	0.826	(0.047)
μ_{C}	0.278	(0.050)	0.286	(0.046)	0.281	(0.048)
σ_{C}	0.640	(0.036)	0.638	(0.032)	0.642	(0.034)

b: *Means and volatilities for the different regimes (in % per month)*

	CAPM		Fama-French Model		Carhart Model	
average ω_i	4.06	–	3.86	–	3.82	–
mean in N and $\mathrm{D_N}$	0.15	(0.028)	0.05	(0.027)	0.09	(0.027)
mean in G and B	3.41	(0.17)	3.05	(0.19)	3.16	(0.18)
mean in C and $\mathrm{D_C}$	−10.64	(0.26)	−10.15	(0.24)	−10.04	(0.25)
volatility in C and $\mathrm{D_C}$	4.68	(0.33)	4.46	(0.30)	4.44	(0.31)

c: *Estimates for the transition probabilities*

	CAPM		Fama-French Model		Carhart Model	
$p_{\mathrm{N,N}}$	0.994	(0.0007)	0.995	(0.0006)	0.994	(0.0006)
$p_{\mathrm{N,C}}$	0.0014	(0.0005)	0.0005	(0.0003)	0.0005	(0.0003)
$p_{\mathrm{N,G_1}}$	0.0050	–	0.0049	–	0.0053	–
$p_{\mathrm{C,N}}$	0.465	(0.1008)	0.354	(0.1388)	0.330	(0.1463)
$p_{\mathrm{C,C}}$	0.530	–	0.641	–	0.665	–
$p_{\mathrm{C,G_1}}$	0.0050	–	0.0049	–	0.0053	–
$p_{\mathrm{B,B}}$	0.833	(0.0230)	0.812	(0.0313)	0.797	(0.0332)
$p_{\mathrm{B,D_C}}$	0.167	–	0.188	–	0.203	–
$p_{\mathrm{D_C,N}}$	0.251	(0.0244)	0.224	(0.0226)	0.238	(0.0231)
$p_{\mathrm{D_C,D_C}}$	0.175	(0.0248)	0.191	(0.0248)	0.176	(0.0232)
$p_{\mathrm{D_C,D_N}}$	0.574	–	0.585	–	0.585	–
$p_{\mathrm{D_N,D_C}}$	0.289	(0.0235)	0.279	(0.0217)	0.268	(0.0211)
$p_{\mathrm{D_N,D_N}}$	0.711	–	0.721	–	0.732	–

This table reports the estimates of the regime switching part of the model in Equation (1). The estimates of the distributions of the different regimes in panel A correspond with Equation (2). Panel B shows the implications for the idiosyncratic part of the return for the average industry conditional on each regime. Panel C shows the estimates for the transition parameters in Table 1. The estimates are based on the returns on the 48 industries of Fama and French (1997). We assume that the innovations $u_{i,t}$, and the regime processes $S_{i,t}$ for the different industries are independent. We estimate the parameters by the EM-algorithm of Dempster et al. (1977). We consider the CAPM, the 3-Factor Model of Fama and French (1993) and the 4-Factor Model of Carhart (1997) as asset pricing models. We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$.

Table 6: Ergodic Probabilities of Different Regimes

Regime	CAPM	Fama-French Model	Carhart Model
N	0.891	0.890	0.885
C	0.0026	0.0013	0.0012
G_1	0.0045	0.0043	0.0047
G_2	0.0045	0.0043	0.0047
G_3	0.0045	0.0043	0.0047
G_4	0.0045	0.0043	0.0047
G_5	0.0045	0.0043	0.0047
G_6	0.0045	0.0043	0.0047
B	0.0267	0.0231	0.0231
D_C	0.0178	0.0194	0.0197
D_N	0.0354	0.0405	0.0430

This table presents the ergodic probabilities of the different regimes: normal (N), crash (C), good news ($G_l, l = 1, 2, \ldots, L$), bubble (B), deflation crash (D_C) and deflation normal (D_N). The probabilities shown in column 1 correspond with the abnormal returns based on the CAPM, column 2 and column 3 correspond with the Fama and French (1993)-Model and Carhart (1997)-Model. We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$.

Table 7: Identification of regimes per industry

	CAPM						Fama-French Model						Carhart Model					
industry	N	C	G	B	D_C	D_N	N	C	G	B	D_C	D_N	N	C	G	B	D_C	D_N
Agric	86.7	0.1	2.7	3.5	2.5	4.4	85.9	0.0	2.7	2.9	2.7	5.8	85.6	0.0	2.6	2.8	2.7	6.2
Food	91.6	0.8	2.9	3.2	0.7	0.8	89.9	0.8	3.2	3.2	1.1	1.8	90.3	0.8	3.1	2.9	1.1	1.8
Soda	88.7	0.5	2.5	2.2	2.1	4.0	88.4	0.4	2.4	1.9	2.2	4.7	87.8	0.3	2.9	2.0	2.2	4.9
Beer	91.0	0.2	1.9	1.0	1.7	4.2	90.2	0.1	2.3	1.2	1.9	4.2	91.7	0.2	1.5	0.7	1.7	4.2
Smoke	88.8	0.0	2.4	3.4	1.8	3.5	88.8	0.0	2.5	3.1	1.8	3.8	88.7	0.0	2.5	2.7	1.9	4.2
Toys	88.0	0.1	3.1	2.1	2.4	4.3	88.5	0.0	2.8	2.0	2.6	4.2	88.4	0.0	3.1	1.9	2.5	4.2
Fun	93.1	0.3	2.3	2.2	0.7	1.3	93.4	0.2	2.3	1.8	0.8	1.5	90.4	0.1	3.1	2.3	1.3	3.0
Books	89.0	0.2	3.7	4.5	1.0	1.6	88.1	0.1	3.8	3.5	1.4	3.1	87.3	0.0	4.3	3.5	1.4	3.5
Hshld	88.4	0.4	2.9	2.7	1.7	3.8	87.6	0.1	3.4	3.6	1.8	3.5	87.7	0.1	3.4	3.5	1.8	3.5
Clths	92.0	1.0	2.3	2.1	0.9	1.7	92.3	0.2	2.1	1.1	1.6	2.7	92.5	0.1	2.3	1.2	1.4	2.5
Health	85.2	0.6	3.5	3.3	2.4	5.1	84.7	0.2	2.9	2.1	2.5	7.6	84.1	0.2	2.9	2.0	2.6	8.1
MedEq	90.8	0.1	2.6	3.6	1.0	2.0	93.1	0.1	2.6	2.4	0.8	1.0	92.4	0.1	2.9	2.4	1.0	1.2
Drugs	78.4	0.1	4.9	4.4	3.2	9.0	92.5	0.0	2.6	1.8	0.9	2.1	92.3	0.0	2.6	1.6	1.1	2.4
Chems	85.9	0.0	3.1	2.5	3.1	5.4	83.9	0.0	3.5	2.7	3.3	6.5	82.5	0.0	3.8	3.0	3.5	7.2
Rubbr	88.4	0.2	3.3	3.8	1.5	2.9	88.7	0.0	2.3	1.5	2.5	4.9	88.5	0.1	2.7	1.7	2.4	4.6
Txtls	91.4	0.1	2.3	2.6	1.4	2.1	89.4	0.0	2.1	0.9	2.6	5.0	93.8	0.0	1.5	1.0	1.4	2.3
BldMt	89.7	0.0	3.1	1.2	2.5	3.5	90.4	0.0	2.2	0.8	2.2	4.3	89.1	0.0	2.5	0.9	2.2	5.2
Cnstr	91.3	0.1	2.8	3.7	1.0	1.1	89.4	0.0	3.1	2.5	1.7	3.4	89.3	0.0	3.1	2.1	1.8	3.7
Steel	90.8	0.4	2.5	2.8	1.2	2.3	86.3	0.2	3.5	4.3	2.1	3.6	85.2	0.1	3.8	4.7	2.2	3.9
FabPr	89.3	0.2	2.6	4.0	1.7	2.2	88.7	0.0	2.8	3.4	2.0	3.0	88.7	0.0	2.9	3.0	2.1	3.2
Mach	88.5	0.1	3.5	2.0	2.2	3.8	86.5	0.0	3.7	2.4	2.6	4.7	84.7	0.0	4.7	3.0	2.5	5.1
ElcEq	96.3	0.1	1.3	0.6	0.6	1.1	95.3	0.0	1.6	0.8	0.8	1.6	95.0	0.0	1.7	0.7	0.8	1.7
Autos	95.6	0.4	1.8	0.9	0.5	0.7	96.3	0.2	1.6	0.5	0.5	0.9	89.1	0.1	3.7	1.5	1.7	4.0
Aero	90.9	0.1	3.5	2.1	1.3	2.2	91.3	0.0	3.1	1.7	1.2	2.6	89.8	0.0	3.9	2.0	1.5	2.8
Guns	97.0	0.0	1.5	0.6	0.3	0.5	97.0	0.0	1.5	0.6	0.3	0.5	97.1	0.0	1.5	0.5	0.3	0.6
Gold	88.7	0.5	2.5	2.2	2.1	4.0	88.4	0.4	2.4	1.9	2.2	4.7	87.8	0.3	2.9	2.0	2.2	4.9
Ships	88.8	0.6	2.8	3.2	1.5	3.1	91.2	0.0	2.6	1.8	1.2	3.2	90.1	0.0	2.8	1.8	1.4	3.8
Mines	89.7	0.1	2.9	3.0	1.3	3.0	81.0	0.1	4.3	4.1	2.9	7.6	80.2	0.1	4.4	3.8	3.1	8.3
Coal	88.5	0.1	4.1	4.1	1.5	1.7	89.1	0.0	3.9	3.6	1.6	1.7	89.5	0.0	3.9	3.0	1.6	1.9
Oil	87.4	0.1	3.6	3.9	1.8	3.3	89.3	0.1	3.0	3.7	1.5	2.4	88.9	0.1	3.0	3.2	1.7	2.9
Util	89.6	0.0	2.4	1.5	1.9	4.6	92.0	0.0	2.1	1.4	1.3	3.2	92.4	0.0	2.0	1.2	1.3	3.1
Telcm	88.2	0.1	1.6	2.8	2.2	5.1	89.1	0.0	1.5	2.1	2.3	5.0	88.4	0.0	1.8	2.4	2.3	5.1
PerSv	69.5	0.1	4.6	6.5	6.1	13.2	73.4	1.2	3.7	4.6	4.9	12.2	73.6	1.2	3.7	4.1	4.9	12.5
BusSv	91.5	0.0	1.5	1.1	1.8	4.1	90.6	0.0	1.2	1.4	1.5	5.3	90.7	0.0	1.2	1.4	1.4	5.3
Comps	92.1	0.1	1.7	2.1	1.7	2.4	90.1	0.0	2.5	2.9	1.8	2.7	90.0	0.0	2.8	2.7	1.8	2.7
Chips	86.3	0.2	2.5	3.3	2.6	5.1	80.9	0.0	4.4	5.8	3.3	5.5	79.9	0.0	4.8	6.1	3.3	5.9
LabEq	93.0	0.2	2.2	1.6	1.0	1.9	94.5	0.1	1.8	1.7	0.8	1.2	94.5	0.1	1.9	1.7	0.7	1.1
Paper	97.1	0.1	1.0	0.5	0.4	0.8	99.4	0.0	0.3	0.2	0.1	0.1	99.1	0.0	0.4	0.2	0.1	0.1
Boxes	76.0	0.1	3.7	4.4	4.4	11.4	75.6	0.1	3.7	4.3	4.4	11.9	75.4	0.1	3.6	4.1	4.4	12.3
Trans	95.5	0.7	1.6	1.2	0.4	0.5	96.3	0.4	1.2	0.7	0.7	0.8	96.4	0.4	1.2	0.8	0.6	0.7
Whshl	85.1	0.2	2.9	3.7	2.1	6.0	81.9	0.0	2.6	2.2	3.2	10.0	81.9	0.0	2.6	2.1	3.2	10.2
Rtail	90.6	0.1	2.5	2.7	1.3	2.8	92.0	0.0	2.1	2.2	1.2	2.5	88.8	0.0	3.2	3.2	1.6	3.1
Meals	78.8	0.2	4.4	6.4	3.2	7.0	80.5	0.0	3.9	5.5	3.3	6.8	79.8	0.0	4.3	5.2	3.4	7.2
Banks	94.2	0.9	1.8	0.7	1.0	1.5	96.4	0.2	1.0	0.3	0.9	1.3	97.3	0.2	1.0	0.4	0.5	0.7
Insur	94.5	0.3	2.4	1.4	0.7	0.7	93.9	0.0	2.6	1.5	0.8	1.1	93.4	0.0	2.8	1.6	0.9	1.2
RlEst	97.2	0.4	0.7	1.4	0.1	0.1	97.5	0.1	0.5	0.3	0.4	1.3	96.8	0.0	0.7	0.4	0.4	1.6
Fin	87.0	0.2	3.6	3.2	2.0	4.1	86.6	0.1	3.9	3.0	2.2	4.3	86.8	0.1	3.9	2.8	2.1	4.3
Other	82.5	0.1	2.5	2.5	4.2	8.2	80.3	0.0	2.9	2.4	4.5	9.8	79.8	0.0	3.1	2.4	4.6	10.1
Average	89.1	0.2	2.7	2.7	1.8	3.5	89.1	0.1	2.6	2.3	1.9	4.0	88.6	0.1	2.8	2.3	1.9	4.2
Min	69.5	0.0	0.7	0.5	0.1	0.1	73.4	0.0	0.3	0.2	0.1	0.1	73.6	0.0	0.4	0.2	0.1	0.1
Max	97.2	1.0	4.9	6.5	6.1	13.2	99.4	1.2	4.4	5.8	4.9	12.2	99.1	1.2	4.8	6.1	4.9	12.5

This table shows the percentage that each industry spends in the different regimes: normal (N), crash (C), good news (G), bubble (B), deflation crash (D_C) and deflation normal (D_N). We calculate this percentage as the sum of the smoothed inference probabilities over time divided by the number of observations. We aggregate the percentages for the L good news regimes. We show the identification for each risk factor model: the CAPM, the 3-Factor Model of Fama and French (1993) and the 4-Factor Model of Carhart (1997). We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$.

Table 8: Optimal weights for strategy with different rebalancing frequencies

(a) Process starts in the bubble regime with certainty, $\Pr[S_t = \text{B}] = 1$

Strategy / Horizon	1	2	3	4	6	12	24	60
Buy-and-hold	21.2	13.7	7.5	2.8	−3.8	−9.3	−12.2	−14.6
$h = 6$	–	–	–	–	0.0	−3.3	−3.5	0.0
$h = 4$	–	–	–	4.0	–	6.5	6.1	5.7
$h = 3$	–	–	7.5	–	10.6	11.1	10.2	8.9
$h = 2$	–	13.7		16.5	17.0	16.7	15.8	14.7
$h = 1$	21.2	26.0	27.1	27.1	27.6	25.5	23.0	22.8

(b) Process starts in the normal regime with certainty, $\Pr[S_t = \text{N}] = 1$

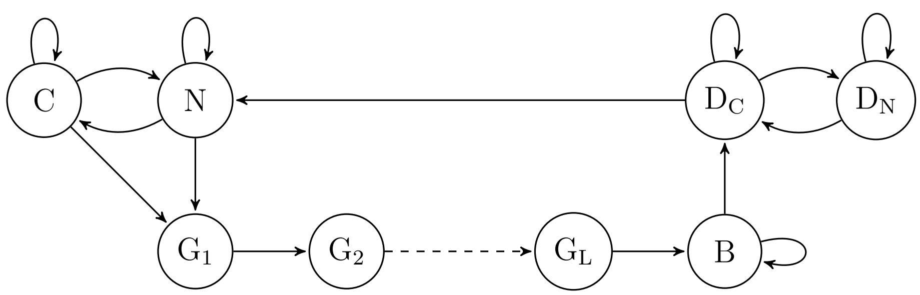
Strategy / Horizon	1	2	3	4	6	12	24	60
Buy-and-hold	8.0	8.8	9.6	10.4	12.0	13.9	14.5	11.6
$h = 6$	–	–	–	–	11.1	11.1	11.1	11.0
$h = 4$	–	–	–	10.8	–	10.8	10.8	10.7
$h = 3$	–	–	9.6	–	8.9	8.9	8.9	8.8
$h = 2$	–	8.8		8.7	8.7	8.7	8.7	8.7
$h = 1$	8.0	6.4	6.4	6.4	6.4	6.4	6.4	6.5

(c) Process starts in the normal or bubble regime with equal probability, $\Pr[S_t = \text{B}] = \Pr[S_t = \text{N}] = 1/2$

Strategy / Horizon	1	2	3	4	6	12	24	60
Buy-and-hold	21.2	13.7	7.5	2.8	−3.8	−9.3	−12.2	−14.6
$h = 6$	–	–	–	–	0.0	−3.3	−3.5	0.0
$h = 4$	–	–	–	4.0	–	6.5	6.1	5.7
$h = 3$	–	–	7.5	–	10.6	11.1	10.2	8.9
$h = 2$	–	13.7		16.5	17.0	16.7	15.8	14.7
$h = 1$	21.2	26.0	27.1	27.1	27.6	25.5	23.0	22.8

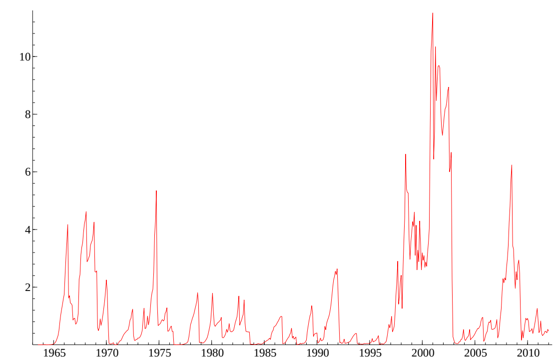
This table shows the optimal allocation to the industry asset for month $t + 1$ (in %) for different investment horizons (in months), different rebalancing frequencies and different starting situations. We report portfolio weights for a buy-and-hold strategy, and strategies with rebalacing every 1, 2, 3, 4 and 6 months. The optimal portfolios are found by numerically solving Equation (22), as described in Appendix A. We construct 10,000 draws per regime, and construct a grid for the inference probabilities with a precision threshold of 0.45. The scale factor is equal to the industry average. The risk-free rate is constant at 0.30% per month. The risk-aversion coefficient is equal to five. The results are based on the Fama-French model.

Figure 1: Graph of the transitions in the bubble model

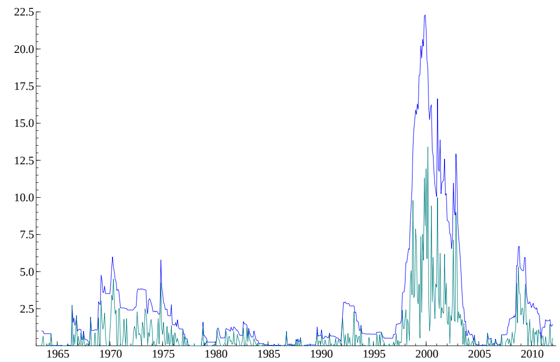


This graph shows the structure of the transition in our model. Each node (drawn as a circle) indicates a different regimes: normal (N), crash (C), good news ($G_l, l = 1, 2, \ldots, L$), bubble (B), deflation crash (D_C) and deflation normal (D_N). The arcs between the nodes indicate possible transitions.

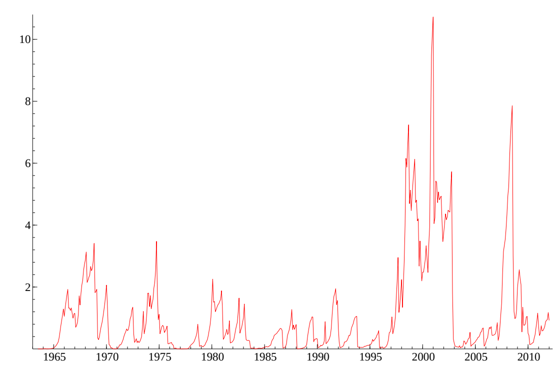
Figure 2: Identification of Regimes over Time



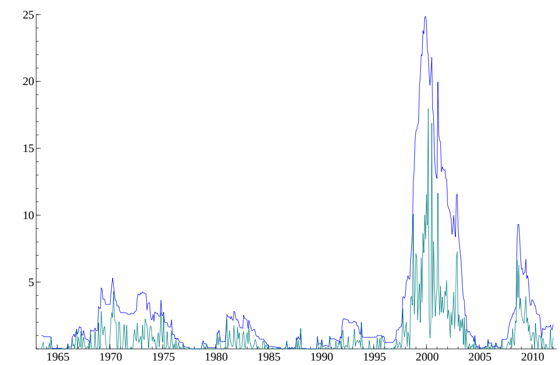
(a) CAPM, bubble regime



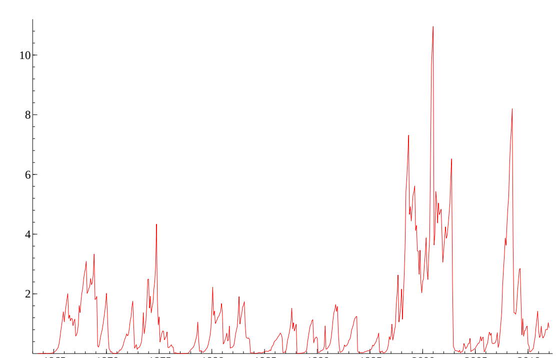
(b) CAPM, deflation regimes



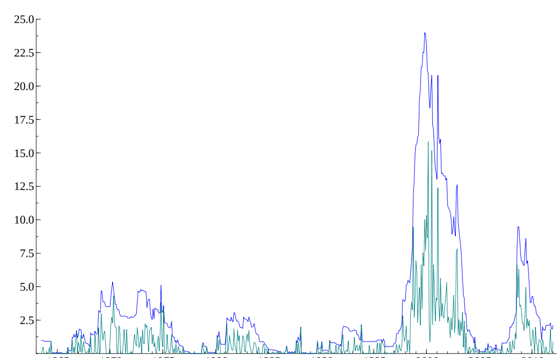
(c) Fama-French Model, bubble regime



(d) Fama-French Model, deflation regimes



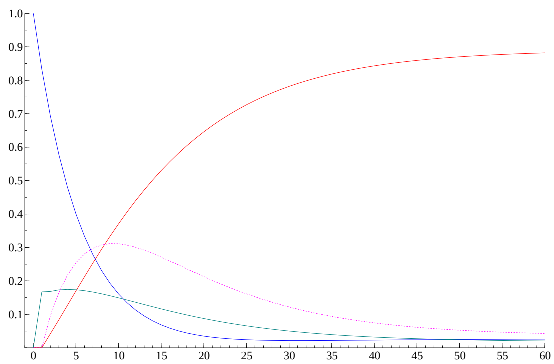
(e) Carhart Model, bubble regime



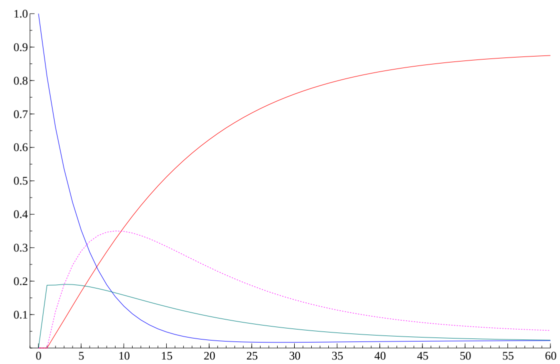
(f) Carhart Model, deflation regimes

This figure shows the identification of the different regimes over time. The red lines in the left panels correspond with the bubble regime, the green lines in the right panels with the deflation crash regime, and the blue lines in the right panels with the deflation crash and normal regimes together. To calculate the values for a regime we sum the smoothed inference probabilities at each point in time over the 48 industries. We show the identification for each risk factor model: (a + b): the CAPM, (c+d) the 3-Factor Model of Fama and French (1993) and (e+f) the 4-Factor Model of Carhart (1997). We put the period of good news equal to $L = 6$ months, and the upper bound for crashes at $k = -1$.

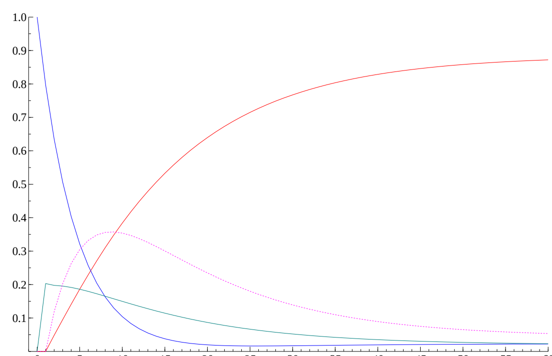
Figure 3: Forecasts of Regime Probabilities from the bubble regime



(a) CAPM



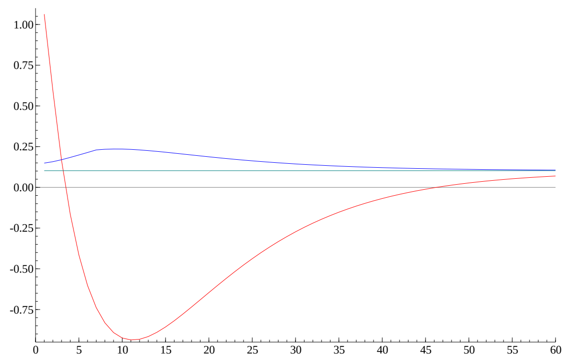
(b) Fama-French Model



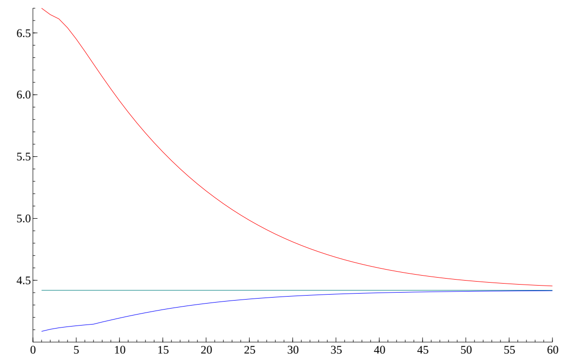
(c) Carhart Model

This figure shows the forecasts of the probabilities with which a regime prevails for horizons from 1 to 60 months. All predictions start from the bubble regime, i.e. $\Pr[S_t = \mathrm{B}] = 1$. The blue line corresponds with the bubble regime, the green line with the deflation crash regime, the purple line with the deflation normal regime, and the red line with the normal regime. The subfigures correspond with the different risk factor models: the CAPM, the 3-Factor model of Fama and French (1993) and the 4-Factor model of Carhart (1997). The transition matrix is given in Table 1 and the estimates in Panel C of Table 5.

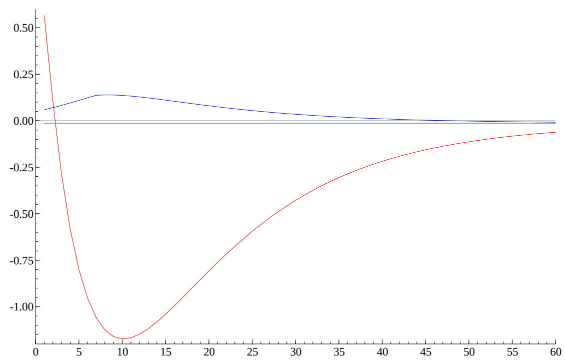
Figure 4: Abnormal Returns Forecasts



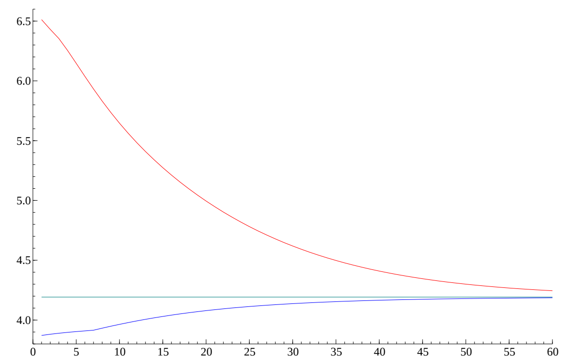
(a) CAPM, expected returns



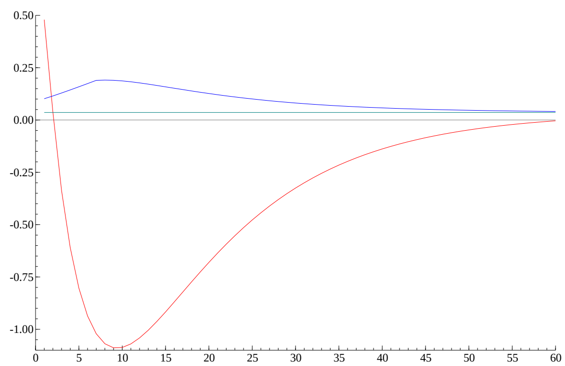
(b) CAPM, volatilities



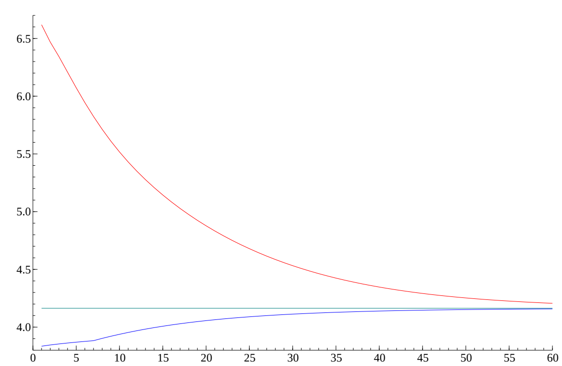
(c) Fama-French model, expected returns



(d) Fama-French model, volatilities



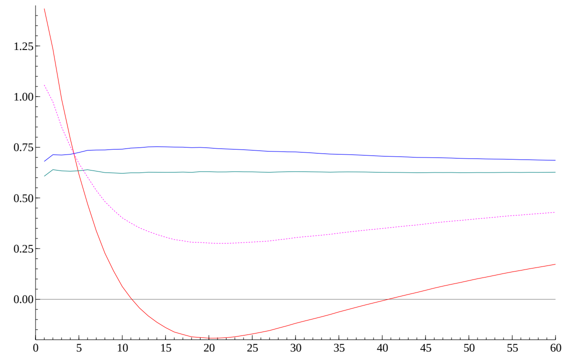
(e) Carhart model, expected returns



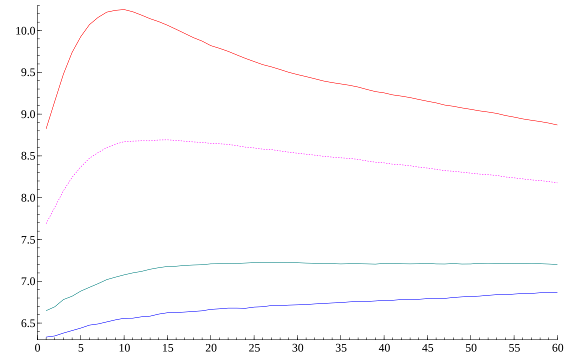
(f) Carhart model, volatilities

This figure shows forecasts of the expected abnormal returns (left panels) and their volatilities (right panels) (both in %) as a function of the forecast horizon (in months). The forecasts are based on estimates for the regime switching models in Table 5 and multiplied by the average scale factor. The subfigures correspond with the different risk factor models: (a,b): the CAPM, (c,d) the 3-Factor model of Fama and French (1993) and (e,f) the 4-Factor model of Carhart (1997). The red lines show the forecasts when the regime process starts with certainty in the bubble regime ($\Pr[S_t = \mathrm{B}] = 1$); the blue line when it starts with certainty in the normal regime ($\Pr[S_t = \mathrm{N}] = 1$), and the green line with the probability distribution at time t equal to the ergodic probabilities.

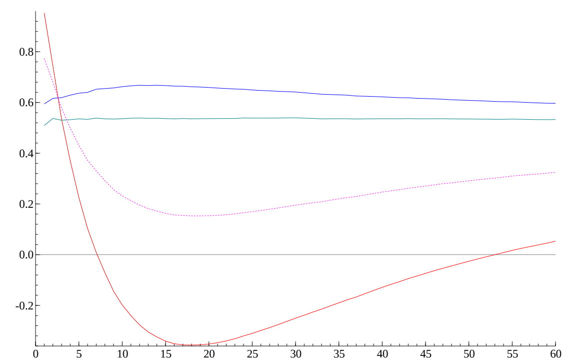
Figure 5: Cumulative Return Forecasts



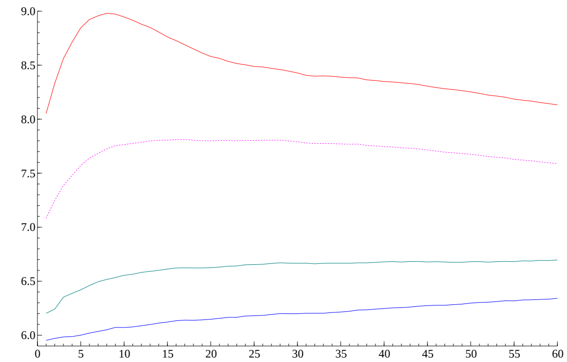
(a) CAPM, expected returns



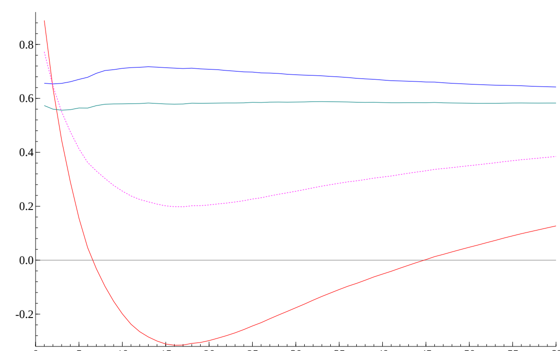
(b) CAPM, volatilities



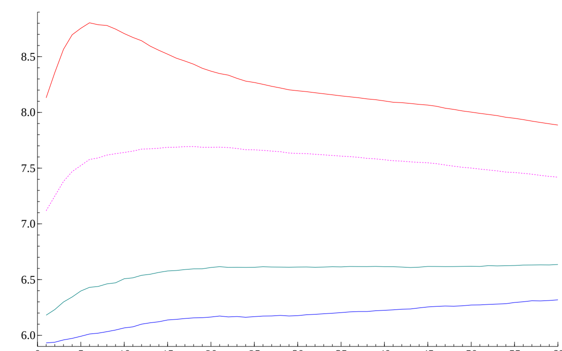
(c) Fama-French model, expected returns



(d) Fama-French model, volatilities



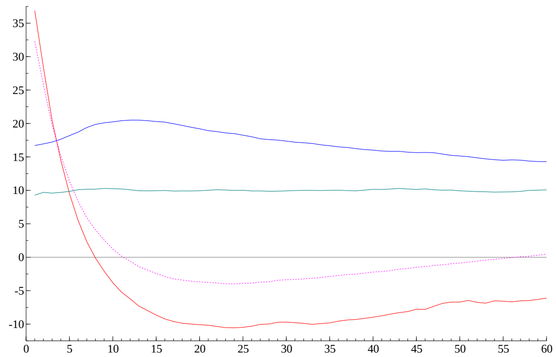
(e) Carhart model, expected returns



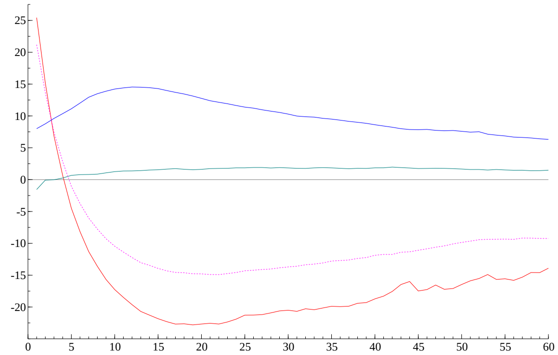
(f) Carhart model, volatilities

This figure shows the expectations (left panels) and volatilities (right panels) of the cumulative returns (in % per month) as a function of the forecast horizon (in months). The red lines show the forecasts when the regime process starts with certainty in the bubble regime ($\Pr[S_t = \text{B}] = 1$); the blue line when it starts with certainty in the normal regime ($\Pr[S_t = \text{N}] = 1$), the purple line when the regime process starts with equal probability in the normal or bubble regime ($\Pr[S_t = \text{B}] = \Pr[S_t = \text{N}] = 1/2$), and the green line when the probability distribution at time t are equal to the ergodic probabilities. The returns are constructed based on 100,000 simulations for one up to sixty consecutive returns, consisting of bootstraps for the excess market return and Monte Carlo draws for the regime process and the distribution of the innovations. The scale factor is equal to the industry average. The risk-free rate is constant at 0.30% per month. For each simulation, we calculate the portfolio value by compounding, starting with an investment of 1. We calculate the average and the variance of the logarithmic final return over the different simulations, and divide them by the holding period to arrive at monthly expected returns and volatilities.

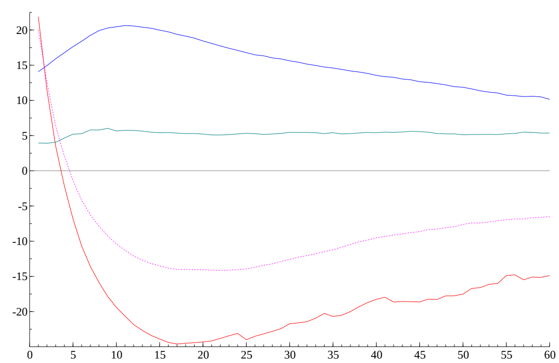
Figure 6: Weights for a buy and hold strategy



(a) CAPM



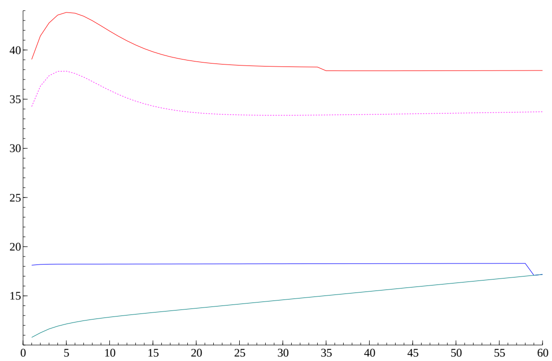
(b) Fama-French Model



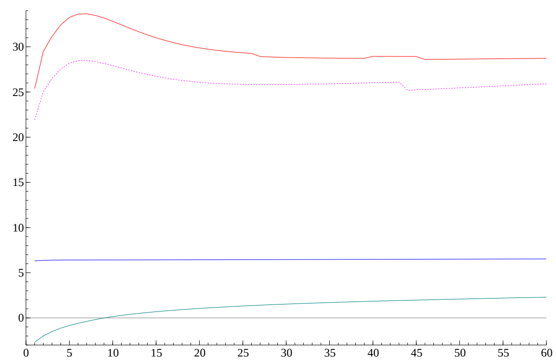
(c) Carhart Model

This figure shows the optimal buy-and-hold allocation to the industry asset (in %) as a function of the investment horizon (in months). The red lines show the portfolio when the regime process for the industry starts with certainty in the bubble regime ($\Pr[S_t = \text{B}] = 1$); the blue line when it starts with certainty in the normal regime ($\Pr[S_t = \text{N}] = 1$), the purple line when it starts with equal probability in the normal or bubble regime ($\Pr[S_t = \text{B}] = \Pr[S_t = \text{N}] = 1/2$), and the green line when the probability distribution at time t is equal to the ergodic probabilities. The optimal portfolios are found by numerically solving Equation (21). We approximate the expectations by constructing the compounded returns from 100,000 simulations for one up to sixty consecutive monthly returns, consisting of bootstraps for the excess market return and Monte Carlo draws for the regime process and the distribution of the innovations. The scale factor is equal to the industry average. The risk-free rate is constant at 0.30% per month. The risk-aversion coefficient is equal to five.

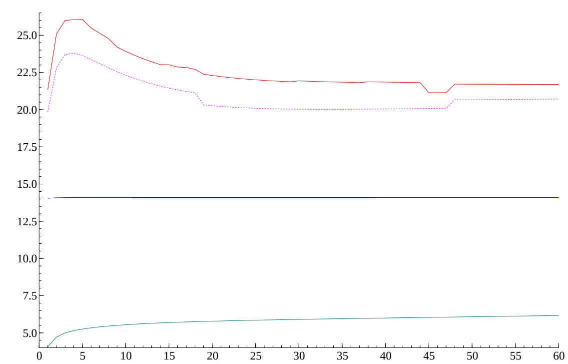
Figure 7: Weights for a strategy with monthly rebalancing



(a) CAPM



(b) Fama-French Model



(c) Carhart Model

This figure shows the optimal allocation to the industry asset for month $t+1$ (in %) as a function of the investment horizon (in months). The investor can rebalance his position every month. The red lines show the portfolio when the regime process for the industry starts with certainty in the bubble regime ($\Pr[S_t = \mathrm{B}] = 1$); the blue line when it starts with certainty in the normal regime ($\Pr[S_t = \mathrm{N}] = 1$), the purple line when it starts with equal probability in the normal or bubble regime ($\Pr[S_t = \mathrm{B}] = \Pr[S_t = \mathrm{N}] = 1/2$), and the green line when the probability distribution at time t is equal to the ergodic probabilities. The optimal portfolios are found by numerically solving Equation (22), as described in Appendix A. We construct 10,000 draws per regime, and construct a grid for the inference probabilities with a precision threshold of 0.45. The scale factor is equal to the industry average. The risk-free rate is constant at 0.30% per month. The risk-aversion coefficient is equal to five.